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Heuristic approach for solving the multi-objective facility layout problem

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A new heuristic approach for the generation of preferred objective weights to solve the multi-objective facility layout problem is presented. By applying a multi-pass halving and doubling procedure, a paired comparison method based on the strength of preference among objectives given by the decision-maker is developed. Furthermore, a 'prior test' is proposed to examine the consistency of the paired comparison matrix. An efficient method to transform the inconsistent matrix into a consistent one so the result can closely approximate the decision-maker's original assessments is also offered. The geometric mean method is then employed to obtain the objective weights and the final solution. There are five phases in the proposed heuristic approach. The first generates a basic solution; the second involves constructing a paired comparison matrix by using the multi-pass halving and doubling procedure; the third identifies the consistency of the paired comparison matrix; the fourth transforms the inconsistent matrix into a consistent one; and the fifth generates the preferred weights and obtains the facility layout solution. An illustrative example is given to demonstrate an application of the proposed approach for solving the multi-objective facility layout problem.

Keywords: Multi-objective facility layout (MOFL) problem; Paired comparison; Multi-pass halving and doubling procedure; Consistency

1. Introduction

The design of the facility layout for a manufacturing system is of tremendous importance for its effective utilization. This fact has been emphasized by Sule (1994) and Tompkins *et al*. (1996). A good solution for the facility layout problem contributes to the overall efficiency of operations. A poor layout can lead to the accumulation of work-in-process inventory, overloading of material handling systems, inefficient set-ups, and longer queues (Chiang and Chiang 1998). Furthermore, the facility layout problem represents a costly, long-term investment; hence, modifications that require large expenditures cannot easily be done.

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Re-layout of facilities is not only time-consuming but also disrupts worker activities and the flow of materials (Sule 1994). The essential impact of the facility layout design on the synergistic benefit of a company should not be ignored.

The facility layout problem has attracted the attention of many researchers because of its practical utility and interdisciplinary importance. Traditionally, facility layout approaches have been divided into two categories. The first category is characterized by a quantitative use of material-handling distances and loads to develop a layout which attempts to minimize the total material-handling cost. For example, one of the classical approaches in this category is the CRAFT (Armour and Buffa 1963, Buffa *et al.* 1964). The second category is characterized by a qualitative approach to maximize the overall subjective closeness ratings between various departments. These subjective factors can comprise different qualitative terms, such as ease of supervision and communication, utilization of manpower, safety of workers, flexibility, etc. An effective method in this category is known as an SLP (systematic layout planning) procedure (Muther 1973). For a comprehensive review of the facility layout problem, see Houshyar and Bringelson (1998), Kusiak and Heragu (1987), Liggett (2000), Meller and Gau (1996), and Welgama and Gibson (1995).

However, the aforementioned approaches are usually used individually for solving a facility layout problem. Many researchers have questioned the appropriateness of selecting a single-criterion objective to solve the facility layout problem because qualitative and quantitative approaches each have advantages and disadvantages (Houshyar 1991). The major limitations on quantitative approaches are that they consider only relationships that can be quantified and do not consider any qualitative factors. The shortcoming of qualitative approaches is their strong assumption that all qualitative factors can be aggregated into one criterion. In reality, the facility layout problem must consider both quantitative and qualitative criteria, thereby falling into the category of a multi-objective facility layout (MOFL) problem.

The primary purpose of a multi-objective approach to the facility layout problem is to generate efficient layouts which are then presented to the decision-maker so that the best layout can be selected (Malakooti 1989a). Rosenblatt (1979), Sayin (1981), Dutta and Sahu (1982), Fortenberry and Cox (1985), Malakooti and Ravindran (1985), Waghodekar and Sahu (1986), Malakooti and D'souza (1987), Malakooti (1987, 1989a, b), Urban (1987, 1989), Houshyar (1991), Harmonosky and Tothero (1992), and Chen and Sha (1999) have all developed multi-objective models that consider both qualitative and quantitative terms. Malakooti (1989a) classified three types of methods for solving the MOFL problem: (1) generating a set of efficient layout alternatives and presenting it to the decision-maker (2) assessing the decision-maker's preferences first, then generating the best layout alternative; and (3) using an interactive method to find the best layout alternative.

For type (1) methods, several reported models specify different objective weights for generating the best layout alternative. Rosenblatt (1979) developed a heuristic algorithm that combined qualitative and quantitative aspects of the facility layout problem into a multi-objective model to minimize total material-handling cost and maximize total closeness rating. He constructed an efficient set of alternatives from randomly generated new alternatives, and then specified different weights for these objectives to generate the best layout. Dutta and Sahu (1982) represented the objective function as the difference between material-handling cost and the closeness rating with predetermined weights being assigned to both the objectives. A heuristic algorithm was developed that takes an initial layout and improves it systematically by using a paired exchange routine to generate efficient layout alternatives. Other studies belonging to the type (1) category include those by Sayin (1981), Fortenberry and Cox (1985), Urban (1987, 1989), Khare *et al*. (1988), Malakooti (1989a), Harmonosky and Tothero (1992), and Chen and Sha (1999). All the aforementioned approaches generate new alternatives by using weighting methods. After an efficient set is generated, it can be presented to the decision-maker for evaluation and selection.

