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A novel hybrid model for portfolio selection

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Abstract

As we know, the performance of the mean–variance approach depends on the accurate forecast of the return rate. However, the conventional method (e.g. arithmetic mean or regression-based method) usually cannot obtain a satisfied solution especially under the small sample situation. In this paper, the proposed method which incorporates the grey and possibilistic regression models formulates the novel portfolio selection model. In order to solve the multi-objective quadric programming problem, multi-objective evolution algorithms (MOEA) is employed. A numerical example is also illustrated to show the procedures of the proposed method. On the basis of the numerical results, we can conclude that the proposed method can provide the more flexible and accurate results.

Keywords: Mean–variance approach; Portfolio selection; Grey model; Possibilistic regression model; Multi-objective evolution algorithms (MOEA)

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1. Introduction

The mean–variance approach was proposed by Markowitz to deal with the portfolio selection problem [1]. A decision-maker can determine the optimal investing rate to each security based on the sequent return rate. The formulation of the mean–variance method can be described as follows [1–3]:

$$\begin{aligned} &\min \sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}x_i x_j \\ &\text{s.t.} \sum_{i=1}^{n}\mu_i x_i \geqslant E, \\ &\quad \sum_{i=1}^{n} x_i = 1, \\ &\quad x_i \geqslant 0 \quad \forall i = 1,\ldots,n. \end{aligned} \tag{1}$$

where μ_i denotes the expected return rate of security 1, σ_{ij} denotes the covariance coefficient between the *i*th security and the *j*th security, E denotes the acceptable least rate of the expected return.

Although the mean–variance model has been widely used in various portfolio selection problems, some issues should be highlighted to increase the accuracy of this model. It is clear that the accuracy of the mean–variance approach depends on the accurate value of the expected return and the variance-covariance matrix. Several methods have been proposed to forecast the adequate excepted return and variance matrix such as arithmetic mean method [1–3] and regression-based method [4]. Since these methods are based on the theory of large sample, they usually can not obtain a satisfied solution in the small sample situation [5].

In this paper, the grey prediction model is used to predict the further return rate. In addition, we divide the portfolio risk into the uncertainty risk and the relation risk. The uncertainty risk measures the possibilistic degree of the future return rate and the relation risk measures the trending degrees of the sequences. These two risks can be calculated using the possibilistic regression model and the grey relation degree. Next, we can formulate the three-objective quadratic programming model (i.e. achieve the maximum return rate and the minimum uncertainty risk and relation risk simultaneously) to obtain the efficient frontier set using multi-objective evolutionary algorithms (MOEA). To summarize the above descriptions, we can depict the proposed method as shown in Fig. 1.

A numerical example is also illustrated to show the proposed method. On this basis of the numerical results, we can conclude that the proposed method can provide the more flexible and accurate portfolio selection alternatives.

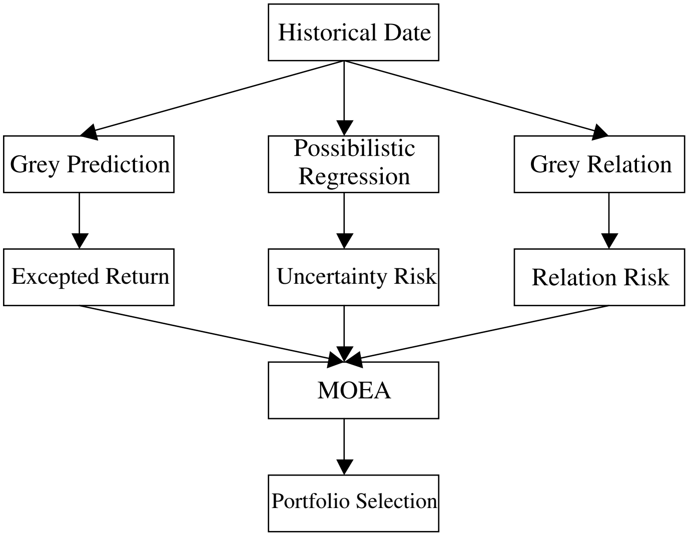

Fig. 1. The procedures of the proposed method.

