Available online at www.sciencedirect.com
SCIENCE DIRECT
Thin-Walled Structures 43 (2005) 1831–1851
THIN-WALLED STRUCTURES
www.elsevier.com/locate/tws

Simplified design approach for cold-formed stainless steel compression members subjected to flexural buckling

S.H. Lin [a,*], S.I. Yen [b], C.C. Weng [c]

[a] *Department of Civil Engineering, Minghsin University of Science and Technology, No. 1, Hsinsin Rd, Hsinfeng, Hsinchu 304, Taiwan, ROC*
[b] *Power Project Group, E&C Engineering Corporation, Taipei 222, Taiwan, ROC*
[c] *Department of Civil Engineering, National Chiao Tung University, Hsinchu 300, Taiwan, ROC*

Received 8 December 2004; accepted 26 August 2005

Abstract

The design criteria of stainless steel compression member are more complicated than those of carbon steels due to the nonlinear stress strain behavior of the material. In general, the tangent modulus theory is used for the design of cold-formed stainless steel columns. The modified Ramberg–Osgood equation given in the ASCE Standard can be used to determine the tangent modulus at specified level of stresses. However, it is often tedious and time-consuming to determine the column buckling stress because several iterations are usually needed in the calculation. This paper presents new formulations to simplify the determination of flexural buckling stress without iterative process. Taylor series expansion theory is utilized in the study for numerical approximations. The proposed design formulas are presented herein and can be alternatively used to calculate the flexural buckling stress for austenitic type of cold-formed stainless steel columns. It is shown that the column strengths determined by using the proposed design formulas have good agreement with those calculated by using the ASCE Standard Specification. A design example is also included in the paper for cold-formed stainless steel column designed by using the ASCE Standard equations and the proposed design formulas.

Keywords: Cold-formed stainless steels; Compression members; Specification; Tangent modulus; Flexural buckling; Numerical approximation

* Corresponding author. Tel.: +886 3 5593142x3302; fax: +886 3 5573718.
E-mail address: lshr@must.edu.tw (S.H. Lin).

0263-8231/$ - see front matter
doi:10.1016/j.tws.2005.08.006

Notation

A	Area of the full, unreduced cross section
A_e	Effective area of the cross section
b	Effective design width of compression element
C	Slenderness ratio, KL/r
C_o	Specified slenderness ratio at $F_n = F_y$
C_1	Limiting slenderness ratio at $F_n = F_1$
E_o	Initial modulus of elasticity
E_t	Tangent modulus
f	Nominal stress
F_y	Specified yield strength
F_1	Specified buckling stress with respect to C_1
F_n	Nominal buckling stress
$F_{n,\mathrm{ASCE}}$	Nominal buckling stress determined from ASCE Standard Specification
$F_{n,\mathrm{prop}}$	Nominal buckling stress determined from the proposed design formulas
I_x	Moment of inertia of x-axis
I_y	Moment of inertia of y-axis
k	Plate buckling coefficient
K	Effective length factor
L	Unbraced length of member
n	Coefficient used for determining the tangent modulus
P_n	Nominal axial strength of member
r	Radius of gyration
t	Thickness of the section
w	Flat width of element exclusive of radii
α	$E_o/E_t - 1$
β	Constant
ϕ_c	Resistance factor for axial strength
λ_o	Parameter used for determining buckling stress
λ_1	$1 - \lambda_o$
ρ	Reduction factor
λ	Slenderness factor

1. Introduction

For the design of cold-formed stainless steel compression members, the ASCE Standard Specification can be used to determine the design axial strength. The current specification was published in 2002 by ASCE as SEI/ASCE 8-02 [1]. This updated edition of the Specification is a revision of the ASCE Standard published in 1991 [2]. Due to

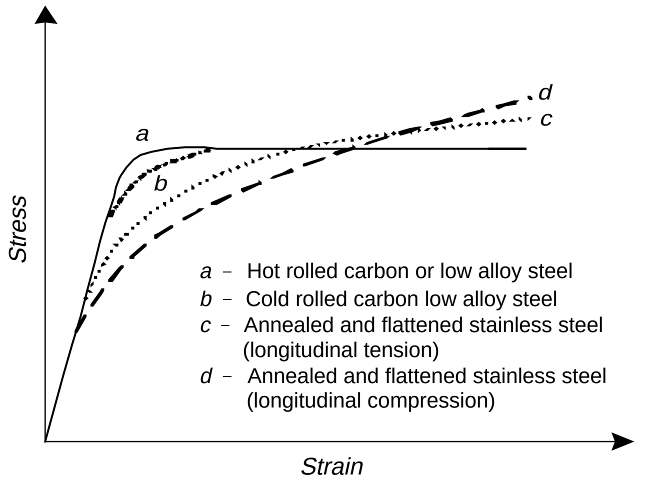


Fig. 1. Stress–strain curves of carbon and stainless steels

the differences in mechanical behavior of stainless steels as compared with carbon steels as shown in Fig. 1, the design of stainless steel compression members is more complex than those of carbon steels. It is noted that stainless steels have gradually yielding type of stress–strain curves with relatively low proportional limits [3,4]. Because of the nonlinear stress-strain behavior, the design of stainless steel compression members is based on the tangent modulus theory [5,6].

Stainless steel has different mechanical properties in longitudinal and transverse directions for tension and compression modes of stress. As a result, different tangent modulus, E_t, are provided for different types of stainless steels in the design tables and figures of the ASCE Specification. Tangent modulus is used to account for the inelastic buckling of stainless steel compression components. Alternatively, it can be determined by using the modified Ramberg–Osgood equation [7,8] given in Appendix B of the ASCE Standard for specified types of stainless steels. Because of the nonlinear nature of tangent modulus, the column buckling stress is determined through an iterative process until the satisfied tolerance is reached [9,10]. This type of calculation is often tedious and time-consuming as compared with that of hot-rolled steel column design.

To simplify the design process, this paper presents a new approach to obtain the design formulas for determining the flexural buckling stress without successive iterations. These formulas are developed from the tangent modulus equation obtained from the modified Ramberg–Osgood equation. A mathematical approximating method utilizing the Taylor series expansion theory is formulated for numerical approximation. New design formulas are proposed for austenitic type of cold-formed stainless steel columns subjected to flexural buckling. It is shown that the approximated solutions have a good agreement as compared with the ASCE Standard solutions. To illustrate the simplified design approach, a design example of cold-formed stainless steel column subjected to flexural buckling is included in Appendix of the paper.

