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Optimization of a multi-response problem in Taguchi's dynamic system☆

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Abstract

Taguchi method was known as an off-line quality control methodology to be used in many industries. Until now, most applications only focus on optimizing a single-response in a static system. Furthermore, due to the increasing complexity of the product design, more than one quality characteristic must be considered simultaneously to improve the production quality. Therefore, there are several studies to address the multi-response problem. In order to satisfy the requirements of the production's design, optimization of a dynamic system been mentioned by Taguchi has received more attentions in the recent years. Hence, optimizing a multi-response problem in a dynamic system becomes an important issue to address the quality improvement.

This study proposes a procedure utilizing the statistic regression analysis and desirability function to optimize the multi-response problem with Taguchi's dynamic system consideration. Firstly, the regression analysis is employed to screen out the control factors significantly affecting the quality variation, and the adjustment factors significantly affecting the sensitivity of a Taguchi's dynamic system. Then, the desirability function will be applied to optimize such a multi-response problem. Finally, the effectiveness of the proposed procedure will be demonstrated by an example of a biological reduction of ethyl acetoacetate process experiment project at the Union Chemical Laboratories of the Industrial Technology Research Institute in Taiwan.

Keywords: Taguchi method; Parameter design; Dynamic system; Multi-response problem; Regression analysis; Desirability function

1. Introduction

In 1960, Dr Taguchi considered that product's quality might lead to the society's loss if the quality cannot achieve the customer's ideal target after the products leaving from the factory to the society. The philosophy of the Taguchi method is not only for the quality being expected to achieve the customer's specification, but the quality's variation must be also taken into consideration (Fowlkes and Creveling, 1995; Peace, 1993; Phadke, 1989). Most related investigations or applications primarily focus on a multiple responses in a static system for manufactured products or processes (Antony, 2000; Derringer & Suich, 1980; Elsayed & Chen, 1993; Hsieh & Tong, 2001; Tong & Hsieh, 2000; Tong & Wang, 2002; Wurl & Albin, 1999). However, many manufactured products have diversified and they

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may cause more than one response and the dynamic characteristics to be simultaneously considered (Chen, 1994; Hong, 1996; Wang and Tong). For instance, semiconductor manufacturing and chemical process must frequently deal with the problem of multi-responses problem with dynamic characteristics consideration. However, the optimization of a multi-response in a dynamic system has seldom been mentioned until now.

The relationship of several responses and the unequal importance may exist in a real case. Optimize separately a particular response may lead to serious conflict of the parameter's settings for a multiple responses problem. Generally, there are two possible shortcomings for optimizing a multi-response problem via using the single-response analysis approach: (1) There is no suitable direction to determine the control factor's settings when conflict incurred; (2) Combining all responses by using the weighted summation may lead to an incorrect judgment. Moreover, the related researches for a multi-response in a dynamic system were mentioned by Hong (Hong, 1996) and Chen (Chen, 1994), there are several shortcomings: (1) the relationship of the quality response and the control factors will be considered when constructing the regression model of each response. It can not completely describe the whole system; (2) The optimum parameter's settings must be determined by performing the confirmation experiment; if there are less control factors which significantly affecting the signal-to-noise ratio (*S/N* ratio), many confirmation experiments are needed. Wang and Tong (Wang and Tong) developed a procedure of optimizing dynamic multiple responses using PCA and multiple criteria evaluation of grey relation model to determine the optimal factor level combination. The multiple criteria evaluation of grey relation model simultaneously considers the ideal solution and negative ideal solution to determine the optimal factor level combination. Consequently, the optimal factor/level combination which the nearest to the ideal solution and the farthest from the negative ideal solution can be explicitly explored. However, the number of response will be determined by the explanation capability of the principle components. If the explanation capability is lower or it cannot be accepted by practitioners, the number of response still can not reduce. From above shortcomings mentioned, an effective optimization for the multi-response problem in a dynamic system must be developed.

This article is organized as follows. The literature review is made in Section 2. In Section 2, we describe clearly the Taguchi's dynamic system, the omega transformation and the desirability function. The proposed approach is represented in Section 3. An illustration example is employed to demonstrate the rationality of the proposed approach in Section 4. The concluding remarks will be made in Section 5.

2. Literature review

2.1. Taguchi method

The Taguchi method, in combining the experimental design techniques with quality loss consideration, is conventionally used for off-line quality control. Three sequential stages will be included for applying Taguchi method into optimizing a product or process: (1) system design, (2) parameter design, and (3) tolerance design. Further details can be found in Peace (Peace, 1993), Fowlkes and Creveling (Fowlkes and Creveling, 1995), and Phadke (Phadke, 1989). Taguchi suggests that we can use the quadratic loss function to measure the loss for the departure of the target. The optimum parameter condition, which makes the product to be more 'robust' for the environment factors and to be more close to the target, is then determined by performing the parameter design. Parameter design is also commonly referred to as 'robust design'. Taguchi's parameter design can be divided into two classes for system's architecture: static and dynamic characteristics. These two classes differ primarily in that the latter employs the signal factor and the former does not. In Taguchi's dynamic method, there are three criterions of the performance: (1) Sensitivity; (2) Linearity and (3) Variability. To measure these three criterions, Taguchi suggest two indexes (Fowlkes and Creveling, 1995; Peace, 1993; Phadke, 1989) to determine the optimum parameter condition and, the two indexes can be defined as the *S/N* ratio (Hong, 1996) and sensitivity (*S*):

$$S/N = 10 \log \frac{\beta^2}{\sigma^2} \tag{2.1}$$

$$S = 10 \log \beta^2 \tag{2.2}$$

where the *S/N* ratio indicates the variability of system and *S* indicates the system's sensitivity. The features of these two indexes are the larger the best for maximizing the system's sensitivity and minimizing the system's variability.

The philosophy of the *S/N* is to consider the corresponding relation between the system's variability when simultaneously adjusting the system's sensitivity and, the philosophy of *S* just only consider the sensitivity.

2.2. Dynamic system

The relationship of the response and the signal factor is generally viewed as linear relationship in Taguchi method, i.e.

$$Y = \beta M + \varepsilon \tag{2.3}$$

where the β represents the system's sensitivity and the ε represents the error term. Wassermam (Wassermam, 1996) considered that the influence of the system's sensitivity and the error term for the different control factor/level combination and, rewrote the term (2.3) to as

$$Y = \beta(\mathbf{d})M + \varepsilon(\mathbf{d}) \tag{2.4}$$

where **d** represents the control factor/level combination, $\beta(\mathbf{d})$ represents the system's sensitivity under $\boldsymbol{d}$ and $\varepsilon(\mathbf{d})$ represents the random error term under **d**.