For type (2) methods, Malakooti and D'souza (1987) developed a paired comparison method based on the strength of preference among alternatives and generated the objective weights of an assumed linear utility function. This method posed a series of questions to formulate a set of constraints and then used linear programming to solve the problem. Subsequently, a heuristic procedure based on the paired exchange of departments was used to generate an efficient solution associated with a given set of objective weights.

For type (3) methods, Malakooti and Ravindran (1985) presented an interactive process which requires the decision-maker to respond to questions posed by the programmer. Malakooti (1987) developed a human/machine interactive method whereby, using the information obtained through the paired comparison of both alternatives and criteria, the optimum alternative layout is obtained in a few questions.

In this study, the proposed approach falls into the category of type (2) methods in terms of generating efficient solutions. By applying a multi-pass halving and doubling procedure, which is a modification of the halving and doubling procedure (Steuer 1986), the preferred weights are generated from the paired comparison matrix of objectives given by a decision-maker. However, an inconsistency in the paired comparison matrix is inevitable, being caused by the decision-maker's preferences (Saaty 1980, 1988). For solving inconsistent assessments, many different methods have been suggested, e.g. Crawford's (1987) (the geometric mean method), Jensen's (1984) (the least-square method), and Saaty's (1977, 1980) (the eigenvector method). In this study, the proposed approach offers a simple and efficient method for transforming the inconsistent matrix into a consistent one so that the result closely approximates the decision-maker's original assessments. Furthermore, a simple method to test the validity of the transformation is also presented.

The main characteristics of the proposed approach as compared to other approaches for the MOFL problem are that (a) the elements of a paired comparison matrix are based on the strength of preference among objectives, while Malakooti and D'souza's (1987) paired comparison matrix is based on the strength of preference among alternatives (b) the proposed paired comparison matrix of objectives is obtained by the multi-pass halving and doubling procedure rather than by direct assessment (c) a continuous ratio scale is adopted in the decision-maker's judgement, in contrast to Saaty's nine-point discrete scale, and (d) a 'prior test' is proposed to examine the consistency of the paired comparison matrix, which is different from the 'posterior test' proposed by Saaty (1980, 1988).

2. Multi-objective facility layout problem

The facility layout problem has traditionally been formulated as a quadratic assignment problem (QAP). This formulation assigns n (equal-sized) facilities to n mutually exclusive sites (locations). The distance between various locations is measured by a rectilinear distance. Note that the QAP is a special case of the facility layout problem because it assumes all facilities have equal areas, the distance from one site to another can be predetermined, and that all locations are fixed and known a priori, etc. (Meller and Gau 1996). Therefore, the QAP approach is not an application for facility layout problems with unequal-sized facilities (Bozer and Meller 1997). However, the MOFL problem often involves non-commensurate and conflicting objectives. These objectives can be classified into two categories, conflicting and congruent ones (Sayin 1981, Waghodekar and Sahu 1986, Khare *et al.* 1988). For example, conflicting objectives aim at minimization of total flow cost and maximization of total closeness rating, whereas congruent objectives aim at minimization of distance-based cost of several attributes, namely flow, closeness rating, hazardous movements, etc. Moreover, generation and evaluation of the various efficient solutions to the MOFL problem is difficult because of the lack of a suitable measure for effectiveness with respect to multiple objectives (Raoot and Rakshit 1993, Chen and Sha 1999).

Owing to the aforementioned illustrations, in this paper, we consider only those congruent objectives where the cost of an inter-department interaction is based on the distance between the two departments; i.e. the congruent distance-based facility layout problem. The MOFL problem can be formulated as the multi-objective QAP (Problem I), which is shown in equations (1–4) (as used in Malakooti 1987, Malakooti 1989a, Malakooti and D'souza 1987).