2. Review of ASCE design criteria

Section 3.4 of the ASCE Standard provides the design requirements to determine the design axial strength for concentrically loaded cold-formed stainless steel compression members. It specifies that the resultant of all loads acting on the members is an axial loads passing through the centroid of the effective section calculated at the stress F_n. The design axial strength, $\phi_c P_n$, is calculated as follows:

$$\phi_c = 0.85$$

$$P_n = A_e F_n \tag{1}$$

where:

A_e = effective area calculated at stress F_n

F_n = the least of the flexural, torsional, and torsional-flexural buckling stress determined according to Sections 3.4.1–3.4.3 of the ASCE Standard, respectively.

Section 3.4.1 of the ASCE Standard specifies that, for doubly symmetric sections, closed cross sections, and any other sections which are not subjected to torsional or torsional–flexural buckling, the flexural buckling stress, F_n, is determined as follows:

$$F_n = \frac{\pi^2 E_t}{(\mathrm{KL}/r)^2} \le F_y \tag{2}$$

where:

E_t = tangent modulus in compression corresponding to buckling stress F_n

KL/r = slenderness ratio

F_y = specified yield strength, as given in Table 1 for austenitic type stainless steels

To calculate the flexural buckling stress in Eq. (2), it is necessary to have a proper value of E_t, which can be obtained from design tables or figures in the ASCE Standard for the assumed stress. Alternatively, it can be determined by using analytical expression, which is based on the modified Ramberg–Osgood equation

$$E_t = \frac{E_o F_y}{F_y + 0.002\, n E_o (F_n/F_y)^{n-1}} \tag{3}$$

Table 1
ASCE specified yield strength F_y for austenitic type stainless steels [1]

Types of stress	F_y, Mpa			
	Types 201, 301, 304, 316			
	Annealed	1/16 Hard	1/4 Hard	1/2Hard
Longitudinal tension	206.9	310.3	517.1	758.5
Transverse tension	206.9	310.3	517.1	758.5
Transverse compression	206.9	310.3	620.6	827.4
Longitudinal compression	193.1	282.7	344.8	448.2

Table 2
ASCE specified initial modulus of elasticity E_o and coefficient n for austenitic type stainless steels [1]

Types of stress	Types 201, 301, 304, 316					
	Annealed and1/16 Hard		1/4Hard		1/2Hard	
	E_o (MPa)	n	E_o (MPa)	n	E_o (MPa)	n
Longitudinal tension	193100	8.31	186200	4.58	186200	4.21
Transverse tension	193100	7.78	193100	5.38	193100	6.71
Transverse compression	193100	8.63	193100	4.76	193100	4.54
Longitudinal compression	193100	4.10	186200	4.58	186200	4.22

in which E_o is the initial modulus of elasticity and n is the coefficient used for determining tangent modulus. Table 2 gives values of E_o and n for austenitic type stainless steels, which are obtained from design tables in the ASCE Standard.

Because the buckling stress, F_n, and the tangent modulus, E_t, are interdependent as self-explanatory in Eqs. (2) and (3), the determination of flexural buckling stress, F_n, requires iterative process. The buckling stress, F_n, is needed to determine E_t in Eq. (3), but it is not known until it has been obtained from Eq. (2). Therefore, to determine a proper value of E_t, a buckling stress is first to be assumed in Eq. (3). Then, this calculated value of E_t is used to determine the buckling stress, F_n, in Eq. (2). In view of the fact that the calculated buckling stress is seldom equal to the first assumed buckling stress, further successive iterations are needed to obtain the final buckling stress. This buckling stress can be achieved when a satisfied convergence of error is reached.

For details of determining the buckling stress by using ASCE Standard design provisions, see the design example in the appendix of the paper.

3. Simplified approach

As discussed above, for the design of cold-formed stainless steel columns, the determination of flexural buckling stress is tiresome due to its iterative process of calculation. As a result, a simplified approach is proposed herein to determine the flexural buckling stress without having iterative calculations. New formulations are developed as targets of the numerical approximation. In order to simplify the calculations, Taylor series expansion is applied to the approximating equation. The following section gives detailed presentations on this subject.

3.1. Initial concept

The flexural buckling stresses determined by using the ASCE design equations for Type 304 stainless steel columns in longitudinal compression (LC) and transverse compression (TC) are shown in Figs. 2 and 3, respectively. Nonlinear buckling stress curves are found typical for those columns when the flexural buckling stresses are less than F_y, which is the upper bond of Eq. (2). Thus, a linear equation is initialized in this study to simplify these nonlinear curves.

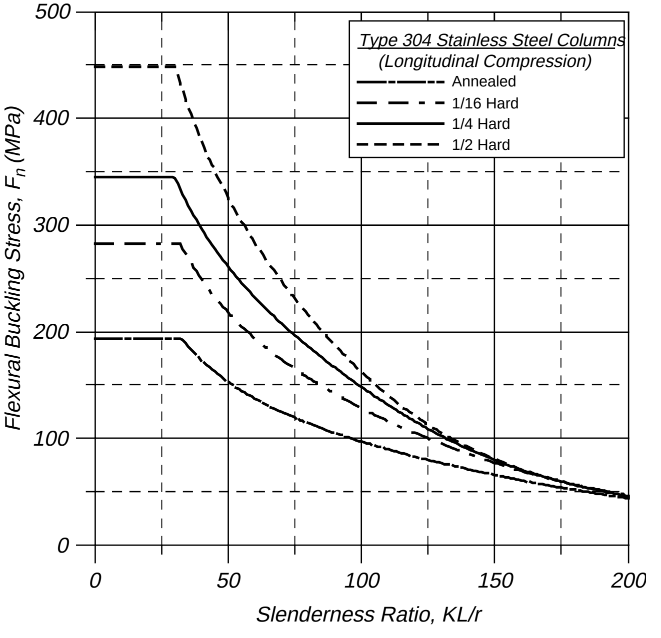


Fig. 2. ASCE Standard flexural buckling stresses for Type 304 stainless steel columns in longitudinal compression.

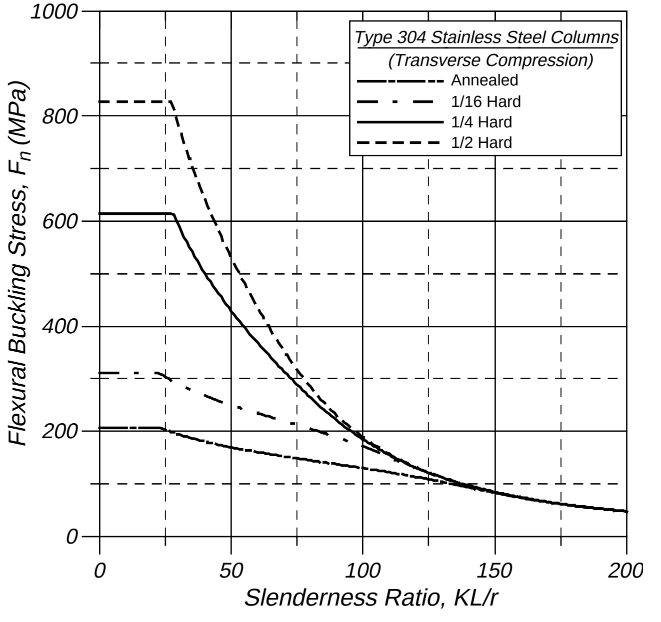


Fig. 3. ASCE Standard flexural buckling stresses for Type 304 stainless steel columns in transverse compression.