The $\beta(\mathbf{d})$ in term (2.4) can be replaced with the regression model which is fitted by employing the control factors to be the regressors, the term is written as:

$$\beta(d) = \beta_0 + \sum_h f_h(x_h) \tag{2.5}$$

where the β_0 indicates the intercept, $f_h(x_h)$ represents the partial regression model of the corresponding control factor x_h. There are three forms of the $f_h(x_h)$ according to the control factor's level:

1. When the factor has 2 levels, then

$$f_h(x_h) = \beta_h x_h \tag{2.6}$$

where β_h represents the regression's coefficient.

2. When the factor has 3 levels, then

$$f_h(x_h) = \beta_h x_h + \beta_{hh} x_h^2 \tag{2.7}$$

where β_h and β_{hh} represent the regression's coefficient, x_h represents the linear effect and x_h^2 represents the quadric effect of factor x_h.

3. When the factor's level excess three levels, then

$$f_h(x_h) = \sum_{i=1}^{a} \beta_{h,i} x_{h,i} \tag{2.8}$$

where $\beta_{h,i}$ denotes the coefficient of regression model; $x_{h,i}$ denotes the *i*-th level for *h*-th control factor. Herein, the $x_{h,i}$ equals to 1 if the control factor with i-th level setting, otherwise the $x_{h,i}$ equals to 0. We can use three independent variables to denote three level of control factor. That is, we can use $(a-1)$ independent variables to represent (a) levels of the control factor.

If the interaction effect of the control factors x_u and x_v are considered, the β(**d**) of the term (2.5) can be added the following term:

$$f_{u,v}(x_u, x_v) = \beta_{uv} x_u x_v \tag{2.9}$$

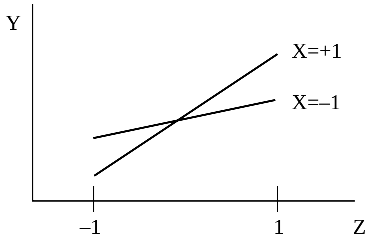

Fig. 1. The interaction effect's diagram of the control factor (X) and the noise factors (Z).

the noise factor's levels are also considered in the error term, then it can be written as following:

$$\varepsilon(d) = \sum_h g_h(z_h) + \varepsilon \tag{2.10}$$

where $g_h(z_h)$ represents the random error of noise factor (z_h). There are four forms of the $g_h(z_h)$ like as the term (2.6) to term (2.9) and, the difference is that the regression's coefficients $\Gamma_h(\mathbf{d})$ of the $g_h(z_h)$ are also a function of control factors.

Term (2.4) can be viewed as a regression perspective for a dynamic system. The interaction effect of the control factors and signal factors will affect the scale of the system's sensitivity. The interaction effect of the control factors and the noise factors can lead to the capability of reducing the system's variability. Fig. 1 can explain the viewpoint.

From the Fig. 1, when the control factor X is at the level -1, the fluctuation of the noise factors Z will lead to smaller variability of the response Y. Therefore, the control factors having the interaction effect with the noise factors are obtained, the corresponding level setting will reduce the response's variability.

2.3. Desirability function and omega transformation

2.3.1. Desirability function

The desirability function is a useful tool to analyze a multi-response problem (Derringer and Suich, 1980). Therefore, the desirability function is employed in this study. The desirability function is primarily proposed by Harrington (Harrington, 1965) and is modified to be more flexible in practical application by Derringer and Suich (Derringer and Suich, 1980). The value of the desirability function, which represents the degree of achieving the target, lies in the interval [0,1] and it can be viewed as the transformation value of the predictor $\hat{y}$ of the observation. There are three forms of the desirability function according to the response's characteristic:

1. The-nominal-the best (*NTB*): the $\hat{y}$ is required to achieve a particular target T. When the $\hat{y}$ equals to T, the desirability value equals to 1; if the departure of $\hat{y}$ excesses a particular range from the target, the desirability value equals to 0 and, such situation represents the worst case. The desirability function of the-nominal-the-best can be written as the term (2.11):

$$\mathrm{d} = \begin{cases} \left(\dfrac{\hat{y}-y_{\min}}{T-y_{\min}}\right)^s, y_{\min} \le \hat{y} \le T, & s \ge 0 \\ \left(\dfrac{\hat{y}-y_{\max}}{T-y_{\max}}\right)^t, T \le \hat{y} \le y_{\max}, & t \ge 0 \\ 0, & \text{otherwise} \end{cases} \tag{2.11}$$

where the $y_{\max}$ and $y_{\min}$ represent the upper/lower tolerance limits of $\hat{y}$ and, s and t represent the weights.

2. The-larger-the best (*LTB*): The value of $\hat{y}$ is expected to the larger the better. When the $\hat{y}$ excess a particular criteria value, which can be viewed as the requirement, the desirability value equals to 1; if the $\hat{y}$ is less than a particular

criteria value, which is unacceptable, the desirability value equals to 0. The desirability function of the-larger-the-best can be written as the term (2.12):

$$d = \begin{cases} 0, & \hat{y} \le y_{\min} \\ \left(\dfrac{\hat{y}-y_{\min}}{y_{\max}-y_{\min}}\right), & y_{\min} - \le \hat{y} \le y_{\max}, r \ge 0 \\ 1, & \hat{y} \ge y_{\min} \end{cases} \tag{2.12}$$

where the $y_{\min}$ presents the lower tolerance limit of $\hat{y}$, the $y_{\max}$ presents the upper tolerance limit of $\hat{y}$ and r represents the weight.

3. The-smaller-the best (*STB*): The value of $\hat{y}$ is expected to be the smaller the better. When the $\hat{y}$ is less than a particular criteria value, the desirability value equals to 1; if the $\hat{y}$ excess a particular criteria value, the desirability value equals to 0. The desirability function of the-smaller-the-best can be written as the term (2.13):

$$d = \begin{cases} 1, & \hat{y} \le y_{\min} \\ \left(\dfrac{\hat{y}-y_{\max}}{y_{\min}-y_{\max}}\right), & y_{\min} \le \hat{y} \le y_{\max}, \quad r \ge 0 \\ 0, & \hat{y} \ge y_{\max} \end{cases} \tag{2.13}$$

where the $y_{\min}$ presents the lower tolerance limit of $\hat{y}$, the $y_{\max}$ presents the upper tolerance limit of $\hat{y}$ and r represents the weight.