Problem I. (multi-objective QAP)

$$\begin{cases} Z_1 = \text{minimize } f_1(\boldsymbol{X}) = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} a_{ijkl}^{1} X_{ij} X_{kl} \\ Z_2 = \text{minimize } f_2(\boldsymbol{X}) = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} a_{ijkl}^{2} X_{ij} X_{kl} \\ \vdots \\ Z_t = \text{minimize } f_t(\boldsymbol{X}) = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} a_{ijkl}^{t} X_{ij} X_{kl} \end{cases} \tag{1}$$

subject to

$$\sum_{i=1}^{n} X_{ij} = 1 \qquad j = 1, \ldots, n \tag{2}$$

$$\sum_{j=1}^{n} X_{ij} = 1 \qquad i = 1, \ldots, n \tag{3}$$

$$X_{ij} \in \{0, 1\} \qquad i, j = 1, \ldots, n \tag{4}$$

where

t = number of objectives,

n = number of departments,

$f_r(\boldsymbol{X})$ = quadratic function of rth objective,

a^r_{ijkl} = coefficient of rth objective function,

$$X_{ij} = \begin{cases} 1 & \text{if department } i \text{ is assigned to location } j. \\ 0 & \text{otherwise.} \end{cases}$$

The problem of finding the layout that minimizes all the objectives can be formulated by a weighted single-objective formulation (Problem II) using the objective weight vector $\boldsymbol{W} = (w_1, w_2, \ldots, w_t)$. Malakooti (1989a) has shown that the solution of Problem II is efficient. Therefore, we can use Problem II for generating the best layout solution associated with the weights.

Problem II. (weighted single-objective QAP)

$$\Phi = \min \left\{ \sum_{r=1}^{t} w_r * f_r(\boldsymbol{X}) \right\} \tag{5}$$

subject to constraints (2–4) of Problem I, where w_r is the constant weight of the rth objective and $\sum_{r=1}^{t} w_r = 1$ ($w_r > 0$, $r = 1, 2, \ldots, t$).

Since solving the QAP for problems with more than fifteen departments takes a lot of computational time by exact methods, heuristics are usually used instead. The approach for solving Problem II might be based on available heuristic methods, such as the paired exchange procedure (e.g. Fortenbery and Cox 1985, Malakooti and D'souza 1987, Chen and Sha 1999). However, evaluation of the various efficient layouts is a difficult task because of the lack of a suitable measure of solution quality with respect to multiple objectives. Chen and Sha's procedure (1999) solved the scaling and measurement problems simultaneously for the MOFL problem. Moreover, they offered also a measure, dominant probability, for assessing the quality of solutions for the MOFL problem. In this paper, therefore, the heuristic procedure of Chen and Sha (1999) is adopted for solving Problem II.

3. Development of a new heuristic procedure

Here, a new five-phase heuristic procedure, belonging to the type (2) methods for solving the MOFL problem, is proposed. There are five phases in the procedure, in which the first generates a basic layout solution through Chen and Sha's procedure (1999) after objective weights and an initial layout are given arbitrarily. The basic layout solution is called the 'Basis'. In the second phase, a paired comparison matrix is constructed by using a multi-pass halving and doubling procedure. The procedure is a modification of one presented by Steuer (1986). The details of the procedure are stated in the Appendix. The third phase tests the consistency of the paired comparison matrix generated in the second phase. The fourth phase constructs a consistent paired comparison matrix when inconsistency occurs. Then the correlation between the original matrix and the resulting one is tested. The fifth phase generates the decision-maker's preferred weights and obtains the final layout solution. The overall structure of the proposed approach for the MOFL problem is illustrated in figure 1. Each phase is further described below.

3.1 *Phase 1: Generation of a basic solution*

In the beginning, initial values for weights and an initial layout are arbitrarily given by the decision-maker. Then Problem II is solved through Chen and Sha's procedure to obtain a basic layout solution called the 'Basis'.