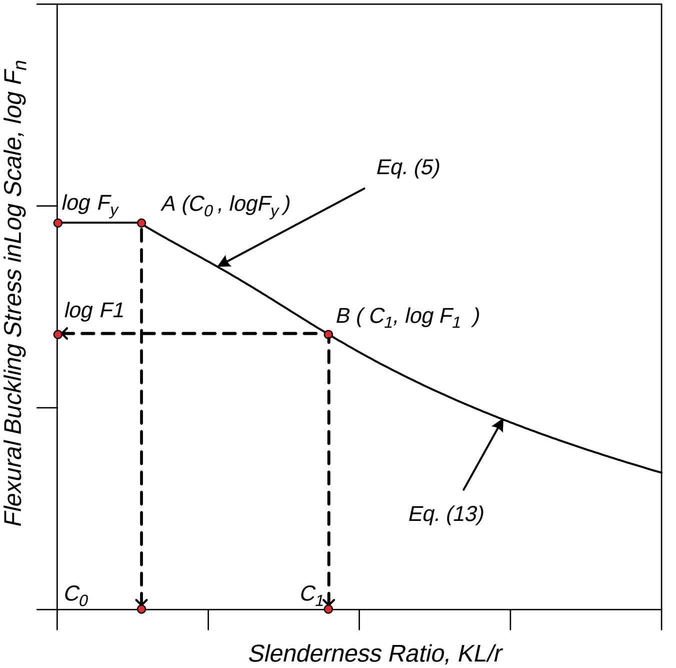


Fig. 4. Simplified flexural buckling stress formulas.

As shown in Fig. 4, it is observed that when applying logarithm to the flexural buckling stress curves, i.e. log(F_n), a portion of the nonlinear buckling stress curve becomes a straight line segment between points A(C_0, logF_y) and B(C_1, logF_1). The linearized portion of the curve can be defined by these two specified points as follows:

$$\frac{\log F_1 - \log F_y}{C_1 - C_0} = \frac{\log F_n - \log F_y}{C - C_0} \tag{4}$$

in which $C = KL/r$ = slenderness ratio, and C_0 and C_1 are two specified slenderness ratios with their corresponding buckling stresses at F_y and F_1 in logarithmic scale, respectively. This linear approximation is beneficial to simplify the procedure of finding the nonlinear buckling stresses.

3.2. Development of formulations

The simplified approach can be achieved by taking the advantage of this linear approximation. New formulations are developed for the corresponding parameters used in the simplified scheme. Eq. (4) can be rearranged as the following simplified form:

$$F_n = F_1^{(C-C_0/C_1-C_0)} \times F_y^{(C_1-C/C_1-C_0)} \tag{5}$$

The parameters in the above equation can be obtained from the flexural buckling stress equation in Eq. (2) and the tangent modulus equation in Eq. (3). As shown in Fig. 4, the slenderness ratio of C_0 in Eq. (5) is determined when F_n is equal to F_y. That is

$$F_n = F_y = \frac{\pi^2 E_y}{(\mathrm{KL}/r)^2} \tag{6}$$

$$\therefore \quad (KL/r)^2 = \frac{\pi^2 E_y}{F_y}$$

Let

$$C_0 = \mathrm{KL}/r = \pi\sqrt{\frac{E_y}{F_y}} \quad (\text{for } F_n = F_y) \tag{7}$$

where E_y is the tangent modulus at yield strength level and is equal to

$$E_y = \frac{E_o}{1 + 0.002\, n \frac{E_o}{F_y}} \tag{8}$$

The buckling stress F_1 given in Eq. (5) is obtained from Eq. (3) by rearranging E_t and F_n and replacing F_n by F_1 as follows:

$$F_1 = \left[\left(\frac{E_o - E_t}{E_t}\right)\frac{F_y}{0.002nE_o}\right]^{1/n-1} \times F_y \tag{9}$$

Let

$$\alpha = \frac{E_o - E_t}{E_t} = \frac{E_o}{E_t} - 1 \tag{10}$$

Then, Eq. (9) becomes

$$F_1 = \left(\frac{\alpha F_y}{0.002nE_0}\right)^{1/n-1} \times F_y \tag{11}$$

The value of F_1 can be considered the proportional limit of stainless steels, which varies with respect to the type of stainless steels.

Based on Eq. (10), the tangent modulus E_t can be expressed in terms of α

$$E_t = \frac{E_o}{(1 + \alpha)} \tag{12}$$

Substitution of Eq. (12) into Eq. (2) yields the following general expression for the flexural buckling stress:

$$\mathrm{F}_n = \frac{\pi^2 E_t}{(\mathrm{KL}/r)^2} = \frac{\pi^2 E_o}{C^2(1 + \alpha)} \tag{13}$$

Then, for the stress level at $F_n = F_1$, the limiting slenderness ratio of C_1 can be obtained as follows:

$$C_1 = \pi\sqrt{\frac{E_o}{F_1(1+\alpha)}} \quad (\text{for } F_n = F_1) \tag{14}$$

The value of C_1 can be considered a limiting slenderness ratio of column buckling as shown in Fig. 4. When the KL/r ratio is greater than this limiting slenderness ratio, the column buckling stress is calculated by Eq. (13); and when the KL/r ratio is smaller than this limiting ratio, the column buckling stress is determined by Eq. (5).

3.3. Approximating expressions

The parameter α plays an important role in finding the buckling stress F_1 in Eq. (11) and the limiting slenderness ratio C_1 in Eq. (14). When the α value is known, the determination of buckling stress F_n in Eqs. (5) and (13) becomes straight forward without iterative calculations. This section presents an approximating technique to evaluate the parameter α used in the equations of the proposed simplified approach.