The s, t and r in the term (2.11) to the term (2.13) indicate the weights and they are defined according to the requirement of the user. If the corresponding response is expected to be closer to the target, the weight can be set the larger value; otherwise, the weight can be set the smaller value.

In a multi-response situation, the ideal case is all responses' desirability value to equal 1 and the whole response's desirability value also equal 1. If any response cannot achieve the requirement, the ideal case of the whole response cannot achieve and that is viewed as the unacceptable case. Moreover, if the desirability value of any response equals to 0, the whole response will be also viewed as the unacceptable case. To complete the requirement, the whole response's desirability value can take the geometric average of all responses' desirability value, i.e.

$$D = (d_1 \times d_2 \times \cdots \times d_m)^{1/m} = \left(\prod_{i=1}^{m} d_i\right)^{1/m} \tag{2.14}$$

where the d_i represents the desirability value of i-th response, $i=1,..m$. That is, the D equals 1 when all responses achieve the target and, the D equals 0 when any one response cannot achieve the requirement.

2.3.2. Omega transformation

When data lies in [0,1], e.g. yield or the desirability value, which may lead to a bad model's additive since the value being more close to 0 or 1. To solve this problem, Taguchi suggest the Omega (Ω) transformation (Phadke, 1989) is employed to transfer the data into an additive mode.

Ω transformation's philosophy is to simultaneously maximize the average of the system and minimize the variation via $S/N = 10 \log(\bar{y}^2/s^2)$. Assume that there is a binary data set $y_1, y_2, \ldots, y_n$, where $y_i = 1$ represents success and $y_i = 0$ represents failure with the probability of success p. The average value of the data set is

$$\hat{p} = \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \quad \text{and} \quad \hat{p} = \frac{1}{n}\sum_{i=1}^{n} y_i^2 \tag{2.15}$$

the variability of this data set is

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i-\bar{y})^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n} y_i^2 - 2n\bar{y} + n\bar{y}^2\right) = \frac{1}{n-1}(n\hat{p} - n\hat{p}^2) \tag{2.16}$$

for most situations, $n>>1$, then the term (2.16) can be rewritten as

$$s^2 \cong \hat{p}(1-\hat{p}) \tag{2.17}$$

The *S/N* ratio for the larger-the-better can be obtained as following:

$$S/N_{p\text{–LTB}} = 10\log\frac{\bar{y}^2}{s^2} = 10\log\left(\frac{\hat{p}^2}{\hat{p}(1-\hat{p})}\right) = 10\log\left(\frac{\hat{p}}{1-\hat{p}}\right) \tag{2.18}$$

The *S/N* ratio for the smaller-the-better can also be obtained by adding a negative signal to the term (4.8) as following:

$$S/N_{p\text{–STB}} = -10\log\left(\frac{\hat{p}}{1-\hat{p}}\right) = 10\log\left(1-\hat{p}\hat{p}\right) \tag{2.19}$$

The terms (2.18) and (2.19) transfer the data with an unadditive mode into the *S/N* ratio with additive mode, i.e. which will transfer the data lying in [0,1] to the range of $(-\infty,\infty)$. This method is called the Ω transformation. It can resolve the problem by summing up the control factor's effect when the data lie outside the interval [0,1].

3. Proposed approach

Kapur and Chen (Kapur & Chen, 1988) had proposed a function to represent the relationship between the quality response Y and the signal factor M in a dynamic system.

$$Y = f(M) + f_e(e) \tag{3.1}$$

Where $f\,(M)$ denotes the predictable part and the effectiveness of the expectable quality, $f_e(e)$ will denote the unpredictable part. As for Eq. (3.1), the interaction between the control factors and signal factors will affect the sensitivity of system. Besides, the interaction between the control factors and noise factors will affect the variation of system. The philosophy of the desirability function is the same as that of Taguchi method for achieving the target, not only for the quality specification. Besides, desirability function can also be viewed as a scale invariant index to be applied into the multi-response optimization. Hence, we will intent to incorporate them to developing a suitable analytic procedure. The thinking of reducing the number of responses to two responses, variation and sensitivity, is then employed. Due to the above thinking, we design a procedure with six steps to optimize a multi-response problem in Taguchi's dynamic system as follow:

Step1. Construct the regression model according to the signal factor and response.

Construct the regression model for different response $\mathrm{Y_i}$ to find the control factor affecting the response and the adjustment factor affecting sensitivity,

$$Y_i = \beta_i(d)M + \varepsilon_i(d), \quad i = 1,\ldots,m \tag{3.2}$$

where the term (2.5) represents the $\beta_i(\mathbf{d})$ and the term (2.10) represents the $\varepsilon_i(\mathbf{d})$, the factor which has significant interaction effect on noise factor will be viewed as the control factor affecting response's variability and, the control factor which has significant interaction effect on signal factor will be viewed as the adjustment factor affecting the sensitivity.

Step2. Estimate the noise factor's coefficient and sensitivity for each experimental trial.

To estimate the noise factor's coefficient $\hat{\Gamma}_{h,i}(d_j)$ and sensitivity $\beta_i(\mathbf{d}_j)$ of the regression model for each response Y_i under each experimental trial $\mathbf{d}_j$.

Step3. Compute the desirability value of each experimental trial for affecting variability and sensitivity.

(1) If the control factor/level combination, which has no influence with the noise factor Z_h, can make the response Y_i more robust. Then, the coefficient $\hat{\Gamma}_{h,i}(d_j)$ of the noise factor Z_h will be close to 0. Therefore, the output of the

response is just only affected by the inputting signal factor and it only lead to less variation. Hence, We can achieve this subjective by minimizing $\Gamma_{h,i}^2(d_j)$. To minimize $\Gamma_{ij}=\sum_h \Gamma_{h,i}^2(d_j)+\sigma_i^2$ (σ_i^2 represents the variation which can not be explained by Eq. (3.2) at Step 1, that is, this error term which is affected by the other factors except the known noise factors) will make the response Y_i more robust. It cannot only lead to any variation by being affected from the noise. Therefore, the-smaller-the-best desirability function can be employed to address this issue:

$$\mathrm{d}(\hat{\Gamma}_{ij}) = \begin{cases} 1, & \hat{\Gamma}_{ij}=0 \\ \left(\dfrac{\mathrm{MSE}_i-\hat{\Gamma}_{ij}}{\mathrm{MSE}}\right)^r, & 0\le \hat{\Gamma}_{ij}\le \mathrm{MSE}_i \\ 0, & \hat{\Gamma}_{ij}\ge \mathrm{MSE}_i \end{cases} \tag{3.3}$$

where, $\hat{\Gamma}_{ij}=\sum_h \hat{\Gamma}_{h,i}^2(d_j)$ and, the MSE_i represents the mean square error of the regression model $Y_i=\beta M$ of response Y_i and r represents the weight of the response.