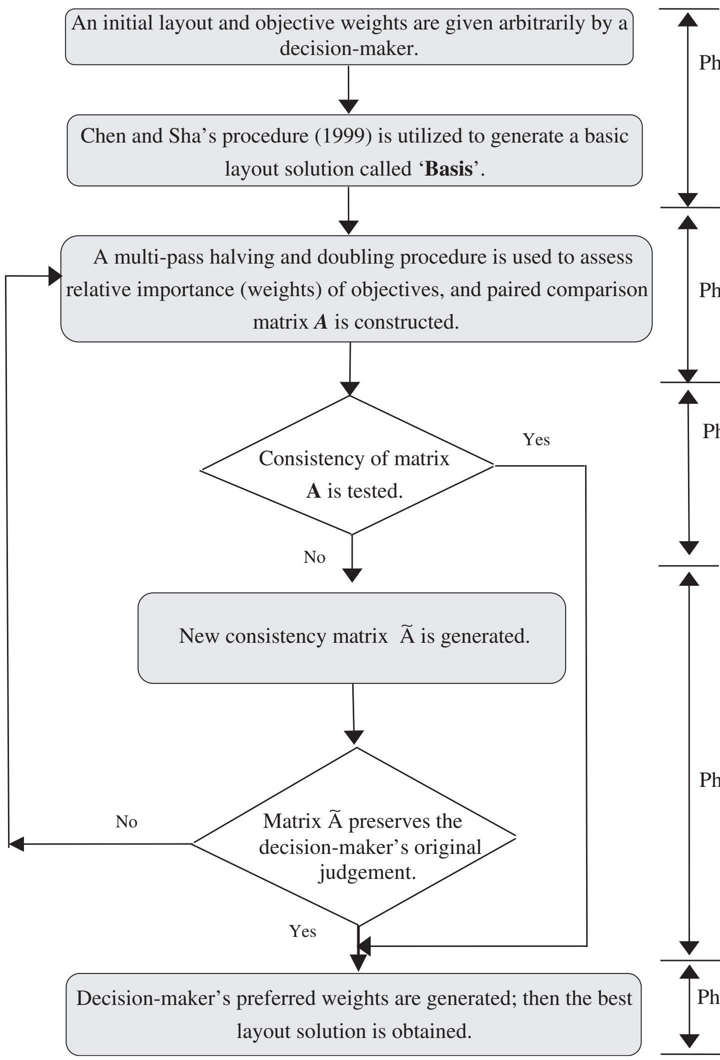

Figure 1. Outline of proposed approach.

3.2 *Phase 2: Construction of paired comparison matrix*

Many methods for solving multi-objective decision-making problems require information about the relative importance of each objective. Saaty (1977, 1980, 1988) developed a method for scaling ratios by using the eigenvector of a paired comparison matrix to generate the weights of the objectives. In Saaty's method, the elements of a paired comparison matrix are all positive and represent the intensity of the decision-maker's preference between individual pairs of objectives. The values of these elements are chosen from a given scale, which usually ranges from 1 to 9 and represents judgmental evaluations such as: 1, equally important; 3, weakly more important; 5, strongly more important; 7, demonstratively more important; and 9, absolutely more important. The even numbers 2, 4, 6 and 8 represent compromising judgements. According to this scale introduced by Saaty (1977, 1980, 1988), the available values for the paired comparison are members of the set: {9, 8, 7, 6, 5, 4, 3, 2, 1, 1/2, 1/3, 1/4, 1/5, 1/6, 1/7, 1/8, 1/9}.

However, it is subjective to assign the intensity between individual pairs of objectives by using Saaty's nine-point discrete scale. In this paper, a new procedure for constructing the paired comparison matrix of objectives by using a multi-pass halving and doubling procedure is proposed. The procedure is interactive for making paired comparisons of objectives rather than direct assessments. Moreover, the decision-maker can use a continuous scale instead of a discrete one so that he/she is capable of making a fine distinction between a pair of objectives.

Let $A = \lfloor a_{ij} \rfloor$ be the positive reciprocal paired comparison matrix of objectives, which is constructed by a decision-maker using the proposed multi-pass halving and doubling procedure. For more details, see the Appendix. The ratio, Δ_i/Δ_j, represents the relative importance between the ith objective and the jth objective.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1t} \\ a_{21} & a_{22} & \cdots & a_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ a_{t1} & a_{t2} & \cdots & a_{tt} \end{bmatrix} = \begin{bmatrix} 1 & \Delta_1/\Delta_2 & \cdots & \Delta_1/\Delta_t \\ \Delta_2/\Delta_1 & 1 & \cdots & \Delta_2/\Delta_t \\ \vdots & \vdots & \ddots & \vdots \\ \Delta_t/\Delta_1 & \Delta_t/\Delta_2 & \cdots & 1 \end{bmatrix}$$

3.3 *Phase 3: Consistency of test*

If the positive paired comparison matrix $\boldsymbol{A}$ constructed in phase 2 is a consistently reciprocal square matrix, it must satisfy the following conditions:

(i) $a_{ij} > 0$ for all i and j,
(ii) $a_{ij} = 1/a_{ji}$ for all i and j,
(iii) $a_{ij} = a_{ir}/a_{jr}$ for all r other than i and j.

However, the judgement in assessing a_{ij} depends on personal experience, knowledge, learning, specific situations and state of mind. Inconsistency might be unavoidable. For example, the decision-maker may consider 'X' to be twice as important as 'Y' and 'Y' three times as important as 'Z', but 'X' only five times as important as 'Z'. This happens due to inconsistency in judgement. Therefore, an efficient procedure for examining the consistency of matrix $\boldsymbol{A}$ is offered. The proposed procedure belongs to a 'prior test', which is different from the 'posterior test' proposed by Saaty (1977, 1980, 1988).