The tangent modulus in Eq. (3) can be rewritten as

$$E_t = \frac{E_o F_y}{F_y + 0.002nE_o(F_n/F_y)^{n-1}} = \frac{E_o}{1 + 0.002n\left(\frac{E_o}{F_y}\right)\left(\frac{F_n}{F_y}\right)^{n-1}} \tag{15}$$

By comparing Eqs. (12) and (15), the parameter α can be expressed as

$$\alpha = 0.002n\left(\frac{E_o}{F_y}\right)\left(\frac{F_n}{F_y}\right)^{n-1} \tag{16}$$

Substituting the value of F_n in Eq. (13), Eq. (16) can be rewritten as

$$\alpha = 0.002n\left(\frac{E_o}{F_y^n}\right)\left[\frac{\pi^2 E_o}{C^2(1+\alpha)}\right]^{n-1} \tag{17}$$

From the above equation, it is noted that the parameter α can be combined to form a new polynomial function as

$$f(\alpha) = \alpha(1+\alpha)^{n-1} = 0.002n\left(\frac{\pi^2}{C^2}\right)^{n-1}\left(\frac{E_o}{F_y}\right)^n \tag{18}$$

Eq. (18) is a function of α with degree of n (n = constant coefficient, used for determining tangent modulus as specified in ASCE Standard). Due to the complexity of the function, approximating polynomial method is used to solve this equation. Therefore, Eq. (18) can be approximately expressed by using Taylor series expansion as follows:

$$\alpha(1+\alpha)^{n-1} = \sum_{i=0}^{N}\frac{f^i(\alpha)}{i!}\alpha^i + \cdots \tag{19}$$

in which $f_i(\alpha)$is the ith derivative of the function $f(\alpha)$.

The correlation between both sides of Eq. (19) is carefully evaluated in order to obtain a better approximated solution. Higher degrees of derivatives are neglected as common engineering practice. Then, Eq. (19) can be approximately expressed as, for $N=2$,

$$\alpha(1+\alpha)^{n-1} \cong \alpha + (n-1)\alpha^2 \tag{20a}$$

and, for $N=3$,

$$\alpha(1+\alpha)^{n-1} \cong \alpha + (n-1)\alpha^2 + \frac{(n-1)(n-2)}{2}\alpha^3 \tag{20b}$$

Eqs. (20a) and (20b) are numerically evaluated for two specified n values as given in Tables 3 and 4. These calculated values contain $f(\alpha)=\alpha(1+\alpha)^{n-1}$,$\alpha+(n-1)\alpha^2$, and $\alpha+(n-1)\alpha^2+(n-1)(n-2)\alpha^3/2$ for α values varies from 0 to 0.2 at intervals of one-hundredth. As compared with the actual values of $f(\alpha)$, the approximating values calculated from Eq. (20b) provide better agreement than those calculated from Eq. (20a). It is an obvious outcome for this typical series expansion, i.e. the more expanded terms are used; the better approximation can be achieved. In this study, Eqs. (20a) and (20b) are separately used to determine the α value for specified column buckling mode.

3.4. Numerical approximation

It is well known that, for columns having larger slenderness ratio, the elastic buckling failure mode frequently controls the column strength. This buckling stress can be

Table 3
Comparisons of calculated values of Eqs. (20a) and (20b) for $n=4.58$

α	$f(\alpha)=\alpha(1+\alpha)^{n-1}$	$\alpha+(n-1)\alpha^2$	$\alpha+(n-1)\alpha^2+((n-1)\times(n-2)/2)\alpha^3$
0.00	0.00000	0.00000	0.00000
0.01	0.01036	0.01036	0.01036
0.02	0.02147	0.02143	0.02147
0.03	0.03335	0.03322	0.03335
0.04	0.04603	0.04573	0.04602
0.05	0.05954	0.05895	0.05953
0.06	0.07392	0.07289	0.07389
0.07	0.08919	0.08754	0.08913
0.08	0.10538	0.10291	0.10528
0.09	0.12253	0.11900	0.12236
0.10	0.14066	0.13580	0.14042
0.11	0.15983	0.15332	0.15946
0.12	0.18005	0.17155	0.17953
0.13	0.20136	0.19050	0.20065
0.14	0.22379	0.21017	0.22284
0.15	0.24739	0.23055	0.24614
0.16	0.27219	0.25165	0.27056
0.17	0.29823	0.27346	0.29615
0.18	0.32554	0.29599	0.32293
0.19	0.35417	0.31924	0.35091
0.20	0.38415	0.34320	0.38015

Table 4
Comparisons of calculated values of Eqs.(20a) and (20b) for $n = 4.76$

α	$f(\alpha) = \alpha(1+\alpha)^{n-1}$	$\alpha + (n-1)\alpha^2$	$\alpha + (n-1)\alpha^2 + ((n-1) \times (n-2)/2)\alpha^3$
0.00	0.00000	0.00000	0.00000
0.01	0.01038	0.01038	0.01038
0.02	0.02155	0.02150	0.02155
0.03	0.03353	0.03338	0.03352
0.04	0.04636	0.04602	0.04635
0.05	0.06007	0.05940	0.06005
0.06	0.07470	0.07354	0.07466
0.07	0.09028	0.08842	0.09020
0.08	0.10685	0.10406	0.10672
0.09	0.12444	0.12046	0.12424
0.10	0.14310	0.13760	0.14279
0.11	0.16286	0.15550	0.16240
0.12	0.18376	0.17414	0.18311
0.13	0.20583	0.19354	0.20494
0.14	0.22913	0.21370	0.22793
0.15	0.25370	0.23460	0.25211
0.16	0.27956	0.25626	0.27751
0.17	0.30678	0.27866	0.30416
0.18	0.33539	0.30182	0.33209
0.19	0.36544	0.32574	0.36133
0.20	0.39696	0.35040	0.39191

determined by using Eq. (13) with a relatively small value of α. As noted in Tables 3 and 4, the approximating values of $\alpha + (n-1)\alpha^2$calculated in Eq. (20a) are very close to the function values of $f(\alpha) = \alpha(1+\alpha)^{n-1}$ when α value becomes smaller. Therefore, in this case, Eq. (20a) is used to calculate the α value for numerical approximation. Thus, it becomes

$$\alpha + (n-1)\alpha^2 = 0.002n\left(\frac{\pi^2}{C^2}\right)^{n-1}\left(\frac{E_o}{F_y}\right)^n \tag{21}$$

This equation is a typical second order equation and can be solved by quadratic formula

$$\alpha = \frac{-1 + \sqrt{1 + 4(n-1)0.002n\left(\frac{\pi^2}{C^2}\right)^{n-1}\left(\frac{E_o}{F_y}\right)^n}}{2(n-1)} \tag{22}$$

This α is used for determining the buckling stress in Eq. (13) as shown in Fig. 4.

On the other hand, a relatively large α value is found to be needed for calculating the buckling stress in the inelastic buckling mode. Because more accurate approximation is needed for this case, Eq. (20b) is used to determine the value of α. However, this equation cannot be solved with a simple formula. A reasonable estimation is needed to minimize the error of the numerical approximation. It is tentatively recommended that the maximum error between the approximating and actual values be limited to $\pm 0.3\%$. To meet this convergence requirement, the following limitation is required:

$$\frac{(n-1)(n-2)\alpha^3/2}{\alpha+(n-1)\alpha^2} \leq 5\% \tag{23}$$

Assume that the maximum value of the parameter α determined from Eq. (23) is equal to β. It yields

$$\alpha_{\max} = \beta = \frac{1+\sqrt{1+\frac{2(n-2)}{0.05(n-1)}}}{n-2} \times 0.05 \tag{24}$$

in which β is a constant, which depends only on the n value for different types of stainless steels. This β value is the maximum value of α and is used to calculate the buckling stress of F_1 in Eq. (11) and the limiting slenderness ratio of C_1 in Eq. (14).