(2) The scale of the $\beta(\mathbf{d})$ is focused on adjusting the system to the ideal input/output relationship. For adjusting the system's sensitivity, the characteristic of the $\beta(\mathbf{d})$ is determined according to the relationship of the inputting signal factor and the outputting response and, the ideal relationship is achieved by employing the corresponding desirability function $\mathrm{d}(\hat{\beta}_i(d_j))$.

Step4. Compute the desirability value of each experiment run for the whole response.

To compute the desirability value N_j of the variability and S_j of the sensitivity for each experiment run $\mathbf{d}_j$.in the whole response.

$$N_j = \left(\prod_{i=1}^{m} \mathrm{d}(\hat{\Gamma}_{ij})\right)^{1/m} \tag{3.4}$$

$$S_j = \left(\prod_{i=1}^{m} \mathrm{d}(\beta_i(d_j))\right)^{1/m} \tag{3.5}$$

Step5.Determine the optimum control factor/level combination.

(1) To transfer initially the desirability value in Step 4 into additive S/N ratio by employing Ω transformation, i.e. ON_j and OS_j,

$$ON_j = 10\log\left(\frac{N_j}{1-N_j}\right) \tag{3.6}$$

$$OS_j = 10\log\left(\frac{S_j}{1-S_j}\right) \tag{3.7}$$

(2) To compute the average value of ON_j and OS_j for each factor's level by using orthogonal array, moreover, to construct the response table and response graph.

(3) Herein, we will apply the philosophy of Taguchi method: 'reduce the variation at first, and then adjust the sensitivity to achieve the optimization' to determine the optimum parameter settings. Firstly, we will determine the level setting of control factors with effect on the variation. Then, the level setting of control factors with effect on the sensitivity can also be determined. However, if there is a conflict for judging the level setting, we will use the philosophy of Taguchi method to make the compromise. Restated, the level setting of control factors can reduce the variation will be the optimum choice. Besides, the experimenter's experience or the engineering's knowledge can also be applied to aid our decision-making.

Step6. Perform the confirmed experiment.

To obtain the estimated S/N ratio η_{ON} and η_{OS} of each response under the optimum parameter's settings by using the control factor affecting response's variability and the adjustment factor affecting system's sensitivity. To make sure the repeatability of the chosen optimum parameter's settings in reality, the confirmed experiment must be performed. If the confirmed experiment's S/N ratio is close to the estimated S/N ratio, the obtained optimum parameter's settings have a well repeatability. If the confirmed experiment's S/N ratio is less than estimated S/N ratio, the optimum parameter's settings obtained have a bad repeatability and it indicates the chosen control factors, signal factors and responses are not suitable in the experiment. Therefore, a plan must be re-performed to find a suitable control factors, signal factors and responses.

4. Illustrative example

4.1. Biological reduction of ethyl acetoacetate process

A biological reduction of ethyl acetoacetate process, which is a working process in the Union Chemical Laboratories of the Industrial Technology Research Institute, will be employed to demonstrate the proposed optimization in this section.

S-4-Chloro-3-hydroxybutyric acid ethyl ester (or S-CHBE), which is a middle optical activity widely used, can employed to synthesize Simvastatin. If the middle components are synthesized by chemical method, which needs more experimental steps and leads more fluctuate reactive conditions, it cannot produce high enantiomeric excess (ee) products. Therefore, the optical features of microorganism can be applied to produce the middle components. The goal of this experiment is to employ yeast to biological reduction of ethyl acetoacetate process. This technique utilizes the yeast for reducing base's ketone to alcohol chemical compound with optical features. However, there are some enzymes which can perform the reduction in yeast cell, e.g. S-type (can produce S-CHBE) dehydrogenase or R-type (can produce R-CHBE) dehydrogenase. The unequal amount of S-CHBE and R-CHBE frequently exist in liquid products for the nonhomogenous or adverse of the enzyme's optical choices. If the process can be careful to control, the S-CHBE enzyme will have the better activity and lead the activity of R-CHBE enzyme to be lower. Next, a high enantiomeric excess (ee) product can be obtained.

The yield and enantiomeric excess (ee) is determined to be the interested responses. Engineers review the related literature, the yield lies in 42–62% and the ee lies in 15–85% for S-CHBE in the previous experience or related reports. Besides, the two responses are the larger the better (*LTB*) according to the application's requirement. For studying the effect on the change of base's concentration, it is determined to be a signal factor (*M*). The obtained reactive condition is then expected to keep high yield and produce high enantiomeric excess (ee) product under different base's concentrations. By performing brainstorm analysis and pre-experiment, eight control factors are chosen: X_1–X_8 (for business secrete). Table 1 lists the levels of these eight factors and the signal factor. The levels of X_4 and X_5 are designed as the fluctuate levels; a better result depends on that the increasing of base's concentration which may lead to increase the amount of X_4 and X_5. Table 2 lists the fluctuate levels of the two factors: X_4 and X_5. The different lot of yeast is viewed as a noise factor and, two lots are considered. The L_{18} OA is employed to perform this experiment. Herein, we use the SAS statistical software to deal with the necessary data analysis including the model constructing.

4.2. Result analysis

The proposed approach is employed herein to demonstrate this illustrated example step by step.

Step 1: Construct the regression model of the signal factor and quality response

Firstly, the factor's levels are coded according to the code -1 and 1 for two levels and, the code -1, 0 and 1 for three levels. Employ the stepwise regression approach to construct the regression model of Y_1 and Y_2. After checking the model's adequacy, the regression models of Y_1 and Y_2 with respect to the significant factors are obtained as

Table 1
The level definition of the control the signal factors

Factor	Level 1	Level 2	Level 3
M	1%	3%	5%
X_1	S	A	
X_2	140 rpm	170 rpm	200 rpm
X_3	0.2%	0.6%	1.0%
X_4	Fluctuate (1)	Fluctuate (2)	Fluctuate (3)
X_5	Fluctuate (1)	Fluctuate (2)	Fluctuate (3)
X_6	7.5	8.0	8.5
X_7	0.3 M	0.4 M	0.5 M
X_8	1 h	2 h	3 h