First, divide row i of matrix $\boldsymbol{A}$ by row j ($i < j$).

row i	Δ_i/Δ_1	Δ_i/Δ_2	$\cdots$	Δ_i/Δ_t
row j	Δ_j/Δ_1	Δ_j/Δ_2	$\cdots$	Δ_j/Δ_t
row i/row j	Δ_i/Δ_j	Δ_i/Δ_j	$\cdots$	Δ_i/Δ_j

If all these values are the same, matrix $\boldsymbol{A}$ satisfies the consistency property (condition iii), which can be stated as follows:

$$a_{ij} = \Delta_i/\Delta_j = \Delta_i/\Delta_r \big/ \Delta_j/\Delta_r = a_{ir}/a_{jr} \quad (1 \le i < j \le t;\ r = 1, \ldots, t)$$

Hence, if the evaluations given by the decision-maker pass this test, his/her assessment is consistent. In that case, the objective weights are generated and the final layout solution is obtained in phase 5; otherwise, a method for constructing a consistent matrix is offered in phase 4.

3.4 *Phase 4: Generation of consistency and evaluation*

3.4.1 Part I. Generation of consistency matrix $\tilde{A}$. If the judgement is inconsistent, the value of a_{ij} may be further revised by the decision-maker. However, while making this correction, the decision-maker may create further inconsistencies. Therefore, it is desirable to provide the decision-maker with a set of consistent evaluations which are close to the original ones. Barzilai *et al.* (1987) have demonstrated that the geometric mean method is the only solution to satisfy the consistency property for the problem of retrieving weights from an inconsistent paired comparison matrix whose elements are the relative importance ratios of the objectives. Here, a consistency matrix is generated by a series of revisions of matrix $\boldsymbol{A}$ with the geometric mean method.

Let $\tilde{A}^{(h)}$ be the hth revised matrix, whose elements are $\tilde{a}_{ij}^{(h)}$; then, $\tilde{A}^{(h)} = \boldsymbol{A}$ when $h = 0$. Each element $\tilde{a}_{ij}^{(h+1)}$ of $\tilde{A}^{(h+1)}$ is calculated by:

$$\tilde{a}_{ij}^{(h+1)} = \sqrt[t]{\prod_{r=1}^{t} \left(\frac{\tilde{a}_{ir}^{(h)}}{\tilde{a}_{jr}^{(h)}} \right)} \qquad \text{for } 1 \le i < j \le t;\ h = 0, 1, 2, \ldots. \tag{6}$$

The revised matrix $\tilde{A}^{(h+1)}$ is then subjected to the test of consistency. If the test fails, $\tilde{A}^{(h+2)}$ is calculated. Calculation is continued until consistency is achieved.

3.4.2 Part II. Evaluation of revised matrix $\tilde{A}$. In part I, a method for generating a revised consistency matrix $\tilde{A}$ is presented. It is expected that the revised matrix ($\tilde{A}$) preserves the maximum information (i.e. minimizes change) in the original evaluations ($\boldsymbol{A}$) given by the decision-maker. Hence, a measure is required to assess the effectiveness of matrix $\tilde{A}$. A non-parametric statistical method, Spearman's rank correlation coefficient (r_s), provides a measure of correlation between $\boldsymbol{A}$ and $\tilde{A}$. The value of r_s always falls between -1 and $+1$, with $+1$ indicating perfect positive correlation and -1 indicating perfect negative correlation. The closer r_s approaches to $+1$ or -1, the greater the correlation between $\boldsymbol{A}$ and $\tilde{A}$. Conversely, the nearer r_s is to 0, the less the correlation (McClave and Sincich 2000). A higher value of r_s would mean greater preservation of the decision-maker's judgement. Generally speaking, when the correlation coefficient is greater than 0.7, there exists a highly positive correlation between the original judgement and the revised assessment (Lind *et al.* 2000). Therefore, if the value of r_s is not statistically significant, the proposed approach should revert to phase 2; otherwise, it advances to phase 5.

3.5 *Phase 5: Weight generation and completion of layout*

Saaty (1987, 1980, 1988) proposed the eigenvector method to solve the paired comparison matrix. Crawford and Williams (1985), Crawford (1987), and Barzilai *et al.* (1987) all preferred the geometric mean method for generating the objective weights. Moreover, Crawford and Williams (1985) and Crawford (1987) derived the geometric mean from statistical considerations and demonstrated that it is preferable to the eigenvector solution in several important respects. Hence, after the consistent paired comparison matrix $\boldsymbol{A}$ (or $\tilde{A}$) is derived, the geometric mean method for determining the objective weights is applied in this phase.