4. Proposed design formulas

The flexural buckling stress of cold-formed stainless steel compression members can be determined based on the above-mentioned simplified approach. By using the proposed formulas, design calculation is no longer tedious because no iterative process is needed. The following design provisions are proposed herein to determine the flexural buckling stress, F_n, for austenitic types of cold-formed stainless steel compression members.

For doubly symmetric sections, closed cross sections, and any other sections which can be shown not to be subjected to torsional or torsional–flexural buckling, the flexural buckling stress, F_n, shall be determined as follows:

For $\mathrm{KL}/r \leq C_1$:

$$F_n = F_y^{\lambda_o} F_1^{\lambda_1} \leq F_y \tag{25}$$

For $KL/r > C_1$:

$$F_n = \frac{\pi^2 E_o}{\left(\frac{\mathrm{KL}}{r}\right)^2 (1+\alpha)} \tag{26}$$

where:

$$\lambda_o = \frac{C_1 - \mathrm{KL}/r}{C_1 - C_o} \tag{27}$$

$$\lambda_1 = 1 - \lambda_o \tag{28}$$

$$C_o = \pi\sqrt{\frac{E_y}{F_y}} \tag{29}$$

$$C_1 = \pi\sqrt{\frac{E_o}{F_1(1+\beta)}} \tag{30}$$

$$E_y = \frac{E_o}{1 + 0.002n\frac{E_o}{F_y}} \tag{31}$$

$$F_1 = F_y\left(\frac{\beta F_y}{0.002nE_o}\right)^{1/n-1} \tag{32}$$

$$\alpha = \frac{-1 + \sqrt{1 + 4(n-1)0.002n\left[\frac{\pi^2}{(KL/r)^2}\right]^{n-1}\left(\frac{E_o}{F_y}\right)^n}}{2(n-1)} \tag{33}$$

$$\beta = \frac{0.05 + \sqrt{0.0025 + \frac{0.1(n-2)}{(n-1)}}}{n-2} \tag{34}$$

5. Comparisons and discussions

In this study, comparisons are made between the predicted flexural buckling stresses of cold-formed stainless steel columns obtained from the ASCE Standard design equations and the proposed simplified design formulas. Eqs. (2) and (3) are used for the ASCE Standard predictions. The proposed design formulas provided in Section 4 of the paper are used to determine the flexural buckling stresses. It shows that, without having iterative calculations, the proposed equations give satisfactory predictions as compared with the ASCE Standard results.

A commonly used Type 304, austenitic stainless steel column is used for the comparison. The specified material properties of the column are previously given in Table 2. The design parameters for the same materials determined from the proposed design equations are listed in Table 5. For columns with this type of stainless steel, the computed buckling stresses, $F_{n,ASCE}$ and $F_{n,prop}$, and the ratios of $F_{n,prop}/F_{n,ASCE}$ for different slenderness ratios, KL/r, in longitudinal and transverse compression are given in Tables 6 and 7, respectively. In these tables, $F_{n,ASCE}$ and $F_{n,prop}$ are predicted flexural

Table 5
Design parameters used in the proposed design formulas for type 304 stainless steels

Type of stress		β	C_0	C_1	F_1 (MPa)
Longitudinal compression	Annealed	0.1500	32.8	176.6	53.12
	1/16 Hard	0.1500	32.0	137.3	87.94
	1/4 Hard	0.1252	29.9	115.0	123.48
	1/2 Hard	0.1429	30.2	98.4	165.91
Transverse compression	Annealed	0.0526	23.2	136.1	97.72
	1/16 Hard	0.0526	22.9	108.2	154.55
	1/4 Hard	0.1179	27.8	80.5	259.62
	1/2 Hard	0.1270	27.2	67.3	373.58

Table 6
Comparisons of flexural buckling stresses for type 304 stainless steel columns in longitudinal compression

	Annealed			1/16 Hard			1/4 Hard			1/2 Hard		
KL/r	$F_{n,\mathrm{ASCE}}$ (MPa)	$F_{n,\mathrm{prop}}$ (MPa)	$F_{n,\mathrm{prop}}/F_{n,\mathrm{ASCE}}$	$F_{n,\mathrm{ASCE}}$ (MPa)	$F_{n,\mathrm{prop}}$ (Mpa)	$F_{n,\mathrm{prop}}/F_{n,\mathrm{ASCE}}$	$F_{n,\mathrm{ASCE}}$ (MPa)	$F_{n,\mathrm{prop}}$ (MPa)	$F_{n,\mathrm{prop}}/F_{n,\mathrm{ASCE}}$	$F_{n,\mathrm{ASCE}}$ (MPa)	$F_{n,\mathrm{prop}}$ (MPa)	$F_{n,\mathrm{prop}}/F_{n,\mathrm{ASCE}}$
20	193.1	193.1	1.00	282.7	282.7	1.00	344.4	344.5	1.00	448.2	448.2	1.00
40	173.4	181.0	1.04	249.1	258.6	1.04	296.3	305.4	1.03	378.5	388.4	1.03
60	137.4	151.2	1.10	193.7	207.2	1.07	232.1	239.9	1.03	283.4	290.3	1.02
80	114.2	126.4	1.11	156.7	166.0	1.06	186.0	188.4	1.01	215.6	217.0	1.01
100	96.9	105.6	1.09	128.4	133.0	1.04	148.2	148.0	1.00	162.2	161.5	1.00
120	82.9	88.3	1.07	105.0	106.5	1.01	116.0	115.7	1.00	121.3	121.3	1.00
140	71.0	73.8	1.04	85.5	85.1	1.00	90.1	90.1	1.00	91.8	91.8	1.00
160	60.7	61.7	1.02	69.4	69.4	1.00	70.6	70.6	1.00	71.1	71.1	1.00
180	51.7	51.5	1.00	56.7	56.6	1.00	56.3	56.3	1.00	56.5	56.5	1.00
200	44.0	43.9	1.00	46.7	46.7	1.00	45.8	45.8	1.00	45.8	45.8	1.00
AVG			1.05			1.02			1.01			1.01
COV			0.041			0.027			0.013			0.011

buckling stresses determined from the ASCE Standard and proposed design equations, respectively.