Table 2
The fluctuate level definition for X_4 and X_5

M		1%	3%	5%
X_4	Fluctuate (1)	40%	80%	120%
	Fluctuate (2)	60%	100%	140%
	Fluctuate (3)	80%	120%	160%
X_5	Fluctuate (1)	1.0 ml/l	1.2 ml/l	1.4 ml/l
	Fluctuate (2)	1.2 ml/l	1.4 ml/l	1.6 ml/l
	Fluctuate (3)	1.4 ml/l	1.6 ml/l	1.8 ml/l

follows:

$$\hat{Y}_1 = (0.39821 - 0.017933x_1 + 0.023587x_3^2 - 0.031206x_4 + 0.012966x_5 - 0.02151x_8)M + 0.10905z, R^2 = 0.97135 \tag{4.1}$$

$$\hat{Y}_2 = (0.11105 - 0.005156x_1 - 0.012232x_2^2 + 0.018344x_4 - 0.009297x_5)M - 0.018378x_3^2 z, R^2 = 0.93132 \tag{4.2}$$

No any interactions exist between the control factors and noise factors, hence, no control factors have the significantly effect on the quality response Y_1. However, the noise factor Z will significantly affect the output of quality response Y_1. That is, the product coming from the different lot will lead to the variance of the experimental results. Furthermore, there are interaction exist between X_1, X_3, X_4, X_5, X_8 and signal factor M from Eq. (4.1), that is, these control factors will affect the sensitivity of quality response of Y_1. Equally, from Eq. (4.2), interaction exist between the control factors X_3 and noise factor Z, the control factor X_3 will be the factor significantly affects the quality response Y_2. At the same time, from Eq. (4.2), control factors X_1, X_2, X_4, X_5 and the signal factor M will have interaction. Hence, these control factors will affect the sensitivity of quality response Y_2.

Step 2: Estimate the sensitivity of the noise factor's coefficient for each experimental run

The coefficients $\hat{\Gamma}_1(d)$ and $\hat{\Gamma}_2(d)$ of the noise factors for each quality response under experimental run can be obtained from the form Eqs. (4.1) and (4.2):

$$\hat{\Gamma}_1(d) = 0.10905 \tag{4.3}$$

$$\hat{\Gamma}_1(d) = -0.018378x_3^2 \tag{4.4}$$

Equally, the sensitivities $\hat{\beta}_1(d)$ and $\hat{\beta}_2(d)$of each quality response under experimental run can also be obtained from the form Eqs. (4.1) and (4.2):

$$\hat{\beta}_1(d) = 0.39821 - 0.017933x_1 + 0.023587x_3^2 - 0.031206x_4 + 0.012966x_5 - 0.02151x_8 \tag{4.5}$$

$$\hat{\beta}_2(d) = 0.11105 - 0.005156x_1 - 0.012232x_2^2 + 0.018344x_4 - 0.009297x_5 \tag{4.6}$$

Step 3: Compute the desirability value of the variation and the sensitivity for each experimental run

Table 3
The desirability values of the variation and the sensitivity for each parameter's combination

X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	N	S
S	140	0.2	1	1	7.5	0.3	1	0.19710	0.64667
S	140	0.6	2	2	8.0	0.4	2	0.20652	0.59653
S	140	1.0	3	3	8.5	0.5	3	0.19710	0.57909
S	170	0.2	1	2	8.0	0.5	3	0.19710	0.62654
S	170	0.6	2	3	8.5	0.3	1	0.20652	0.61751
S	170	1.0	3	1	7.5	0.4	2	0.19710	0.55153
S	200	0.2	2	1	8.5	0.4	3	0.19710	0.58270
S	200	0.6	3	2	7.5	0.5	1	0.20652	0.57476
S	200	1.0	1	3	8.0	0.3	2	0.19710	0.66534
A	140	0.2	3	3	8.0	0.4	1	0.19710	0.58999
A	140	0.6	1	1	8.5	0.5	2	0.20652	0.59786
A	140	1.0	2	2	7.5	0.3	3	0.19710	0.58023
A	170	0.2	2	3	7.5	0.5	2	0.19710	0.60256
A	170	0.6	3	1	8.0	0.3	3	0.20652	0.50001
A	170	1.0	1	2	8.5	0.4	1	0.19710	0.63672
A	200	0.2	3	2	8.5	0.3	2	0.19710	0.55875
A	200	0.6	1	3	7.5	0.4	3	0.20652	0.61551
A	200	1.0	2	1	8.0	0.5	1	0.19710	0.59363

The variance of quality response of Y_1 and Y_2 can be estimated from the Mean Square Error (MSE) of Eqs. (4.1) and (4.2). Then, the $\hat{\Gamma}_1$ and $\hat{\Gamma}_2$ can be computed by using Eqs. (4.3) and (4.4). The desirability value of $d(\hat{\Gamma}_1)$ and $d(\hat{\Gamma}_2)$ can be obtained by employing the formula (3.3). Next, we can use the result obtained from Eqs. (4.5) and (4.6) to compute the desirability value of sensitivity $d(\hat{\beta}_1(d))$ and $d(\hat{\beta}_2(d))$. Herein, we will use Eqs. (2.12) and (2.13) to compute the desirability value for response Y_1 with LTB characteristic and response Y_2 with STB characteristic. Furthermore, the importance of these two responses have the same, the weight r of both will be set to 1.

Step 4: Compute the whole desirability value of each experimental run

The desirability value N and S of the variance and the sensitivity can be computed by inputting the $d(\hat{\Gamma}_1)$ and $d(\hat{\Gamma}_2)$, $d(\hat{\beta}_1)$and $d(\hat{\beta}_2)$ to Eqa. (3.4) and Eqa. (3.5). Table 3 lists all results

Step 5: Determine the optimum control factor/level combination

The desirability value N and S from the form Eqs. (3.4) and (3.5) are firstly employed to transfer the both value into the additive *S/N* ratio. Then, the effect of the factor's level can be determined. Tables 4 and 5 represent the response table. The response graph can be represented as Fig. 2 and Fig. 3. Reviewing Table 4 and Fig. 2, the significant factors affecting the variance of the whole quality **N** is X_3. Reviewing Table 5 and Fig. 3, the significant factors affecting the sensitivity of the system will be the factors X_1, X_2, X_3, X_4, X_5, X_7 and X_8. Finally, we will apply the philosophy of Taguchi method, reduce the variation at first and then adjust the sensitivity, to determine the optimum parameter settings. The optimum factor/level combination can be determined as: $X_1=S$, $X_2=200$, $X_3=0.6$, $X_4=$fluctuate level one, $X_5=$fluctuate level three, $X_6=8.0$, $X_7=0.3$ and $X_8=1$.