Let λ_i be the ith element of the weight vector, which can be calculated:

$$\lambda_i = \begin{cases} \sqrt[t]{\prod_{j=1}^{t} a_{ij}} & \text{if } \boldsymbol{A} \text{ is a consistent matrix} \\ \sqrt[t]{\prod_{j=1}^{t} \tilde{a}_{ij}} & \text{if } \boldsymbol{A} \text{ is not a consistent matrix} \end{cases} \quad \text{for all } i = 1, 2, \ldots, t \qquad (7)$$

The normalized weight w_i can be calculated by

$$w_i = \frac{\lambda_i}{\sum_{j=1}^{t} \lambda_j} \quad \text{for all } i = 1, 2, \ldots, t \qquad (8)$$

4. Illustrative example

To illustrate the use of the proposed approach, an illustrative example is presented and solved. The sample problem is adapted from Waghodekar and Sahu (1986), with additional data generated as needed. The objectives include (1) minimizing the material-handling cost (2) minimizing the numerical rating (3) minimizing material-movement time and (4) minimizing hazardous-material movement distances. Of course, these objectives are merely assumed for demonstrating the proposed procedure.

The five phases of the proposed approach are demonstrated below as a 2×4 structure. The values of the workflow, closeness rating, material-handling time and hazardous-material movement between departments are given in table 1. The distance between department locations is rectilinear, the width of each location being one unit. For computational simplicity, it is assumed that the elements of the move-cost chart are equal to one.

A	B	C	D
E	F	G	H

Phase 1. An initial layout and a weighted vector $\boldsymbol{W} = (0.25, 0.25, 0.25, 0.25)$ are given arbitrarily. By Chen and Sha's procedure (1999) to solve problem II,

Table 1. Example data for a facility layout problem of size 8.

Department	1	2	3	4	5	6	7	8
Workflow								
1	–	6	1	1	8	2	4	4
2	6	–	1	2	3	3	6	2
3	1	1	–	5	2	3	1	10
4	1	2	5	–	2	8	3	3
5	8	3	2	2	–	4	10	10
6	2	3	3	8	4	–	8	8
7	4	6	1	3	10	8	–	2
8	4	2	10	3	10	8	2	–
Closeness rating								
1	–	6	5	5	6	4	5	2
2	6	–	3	5	3	2	6	2
3	5	3	–	6	3	1	2	2
4	5	5	6	–	2	2	3	1
5	6	3	3	2	–	5	6	6
6	4	2	1	2	5	–	6	6
7	5	6	2	3	6	6	–	4
8	2	2	2	1	6	6	4	–
Material-handling time								
1	–	1.5	0.5	1.4	1.5	0.5	1.0	0.6
2	1.5	–	1.5	1.6	1.5	1.0	2.0	1.8
3	0.5	1.5	–	2.0	0.7	3.0	1.5	1.6
4	1.4	1.6	2.0	–	2.2	1.0	0.3	2.0
5	1.5	1.5	0.7	2.2	–	1.5	2.0	0.8
6	0.5	1.0	3.0	1.0	1.5	–	1.4	2.2
7	1.0	2.0	1.5	0.3	2.0	1.4	–	2.5
8	0.6	1.8	1.6	2.0	0.8	2.2	2.5	–
Hazardous movement								
1	–	4	0	0	4	0	0	0
2	4	–	1	0	0	0	4	2
3	0	1	–	0	0	3	0	3
4	0	0	0	–	3	6	2	0
5	4	0	0	3	–	0	0	5
6	0	0	3	6	0	–	2	0
7	0	4	0	2	0	2	–	2
8	0	2	3	0	5	0	2	–

the Basis, whose objective vector $\boldsymbol{Z}^0 = (201, 220, 288.5, 57)$, is obtained. The layout solution for the Basis is as follows:

4	8	5	1
6	3	7	2

Basis

Phase 2. Now the multi-pass halving and doubling procedure (shown in the Appendix) is used to construct the paired comparison matrix $\boldsymbol{A}$.

Iteration 1. Let $\Delta_1 = 10$ and $\Delta_2^{(1)} = 4$. Thus, $\boldsymbol{Z}' = (211, 216, 288.5, 57)$. If $\boldsymbol{Z}^0$ is preferred to $\boldsymbol{Z}'$ then the value of $\Delta_2^{(1)}$ is increased.

Iteration 2. The value of $\Delta_2^{(1)}$ is increased by four so that $\Delta_2^{(2)} = 2\Delta_2^{(1)} = 8$. Thus, $\boldsymbol{Z}' = (211, 212, 288.5, 57)$. If $\boldsymbol{Z}'$ is preferred to $\boldsymbol{Z}^0$, then the value of $\Delta_2^{(2)}$ is decreased.

Iteration 3. The value of $\Delta_2^{(2)}$ is decreased by 2 so that $\Delta_2^{(3)} = 8 - 1/2\delta_2^{(2)} = 8 - 1/4 * \Delta_2^{(2)} = 8 - 2 = 6$. Thus, $\boldsymbol{Z}' = (211, 214, 288.5, 57)$. If $\boldsymbol{Z}^0$ is preferred to $\boldsymbol{Z}'$, then the value of $\Delta_2^{(3)}$ is increased.