Based on the results of comparison, it is observed that the proposed design formulas predict slightly larger buckling stresses than those determined by the ASCE design provisions for Type 304 annealed and 1/16 Hard stainless steel columns having medium range of *KL/r*. The maximum ratio of $F_{n,\mathrm{prop}}/F_{n,\mathrm{ASCE}}$ for annealed type is equal to 1.11 at $KL/r=80$, and for 1/16 Hard, the maximum ratio of $F_{n,\mathrm{prop}}/F_{n,\mathrm{ASCE}}$ is equal to 1.07 at $KL/r=60$ as given in Table 6. These larger predictions of $F_{\mathrm{n,prop}}$ for annealed and 1/16 Hard types are mainly due to the relative low values of F_y and n specified in ASCE Standard. However, for 1/4 Hard and 1/2 Hard, the proposed design formulas can provide good predictions as compared with those obtained from the ASCE design equations. Their

Table 7
Comparisons of flexural buckling stresses for Type 304 stainless steel columns in transverse compression

	Annealed			1/16 Hard			1/4 Hard			1/2 Hard		
KL/r	$F_{n,\mathrm{ASCE}}$ (MPa)	$F_{n,\mathrm{prop}}$ (MPa)	$F_{n,\mathrm{prop}}/F_{n,\mathrm{ASCE}}$	$F_{n,\mathrm{ASCE}}$ (MPa)	$F_{n,\mathrm{prop}}$ (MPa)	$F_{n,\mathrm{prop}}/F_{n,\mathrm{ASCE}}$	$F_{n,\mathrm{ASCE}}$ (MPa)	$F_{n,\mathrm{prop}}$ (MPa)	$F_{n,\mathrm{prop}}/F_{n,\mathrm{ASCE}}$	$F_{n,\mathrm{ASCE}}$ (MPa)	$F_{n,\mathrm{prop}}$ (MPa)	$F_{n,\mathrm{prop}}/F_{n,\mathrm{ASCE}}$
20	206.9	206.9	1.00	310.3	310.3	1.00	613.7	613.7	1.00	827.4	827.4	1.00
40	180.2	185.1	1.03	267.4	269.8	1.01	499.8	504.4	1.01	641.0	641.6	1.00
60	160.3	162.0	1.01	234.3	229.2	0.98	368.1	364.9	0.99	434.9	431.6	0.99
80	144.8	141.9	0.98	204.9	194.6	0.95	264.4	264.0	1.00	284.1	284.0	1.00
100	130.0	124.2	0.96	171.0	165.3	0.97	184.6	184.5	1.00	188.5	188.5	1.00
120	113.6	108.8	0.96	130.5	130.5	1.00	131.2	131.2	1.00	131.9	131.9	1.00
140	93.7	93.6	1.00	97.1	97.1	1.00	97.0	97.0	1.00	97.1	97.1	1.00
160	74.0	74.0	1.00	74.4	74.4	1.00	74.4	74.4	1.00	74.4	74.4	1.00
180	58.8	58.8	1.00	58.8	58.8	1.00	58.8	58.8	1.00	58.8	58.8	1.00
200	47.6	47.6	1.00	47.7	47.7	1.00	47.6	47.6	1.00	47.6	47.6	1.00
AVG			0.99			0.99			1.00			1.00
COV			0.023			0.019			0.004			0.003

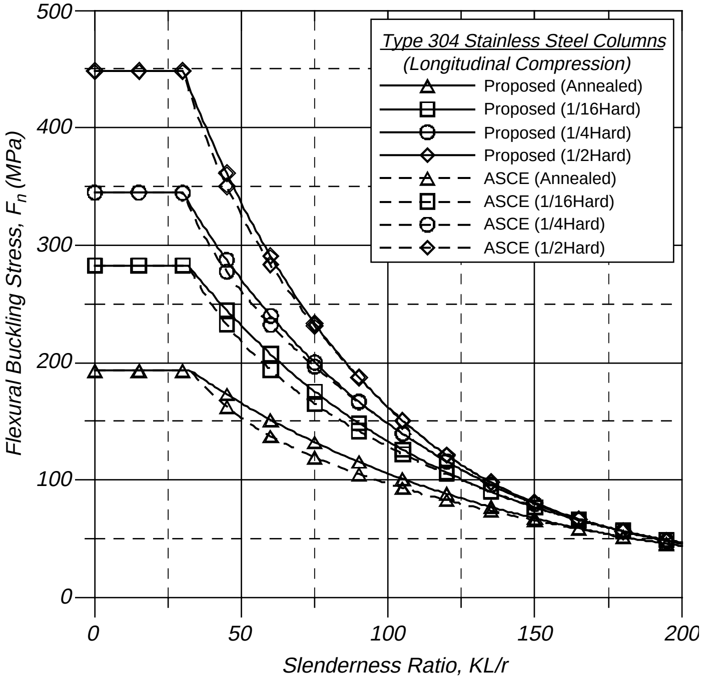


Fig. 5. Comparisons of flexural buckling stress curves in longitudinal compression.

maximum ratio of $F_{n,\mathrm{prop}}/F_{n,\mathrm{ASCE}}$ is equal to 1.03 as given in Table 6. The computed flexural buckling stresses versus slenderness ratios for Type 304 stainless steel columns, annealed, 1/16 Hard, 1/4 Hard and 1/2 Hard in longitudinal compression, are also shown in Fig. 5.

Table 7 gives predicted flexural buckling stresses and their corresponding stress ratios for Type 304 stainless steels columns in transverse compression. It is found that the proposed design equations provide good agreements with the ASCE design formulas for annealed, 1/16 Hard, 1/4 Hard and 1/2 Hard types. The lowest ratio of $F_{n,\mathrm{prop}}/F_{n,\mathrm{ASCE}}$ for annealed type is equal to 0.96 at $KL/r=100$ and 120, and for 1/16 Hard, the lowest ratio of $F_{n,\mathrm{prop}}/F_{n,\mathrm{ASCE}}$ is equal to 0.95 at $KL/r=80$ as given in Table 7. For 1/4 Hard and 1/2 Hard, it is noted that the proposed design formulas provide good predictions as compared with those obtained from the ASCE design equations. Fig. 6 shows the predicted flexural buckling stresses, F_n, versus the slenderness ratio, KL/r, for Type 304 stainless steel columns, annealed, 1/16 Hard, 1/4 Hard and 1/2 Hard in transverse compression.

To demonstrate the adequacy of proposed design formulas discussed above, a design example is included in the appendix of the paper. It provides detailed calculations for determining a cold-formed stainless steel column buckling strength by using the ASCE Standard design equations and the proposed simplified formulas.