Table 4
The response table for the *S/N* ratio of the variation

Control factor	Level one	Level two	Level three
X_1	−6.01507	−6.01507	
X_2	−6.01507	−6.01507	−6.01507
X_3	−6.09981	−5.84561	−6.09981
X_4	−6.01507	−6.01507	−6.01507
X_5	−6.01507	−6.01507	−6.01507
X_6	−6.01507	−6.01507	−6.01507
X_7	−6.01507	−6.01507	−6.01507
X_8	−6.01507	−6.01507	−6.01507

Table 5
The response table for the *S/N* ratio of the sensitivity

Control factor	Level 1	Level 2	Level 3
X_1	1.85119	1.51864	
X_2	1.73626	1.57557	1.74292
X_3	1.78641	1.47548	1.79285
X_4	2.34041	1.68123	1.03310
X_5	1.39030	1.68420	1.98024
X_6	1.68144	1.68991	1.68340
X_7	1.68683	1.68461	1.18330
X_8	1.94623	1.68929	1.41922

4.3. Confirmation experiments

The factors having the significant effect on both of the variation and the sensitivity for response Y_1 and Y_2 in Step1 are employed to estimate the *S/N* ratio:

1. The concentration of the S-CHBE(Y_1)

Because no significant factor having effect on the variation in this experiment, hence,

$$\eta_{\mathrm{ON_{opt}}} = \bar{\eta}_{\mathrm{ON}} = -6.0158$$

Besides, the estimated desirability value can also computed as 0.2002.

2. The factors X_1, X_3, X_4, X_5 and X_8 have the significant effect on the sensitivity:

$$\begin{aligned}\eta_{\mathrm{OS_{opt}}} &= \bar{\eta}_{\mathrm{OS}} + (\bar{\eta}_{X_1=S(\mathrm{OS})} - \bar{\eta}_{\mathrm{OS}}) + (\bar{\eta}_{X_3=0.6(\mathrm{OS})} - \bar{\eta}_{\mathrm{OS}}) + (\bar{\eta}_{X_4=1(\mathrm{OS})} - \bar{\eta}_{\mathrm{OS}}) + (\bar{\eta}_{X_5=3(\mathrm{OS})} - \bar{\eta}_{\mathrm{OS}}) + (\bar{\eta}_{X_8=1(\mathrm{OS})} - \bar{\eta}_{\mathrm{OS}}) \\ &= \bar{\eta}_{X_1=S(\mathrm{OS})} + \bar{\eta}_{X_3=0.6(\mathrm{OS})} + \bar{\eta}_{X_4=1(\mathrm{OS})} + \bar{\eta}_{X_5=3(\mathrm{OS})} + \bar{\eta}_{X_8=1(\mathrm{OS})} - 4\times\bar{\eta}_{\mathrm{OS}} \\ &= 1.85119 + 1.47548 + 2.34041 + 1.98024 + 1.94623 - 4(1.64892) = 2.99786\end{aligned}$$

And, the estimated desirability value can also computed as 0.9990.

3. The concentration of the R-CHBE(Y_2)

The factor X_3 is the significant factor for the variation:

$$\eta_{\mathrm{ON_{opt}}} = \bar{\eta}_{\mathrm{ON}} + (\bar{\eta}_{X_3=0.6(\mathrm{on})} - \bar{\eta}_{\mathrm{ON}}) = \bar{\eta}_{X_3=0.6(\mathrm{on})} = -5.8456$$

Then, the estimated desirability value can also computed as 0.2065.

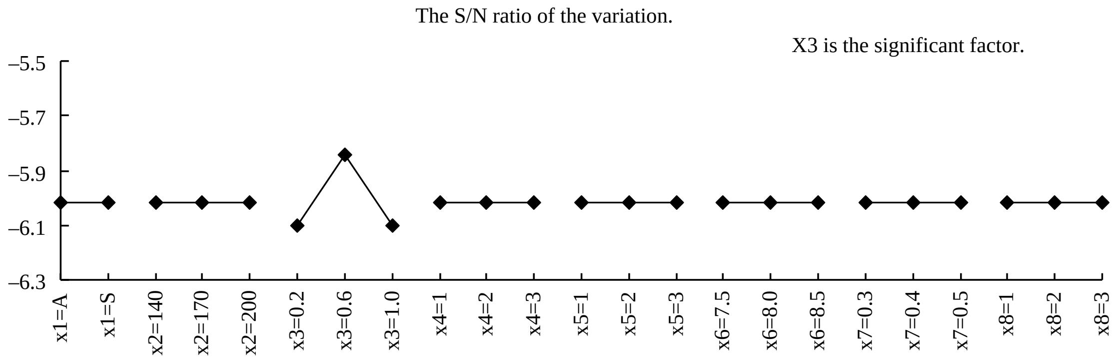

Fig. 2. The response diagram for the S/N ratio of the variation.

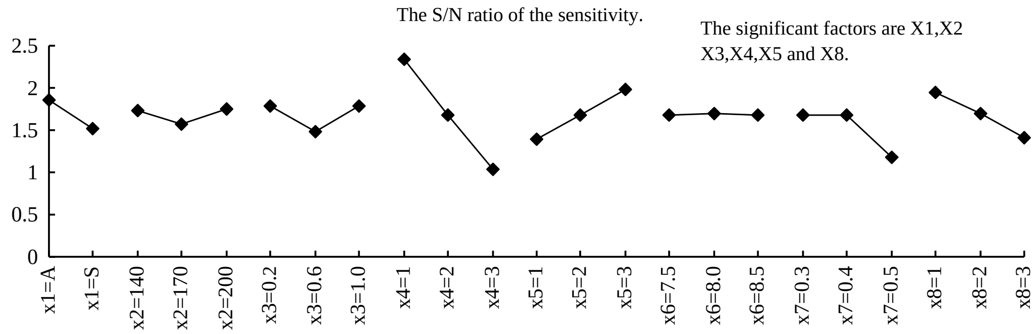

Fig. 3. The response diagram for the S/N ratio of the sensitivity.