Iteration 4. The value of $\Delta_2^{(3)}$ is increased by 1 so that $\Delta_2^{(4)} = \Delta_2^{(3)} + 1/2 * \delta^{(3)} = 6 + 1/2 * (1/2 * \delta^{(2)}) = 6 + 1 = 7$. Thus, $\boldsymbol{Z}' = (211, 213, 288.5, 57)$. If the decision-maker is indifferent between $\boldsymbol{Z}^0$ and $\boldsymbol{Z}'$, $\Delta_2 = \Delta_2^{(4)} = 7$ is taken. Hence, $a_{12} = \Delta_1/\Delta_2 = 10/7$ is obtained. Continue the procedure until all a_{ij} values are calculated. The paired comparison matrix $\boldsymbol{A}$ is assumed to be constructed and given as follows:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 1.4286 & 1.6667 & 1.25 \\ 1/1.4286 & 1 & 1.4 & 0.7778 \\ 1/1.6667 & 1/1.4 & 1 & 1 \\ 1/1.25 & 1/0.7778 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 10/7 & 10/6 & 10/8 \\ 7/10 & 1 & 7/5 & 7/9 \\ 6/10 & 5/7 & 1 & 5/5 \\ 8/10 & 9/7 & 5/5 & 1 \end{bmatrix}$$

Phase 3. In this phase, the consistency of matrix $\boldsymbol{A}$ is examined. Now, row 1 of the matrix $\boldsymbol{A}$ is divided by row 2, the resulting values being 1.4286, 1.4286, 1.1905, and 1.6071. By continuing this procedure, all pairs of rows are calculated, the results of which are shown in table 2. Since elements of each row are not all identical, an inconsistency in matrix $\boldsymbol{A}$ occurs. Hence, it is necessary to generate the revised matrix $\tilde{A}$ in phase 4 by using equation (6).

Phase 4. *Part I*. According to the revised values in table 2, the revised evaluation matrix $\tilde{A}^{(1)}$ is generated as follows:

$$\tilde{A}^{(1)} = \begin{bmatrix} 1 & 1.4057 & 1.6234 & 1.3042 \\ 1/1.4057 & 1 & 1.1548 & 0.9278 \\ 1/1.6234 & 1/1.1548 & 1 & 0.8034 \\ 1/1.3042 & 1/0.9278 & 1/0.8034 & 1 \end{bmatrix}$$

The procedure for checking consistency in phase 3 is repeated in the revised evaluation matrix $\tilde{A}^{(1)}$, the results of which are given in table 3, which shows that matrix $\tilde{A}^{(1)}$ is consistent.

Part II. When the consistency matrix $\tilde{A}^{(1)}$ is generated, Spearman's rank correlation coefficient (r_s) is used to assess the preservation between the original evaluations given by the decision-maker and those revised by the proposed

Table 2. Test of consistency (iteration 1).

Ratio pair	Values				Revised value
row 1/row 2	1.4286	1.4286	1.1905	1.6071	1.4057
row 1/row 3	1.6667	2	1.6667	1.25	1.6234
row 1/row 4	1.25	1.1111	1.6667	1.25	1.3042
row 2/row 3	1.1667	1.4	1.4	0.7778	1.1548
row 2/row 4	0.875	0.7778	1.4	0.7778	0.9287
row 3/row 4	0.75	0.5556	1	1	0.8034

Table 3. Test of consistency.

Ratio pair	Values			
row 1/row 2	1.4057	1.4057	1.4058	1.4057
row 1/row 3	1.6234	1.6233	1.6234	1.6233
row 1/row 4	1.3042	1.3042	1.3042	1.3042
row 2/row 3	1.1549	1.1548	1.1548	1.1548
row 2/row 4	0.9278	0.9278	0.9278	0.9278
row 3/row 4	0.8034	0.8034	0.8034	0.8034

Table 4. Evaluation of preservation of judgement.

Pair	a_{ij}	$\tilde{a}_{ij}^{(1)}$
(1,2)	1.4286	1.4057
(1,3)	1.6667	1.6233
(1,4)	1.2500	1.3042
(2,3)	1.400	1.1548
(2,4)	0.7778	0.9278
(3,4)	1.0000	0.8034

Spearman's rank correlation coefficient (r_s) = 0.8857, $p = 0.025$ (level of significance set at $\alpha = 0.05$).

Table 5. Summary of basis and final layout.