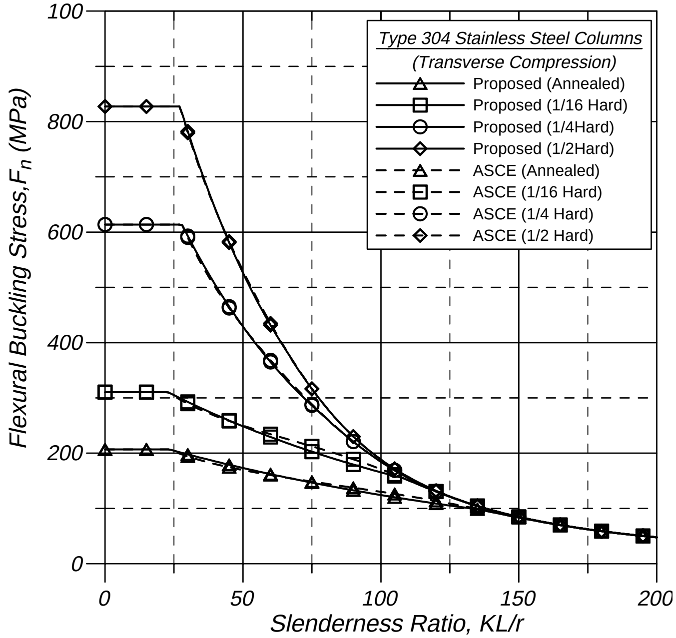


Fig. 6. Comparisons of flexural buckling stress curves in transverse compression.

6. Conclusions

For the design of cold-formed stainless steel compression members, the flexural buckling stress is determined based on the tangent modulus theory. Because of the nonlinear stress-strain behavior of the materials, the determination of flexural buckling stress usually requires iterative process, which is often tedious and time-consuming for a typical column design. In order to simplify the iterative calculation, newly developed formulas utilizing the Taylor series expansion theory are proposed herein to determine flexural buckling stress for austenitic type of cold-formed stainless steel columns. This paper presents detailed derivations of the mathematical formulation and numerical approximation. Comparisons are made between the predicted column flexural buckling stresses determined from he ASCE design formulas and the proposed design equations. It shows that the predicted flexural buckling stresses determined by the proposed design equations are in good agreement with those calculated by the ASCE design formulas. The proposed equations can be used as an alternative to determine the flexural buckling stress for austenitic type of cold-formed stainless steel columns.

Acknowledgements

The financial support provided by the Minghsin University of Science and Technology through the Project Number MUST 93-CE-006 is gratefully acknowledged.

Appendix. Design example

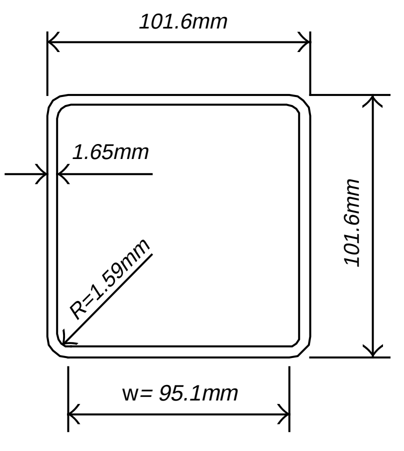
The following design example is to determine the design axial strength of a cold-formed stainless steel column subjected to flexural buckling. The flexural buckling stress used to calculate the design axial strength is determined by the following two methods: (A) the design equations specified in the ASCE Standard Specification, and (B) the proposed simplified design approach presented in the paper.

Given:

1. Square tube dimensions: 101.6 mm × 101.6 mm × 1.65 mm as shown in Fig. 7.
2. Sectional properties: $A = 652.9\ \text{mm}^2$, $I_x = I_y = 1{,}081{,}369\ \text{mm}^4$, $r_x = r_y = 40.7$ mm.
3. Material properties: Type 304, 1/4-Hard stainless steel, $F_y = 344.8$ MPa, $E_o = 186{,}200$ MPa and $n = 4.58$ in longitudinal compression.
4. Effective length: $K_xL_x = K_yL_y = 3048$ mm; and $K_xL_x/r_x = K_yL_y/r_y = 74.9$.

The sectional properties of square tubular section given above are determined by using the information given in Part I of the AISI Cold-Formed Steel Design Manual (2002) [11].

Solution:



Sectional Properties: A = 652.9 mm^2

$I_x = I_y$ = 1081369 mm^4

$r_x = r_y$ = 40.7 mm

Effective Length: $K_xL_x = K_yL_y$ = 3048 mm

Fig. 7. Square tube used for design example.

(A). The ASCE standard specification

The square tube is a doubly symmetric closed section, which is not subjected to torsional or torsional-flexural buckling. The flexural buckling stress, F_n, can be determined by Equation 3.4.1−1 of the ASCE Standard as given in Eq. (2) above. The tangent modulus E_t used for this example is determined by using the Modified Ramberg–Osgood equation as given in Eq. (3). Due to the nature of the iterative process, try-and-error calculations are necessary to determine the buckling stress, F_n.

For the first try, assume a buckling stress of $F_\text{n} = 230$ MPa and its corresponding tangent modulus in Eq. (3) is calculated as

$$E_t = (186200 \times 344.8)/[344.8 + 0.002 \times 4.58 \times 186200 \times (230/344.8)^{3.58}]$$

$$\cong 86167 \text{ MPa}$$

Thus, the computed buckling stress in Eq. (2) is equal to

$$(F_\text{n})_1 = (\pi^2 \times 86167)/(74.9)^2 = 151.6 \text{ MPa} < \text{assumed buckling stress 230 MPa}$$

(N.G.)

Because the computed buckling stress is not close enough to the assumed value, further iteration is needed.

For the second try, it is assumed that $F_\text{n} = 191$ MPa. The corresponding tangent modulus becomes

$$E_\text{t} = (186200 \times 344.8)/[344.8 + 0.002 \times 4.58 \times 186200 \times (191/344.8)^{3.58}]$$

$$\cong 116599 \text{ MPa}$$

The computed buckling stress is equal to

$$(F_\text{n})_2 = (\pi^2 \times 116599)/(74.9)^2 = 205.1 \text{ MPa} > \text{assumed buckling stress 191 MPa}$$

(N.G.)