4. The factors X_1, X_2, X_4 and X_5 have the significant effect on the sensitivity:

$$\begin{aligned}\eta_{\mathrm{OS}_{\mathrm{opt}}} &= \bar{\eta}_{\mathrm{OS}} + (\bar{\eta}_{X_1=S(\mathrm{OS})} - \bar{\eta}_{\mathrm{OS}}) + (\bar{\eta}_{X_2=200(\mathrm{OS})} - \bar{\eta}_{\mathrm{OS}}) + (\bar{\eta}_{X_4=1(\mathrm{OS})} - \bar{\eta}_{\mathrm{OS}}) + (\bar{\eta}_{X_5=3(\mathrm{OS})} - \bar{\eta}_{\mathrm{OS}}) \\ &= \bar{\eta}_{X_1=S(\mathrm{OS})} + \bar{\eta}_{X_2=200(\mathrm{OS})} + \bar{\eta}_{X_4=1(\mathrm{OS})} + \bar{\eta}_{X_5=3(\mathrm{OS})} - 3\times\bar{\eta}_{\mathrm{OS}} \\ &= 1.855119 + 1.74292 + 2.3404 + 1.98024 - 3\times 1.6849 = 2.86\end{aligned}$$

And, the estimated desirability value can also computed as 0.9986.

Perform the confirmed experiment according to the optimum parameter's setting we obtained. Table 6 and Table 7 list the results of the confirmed experiment and the *S/N* value of the confirmed experiment. The related data of Table 6 can be sent to the proposed approach from step 2 to Step 4. Then, the *S/N* value can be then computed. To compare the *S/N* ratio of the confirmed experiment and the estimated *S/N* ratio, only the sensitivity's *S/N* ratio of S-CHBE is lower than the estimated *S/N* ratio. The other S/N ratios are higher than the estimated *S/N* ratio. This indicates that the

Table 6
The results of the confirmed experiment

M	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	Yield(%)		ee(%)		Y_1		Y_2	
1	S	200	0.6	40	1.4	8.0	0.3	1	72.10	69.50	88.63	83.88	0.680	0.639	0.041	0.056
									69.90	69.60	88.84	85.63	0.660	0.646	0.039	0.050
									76.10	71.80	89.22	83.29	0.720	0.658	0.041	0.060
									72.80	78.20	80.40	74.42	1.970	2.046	0.214	0.300
3	S	200	0.6	80	1.6	8.0	0.3	1	67.90	76.03	82.23	78.34	1.856	2.034	0.181	0.247
									66.30	77.30	80.59	78.01	1.796	2.064	0.193	0.255
									54.30	65.18	75.47	62.32	2.382	2.645	0.333	0.614
5	S	200	0.6	120	1.8	8.0	0.3	1	56.20	63.58	76.73	63.57	2.483	2.600	0.327	0.579
									52.10	64.38	78.12	62.97	2.320	2.623	0.285	0.596

p.s: Where the yield and ee value can be computed as the following formulas: Yield $= [(Y_1 + Y_2)/M] * 100$, $ee = [(Y_1 - Y_2)/(Y_1 + Y_2)] * 100$.

Table 7
The S/N ratios of the confirmed experiment

	S/N ratio of the variation		S/N ratio of the sensitivity	
	Confirmed result	Estimated result	Confirmed result	Estimated result
Y_1	−5.231	−6.0158	1.040	2.99786
Y_2	−2.367	−5.8456	8.304	2.86

p.s: the S/N values of the confirmed experiment can be computed by inputting the confirmed result into the step 2∼step 4 for the proposed procedure.

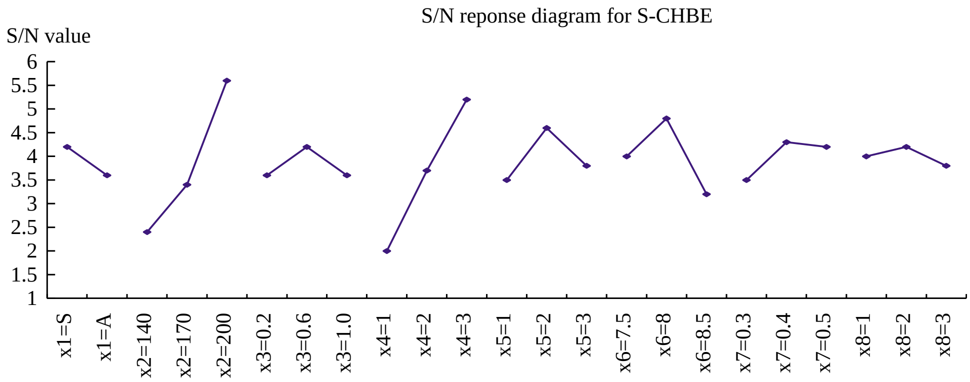

Fig. 4. The S/N response diagram of variation for the S-CHBE.

optimum parameter's setting represents a well repeatability. Only one optimum parameter's setting can be obtained and it can achieve the target: the higher yield and high enantiomeric excess product.

4.4. The comparison result of the proposed approach and the Taguchi's dynamic method for a single response

This experiment can be viewed as an experiment combined with two responses. The Taguchi's dynamic method for a single response is then employed to analyze the same experimental data.

When the Taguchi's dynamic method is employed to response Y_1 (the concentration of S-CHBE). The factors X_2 and X_4 are the significant factors affecting the *S/N* ratio of Y_1 (we can screen out from Fig. 4). The factors X_4 and X_8 are the significant factors having the significant effect on the sensitivity of Y_1 (we can screen out from Fig. 5). Hence, the optimum parameter's settings can be determined as X_1 = S, $X_2 = 170$, $X_3 = 1.0$, X_4 = fluctuate level two, X_5 = fluctuate level three, $X_6 = 8.0$, $X_7 = 0.5$ and $X_8 = 2$. Employing Taguchi's dynamic method to Y_2 (the concentration of R-CHBE), the factors X_2, X_4, X_5, X_6 and X_8 are the factors having the significant effect on the *S/N* ratio (we can screen out from Fig. 6) and, the factors X_4 and X_5 are the significant factors affecting the sensitivity (we can screen out from Fig. 7). Therefore, the optimum parameter's setting for the concentration of R-CHBE can be determined as X_1 = A, $X_2 = 140$, $X_3 = 0.6$, X_4 = fluctuate level two, X_5 = the fluctuate level one, $X_6 = 8.5$, $X_7 = 0.4$ and $X_8 = 2$.