Solution	Layout				W_1 / W_3	W_2 / W_4	Z_1 / Z_3	Z_2 / Z_4	Φ	Dominant probability (%)
Basis	4	8	5	1	0.25	0.25	201	220	191.625	99.9984
	6	3	7	2	0.25	0.25	288.5	57		
Final layout	2	7	6	4	0.3243	0.2307	179	202	172.036	≈100.00
	1	5	8	3	0.1998	0.2452	262.4	61		

method. After the calculation, the value of r_s and the p-value are shown in table 4. Accordingly, a significantly positive correlation is obtained, meaning that the revised evaluation matrix can be accepted by the decision-maker. Subsequently, the weights of the objectives can be generated in phase 5.

Phase 5. By using the revised matrix $\tilde{A}^{(1)}$ and equation (7), the weighted vector $\boldsymbol{\lambda} = (1.3134, 0.9344, 0.8091, 0.9930)$ is obtained. To normalize $\boldsymbol{\lambda}$ using equation (8), the normalized weighted vector $\boldsymbol{W} = (0.3243, 0.2307, 0.1998, 0.2452)$ is obtained. According to $\boldsymbol{W}$ and an arbitrarily initial layout, Problem II is resolved, wherein the best layout solution is obtained. The final layout solution is shown below. Table 5 constitutes a summary of the values obtained for the Basis and the final layout. Table 5 shows that the dominant probability of final layout is extremely approximate to 1, and the decision-maker can consider accepting the solution.

2	7	6	4
1	5	8	3

Final layout

5. Conclusion

This paper has presented a simple and effective approach, belonging to the type (2) methods, for assisting the layout planner in the selection of the best alternative for the MOFL problem. A paired comparison matrix based on the relative importance among objectives was developed. The multi-pass halving and doubling procedure was used to construct the paired comparison matrix. Furthermore, an effective procedure was offered to examine the consistency of the paired comparison matrix. A consistency evaluation matrix was constructed for an existing inconsistency. The correlation between the resulting matrix and the original one was tested to confirm the preservation of the decision-maker's judgement.

The proposed approach is computationally simple, and its underlying concept is rational and comprehensible, while Malakooti and D'souza's (1987) method is sensitive to the ranking of alternative and consistency with the strength of preference. Therefore, it is helpful for assisting the layout planner in selecting good-quality solutions for practical facility layout problems.

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A.1. Appendix

The proposed procedure (multi-pass halving and doubling procedure) is a modification of the halving and doubling procedure (Steuer 1986). This procedure is illustrated for details as follows, with notation identical to that of Steuer (1986) for easy reference.

Step 0: Let $\boldsymbol{Z}^0 = (z_1, \ldots, z_i, \ldots, z_t)$ be the objective vector of the 'Basis'. Let $r = 1$.
Step 1: Specify Δ_r and let $i = r + 1$. Let $a_{rr} = 1$.
Step 2: Let $h = 0$ and specify an initial $\Delta_i^{(1)}$.
Step 3: Let $h = h + 1$. Compare $\boldsymbol{Z}^0$ and $\boldsymbol{Z}' = (z_1, \ldots, z_r + \Delta_r, \ldots, z_i - \Delta_i^{(h)}, \ldots, z_t)$.
Step 4: If $\boldsymbol{Z}^0$ is preferred, decrease the desirability of $\boldsymbol{Z}'$ by letting $\Delta_i^{(h+1)} = 2\Delta_i^{(h)}$. Go to Step 3. If $\boldsymbol{Z}'$ is preferred, let $\delta^{(h)} = \Delta_i^{(h)}/2$ and go to Step 6. If the decision-maker is indifferent between $\boldsymbol{Z}^0$ and $\boldsymbol{Z}'$, let $\Delta_i = \Delta_i^{(h)}$. Go to Step 8.
Step 5: Let $\delta^{(h+1)} = \delta^{(h)}/2$ and let $h = h + 1$. Compare $\boldsymbol{Z}^0$ and $\boldsymbol{Z}'$. If $\boldsymbol{Z}'$ is preferred, go to Step 6. If $\boldsymbol{Z}^0$ is preferred, go to Step 7. If the decision-maker is indifferent between $\boldsymbol{Z}^0$ and $\boldsymbol{Z}'$, let $\Delta_i = \Delta_i^{(h)}$. Go to Step 8.
Step 6: Increase the desirability of $\boldsymbol{Z}'$ by letting $\Delta_i^{(h+1)} = \Delta_i^{(h)} - \delta^{(h)}/2$ and go to Step 5.
Step 7: Decrease the desirability of $\boldsymbol{Z}'$ by letting $\Delta_i^{(h+1)} = \Delta_i^{(h)} + \delta^{(h)}/2$ and go to Step 5.
Step 8: Let $a_{ri} = \Delta_r/\Delta_i$. If $i < t$, let $i = i + 1$ and go to Step 2. Otherwise, go to Step 9.
Step 9: If $r \le t - 1$, let $r = r + 1$ and go to Step 1. Otherwise, stop.

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