For the third try, it is assumed that $F_\text{n} = 198.1$ MPa. The corresponding tangent modulus is calculated as follows:

$$E_\text{t} = (186200 \times 344.8)/[344.8 + 0.002 \times 4.58 \times 186200 \times (198.1/344.8)^{3.58}]$$

$$\cong 110817 \text{ MPa}$$

The computed buckling stress is

$$(F_\text{n})_3 = (\pi^2 \times 110817)/(74.9)^2 = 195.0 \text{ MPa} < \text{ assumed stress 198.1 MPa(N.G.)}$$

The buckling stress is repeatedly assumed until it reaches the minimum convergence to the calculated value. After several try-and-error iterations, a buckling stress of $F_\text{n} =$

196.8 MPa is obtained, and the tangent modulus is calculated as

$$E_t = (186200 \times 344.8)/[344.8 + 0.002 \times 4.58186200 \times (196.8/344.8)^{3.58}]$$

$$\cong 111872 \text{ MPa}$$

Thus, the buckling stress is equal to

$$F_n = (\pi^2 \times 111872)/(74.9)^2 = 196.8 \text{ MPa} \qquad \text{(O.K.)}$$

To determine the effective area of the member, the design provisions of Section 2.2 in the ASCE Standard are used to calculate the effective widths of flanges and webs of the tubular section. Based on the results of calculation, it is found that the flanges and webs of the section are not fully effective. The effective area of the tubular section is calculated in accordance with the ASCE Standard provisions as follows:

$$k = 4.0, \quad f = F_n = 196.8 \text{ MPa}$$

$$\lambda = \left(\frac{1.052}{\sqrt{k}}\right)\left(\frac{w}{t}\right)\left(\sqrt{\frac{f}{E_o}}\right) = \left(1.052/\sqrt{4.0}\right)(95.1/1.65)\sqrt{196.8/186200}$$

$$= 0.986 > 0.673 \qquad \text{(A1)}$$

$$\rho = \frac{1 - 0.22/\lambda}{\lambda} = (1 - 0.22/0.986)/0.986 = 0.788 \qquad \text{(A2)}$$

$$b = \rho_w = 0.788 \times 95.1 = 75.2 \text{ mm} \qquad \text{(A3)}$$

$$A_e = A - 4(w - b)t = 652.9 - 4(95.1 - 75.2) \times 1.65 = 521.6 \text{ mm}^2$$

The design axial strength, $\phi_c P_n$, is determined from Eq. (1) as

$$\phi_c = 0.85, \quad P_n = A_e \quad F_n = 521.6 \times 196.8 = 102651 \quad \text{N} = 102.7 \text{ kN}$$

$$(\phi_c P_n)_{\text{ASCE}} = 0.85 \times 102.7 = 87.3 \text{ kN}$$

(B). The proposed simplified approach

By using the proposed design formulas presented in Section 4 of this paper, the flexural buckling stress of cold-formed stainless steel column can be easily calculated without iterations.

(1) Calculate the design parameters:

Design parameters used in this example are calculated from the proposed formulas as follows:

$$\beta = \frac{0.05 + \sqrt{0.0025 + \frac{0.1(4.58-2)}{(4.58-1)}}}{4.58 - 2} = 0.1252$$

$$E_y = \frac{186200}{1 + 0.002 \times 4.58 \times \frac{186200}{344.8}} = 31312 \text{ MPa}$$

$$C_o = \pi\sqrt{\frac{31312}{344.8}} = 29.9$$

$$F_1 = 344.8\left(\frac{0.1252 \times 344.8}{0.002 \times 4.58 \times 186200}\right)^{1/4.58-1} = 123.5 \text{ MPa}$$

$$C_1 = \pi\sqrt{\frac{186200}{123.5(1 + 0.1252)}} = 115$$

(2) Calculate the flexural buckling stress:

Since the column slenderness ratio $KL/r = 74.9 < C_1$, the flexural buckling stress is determined from Eq. (25) as follows:

$$F_n = F_y^{\lambda_o} F_1^{\lambda_1} \le F_y$$

where:

$$\lambda_o = \frac{C_1 - \text{KL}/r}{C_1 - C_o} = \frac{115 - 74.9}{115 - 29.9} = 0.471, \quad \text{and } \lambda_1 = 1 \quad \lambda_o = 0.529$$

Thus,

$$F_n = 344.8^{0.471} \times 123.5^{0.529} = 200.3 \text{ MPa}$$

(3) Determine the effective area:

The effective area of the tubular section is calculated as follows:

$$k = 4.0, \quad f = F_n = 200.3 \text{ MPa}$$

$$\lambda = \left(\frac{1.052}{\sqrt{k}}\right)\left(\frac{w}{t}\right)\left(\sqrt{\frac{f}{E_o}}\right) = \left(1.052/\sqrt{4.0}\right)(95.1/1.65)\sqrt{200.3/186200}$$

$$= 0.994 > 0.673$$

$$\rho = \frac{1 - 0.22/\lambda}{\lambda} = (1 - 0.22/0.994)/0.994 = 0.783$$

$$b = \rho w = 0.783 \times 95.1 = 74.5 \text{ mm}$$

$$A_e = A - 4(w - b)t = 652.9 - 4(95.1 - 74.5) \times 1.65 = 516.9 \text{ mm}^2$$

(4) Find the design axial strength:

The design axial strength, $\phi_c P_n$, is determined as follows:

$$\phi_c = 0.85, \quad P_n = A_e \quad F_n = 516.9 \times 200.3 = 103535 \quad \mathrm{N} = 103.5\ \mathrm{kN}$$

$$(\phi P_n)_{\mathrm{prop.}} = 0.85 \times 103.5 = 88.0\ \mathrm{kN}$$

This design example provides detailed calculations to determine the design axial strength for a typical cold-formed stainless steel tubular column. Two methods are used in this example: (A) the ASCE Standard design equations and (B) the proposed design formulas presented herein. There is no iterative calculation used in the proposed design formulas. The flexural buckling stress calculated from the proposed design formulas is easier than that determined from ASCE Standard design equations. For this design example, the design axial strength determined by using the proposed design formulas is 0.8% larger than that found by using the design equations stipulated in the ASCE Standard.

References

[1] Specification for the design of cold-formed stainless steel structural members, (SEI/ASCE 8-02), American Society of Civil Engineers (ASCE); 2002.
[2] Specification for the design of cold-formed stainless steel structural members, (ANSI/ASCE 8-90), American Society of Civil Engineers (ASCE); 1991.
[3] Johnson AL, Kelsen GA. Stainless steel in structural applications, stainless steel for architecture. American Society for Testing and Materials, STP 454; Aug. 1969.
[4] Yu WW. Cold-formed steel design. New York, NY: Wiley–Interscience; 2000.
[5] Johnston BG, editor. Guide to stability design criteria for metal structures. 3rd ed. New York, NY: Wiley; 1976.
[6] Galambos TV. Structural members and frames. Englewood Cliffs, NJ: Prentice-Hall; 1968.
[7] Ramberg W, Osgood WR. Description of stress-strain curves by three parameters. NACA technical note, No. 902; July 1943.
[8] Hill HN. Determination of stress–strain relations from offset yield strength values. NACA tech. note, No. 927; Feb 1944.
[9] Rasmussen KJR, Rondal J. Column curves for stainless steel alloys. J Construct Steel Res 2000;54:89–107.
[10] Rasmussen KJR, Rondal J. Explicit approach to design of stainless steel columns. J Struct Eng Am Soc Civil Eng 1997;123(7):857–63.
[11] Cold-formed steel design manual. Washington, DC: American Iron and Steel Institute; 2002.