Analyzing these results, we can find out that there are serious conflicts between the parameter's settings except the factors X_4 and X_8. Hence, determining the optimum parameter's settings of the whole system will be more difficult. However, for making the comparison, one optimum parameter's setting must be determined by making compromise for several senior engineers as: X_1 = S, $X_2 = 170$, $X_3 = 0.6$, X_4 = the fluctuate level two, X_5 = the fluctuate level three, $X_6 = 8.0$, $X_7 = 0.5$ and $X_8 = 2$. Table 8 lists the results of the Taguch's method (by compromise) and that of

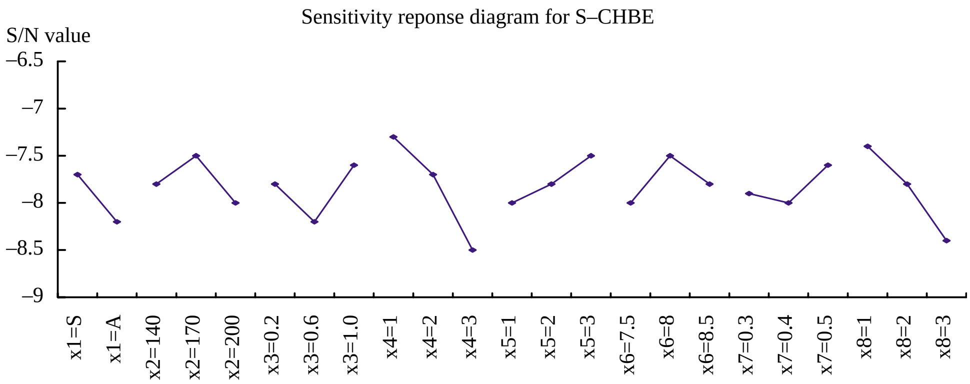

Fig. 5. The S/N response diagram of sensitivity for the S-CHBE.

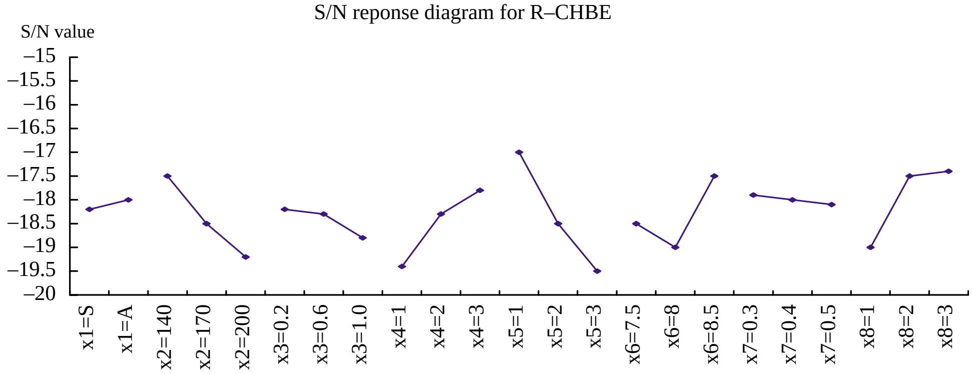

Fig. 6. The S/N response diagram of variation for the R-CHBE.

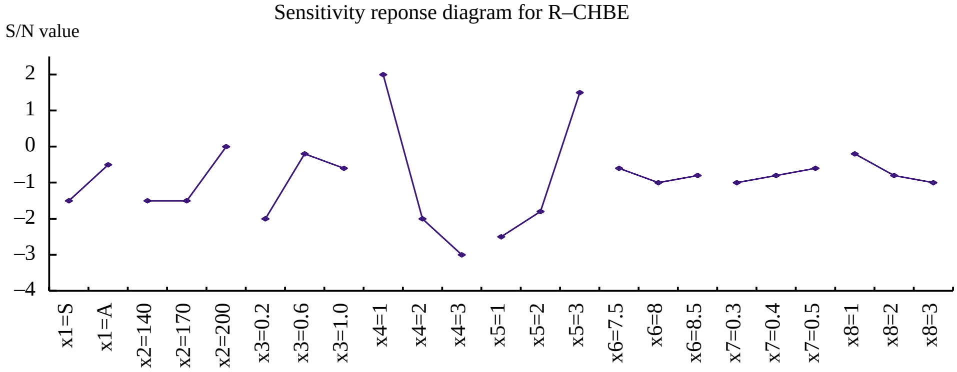

Fig. 7. The S/N response diagram of sensitivity for the R-CHBE.

the proposed approach. The settings of X_2, X_4, X_7 and X_8 have the significant difference. Furthermore, Table 9 lists the estimated *S/N* ratios of the two methods. From the comparison of Table 9, only the estimated *S/N* ratio of the variation for the concentration of R-CHBE is less than Taguchi's approach. The others estimated *S/N* ratios are larger than Taguchi's method. Hence, the proposed approach will be efficiency. It can effectively achieve the quality improvement.

Table 8
The comparison result of the optimum parameter's settings for the proposed approach and the Taguchi's method

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8
The proposed approach	S	200	0.6	Level 1	Level 3	8.0	0.3	1
The Taguchi's dynamic method	S	170	0.6	Level 2	Level 3	8.0	0.5	2

Table 9
The comparison of the estimated S/N ratios for the proposed approach and the Taguchi's method

	The *S/N* ratio of the variation		The *S/N* ratio of the sensitivity	
	The proposed approach	The Taguchi's method	The proposed approach	The Taguchi's method
Y_1	−6.0158	−7.365	2.99786	−1.319
Y_2	−5.8456	−4.574	2.86	1.977

5. Concluding remarks

The related literature for the optimization of a multi-response problem with Taguchi's dynamic system consideration has seldom been mentioned. Wasermam (Wassermam, 1996) has demonstrated the operation of a single response problem in dynamic system by employing the regression's perspective; however, there is not a suitable approach to a multi-response problem. The most difficulty is that the optimum parameters' settings for different response are usually conflict and, the weight value of response usually depend on the engineers' subjective judgment. In such situations, the final optimum parameters' setting will be more difficult to determine. In this study, an optimization approach incorporating the regression analysis and the desirability function perspective in a multi-response with Taguchi's dynamic system consideration is proposed. Moreover, the proposed approach cannot be employed in a dynamic system, but also can be employed in a static system. The proposed approach can provide several metrics:

(1) Our proposed approach can effectively departure those control factors which significant affect the response's variability or system's sensitivity and, the requirement of minimizing variability and adjusting system's sensitivity can be achieved;
(2) For real applications, linear relationship between the response and signal factor may be not necessary, the proposed approach with great flexibility can be employed for non-linear relationship;
(3) In a dynamic system, the number of signal factor sometimes excess one signal factor, the proposed approach can also be employed to a dynamic system with multiple signal factors;
(4) The proposed approach employing the desirability function can not only consider the unequal importance between responses, but also represent the requirement for the response's quality;
(5) The proposed approach cannot only be employed to optimize the multi-response problem in a dynamic system, but also can be employed to optimize the multi-response problem in a static system.

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