The remainder of this paper is organized as follows. The grey and possibilistic regression models are discussed in Section 2. Multi-objective evolutionary algorithms is proposed in Section 3. A numerical example is used to illustrate the proposed method in Section 4. The discussions of the numerical results are presented in Section 5 and the conclusions are presented in the last section.

2. Grey and possibilistic regression models

The grey prediction model is proposed to fit the sequence curve under the small sample [6–8] and this method has been recently used in various applications such as stock price [9], and control system [10]. In this paper, the GM (1,1) model, which is most commonly used, is employed to predict the future return rate.

Assume a sequence can be represented as $\boldsymbol{x}^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n))$, then the corresponding first order accumulated generating operation (AGO) series and mean generating operation can be represented as $\boldsymbol{x}^{(1)} = (x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n))$ and $z^{(1)}(k) = 0.5(x^{(1)}(k) + x^{(1)}(k-1))$. Therefore, the grey differential equation of GM (1,1) can be described as

$$x^{(0)}(k) + az^{(1)}(k) = b; \quad \forall k \in \{2, 3, \ldots, k\}.$$

Using the ordinal least square (OLS) method, we can obtain the grey parameter matrix

$$\hat{\boldsymbol{a}} = (\boldsymbol{B}'\boldsymbol{B})^{-1}\boldsymbol{B}'\boldsymbol{Y}_n \quad \text{where} \quad \boldsymbol{Y}_n = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix}, \quad \hat{\boldsymbol{a}} = \begin{bmatrix} a \\ b \end{bmatrix}. \tag{2}$$

Last, the solution of the prediction value can be derived as

$$x^{(0)}(k) = \left(\frac{1-0.5}{1+0.5}\right)^{k-2} \frac{b - ax^{(0)}(1)}{1+0.5a}. \tag{3}$$

Using the grey prediction model, we can predict the future return rate more accurately under the restriction of the small sample.

Next, we use the possibilistic regression [11] to obtain the uncertainty risk of the future return rate. The form of a possibilistic regression can be expressed as

$$\boldsymbol{y} = A_0 + A_1x_1 + \cdots + A_nx_n = \boldsymbol{A}'\boldsymbol{x} \tag{4}$$

where A_i is a symmetrical fuzzy number denoted as $(a_i, c_i)_L$, and the form of the membership function [12] of Eq. (4) can be obtained for $\boldsymbol{x} \neq \boldsymbol{0}$ as

$$\boldsymbol{\mu}_Y(y) = L((y - \boldsymbol{x}'\boldsymbol{a})/\boldsymbol{c}'|\boldsymbol{x}|) \tag{5}$$

for $\boldsymbol{x} = \boldsymbol{0}$ and $y = 0$, $\mu_Y(y) = 1$, and for $\boldsymbol{x} = \boldsymbol{0}$ and $y \neq 0$, $\boldsymbol{\mu}_Y(y) = 0$. The h-level set of y denoted as $[y]_h$ can be obtained as following setting:

$$L((y - \boldsymbol{x}'\boldsymbol{a})/\boldsymbol{c}'|\boldsymbol{x}|) = h \tag{6}$$

Then, $[y]_h$ can be obtained as

$$[y]_h = \left[\left(\boldsymbol{x}'\boldsymbol{a} - |L^{-1}(h)|\boldsymbol{c}'|\boldsymbol{x}|\right), \left(\boldsymbol{x}'\boldsymbol{a} + |L^{-1}(h)|\boldsymbol{c}'|\boldsymbol{x}|\right)\right] = [\boldsymbol{x}^-, \boldsymbol{x}^+] \tag{7}$$

On the basis of the above conditions, we can obtain the formulation of a possibilistic regression model as follows:

$$\min_{a,c} J = \sum_{j=1,\ldots,m} h_j\boldsymbol{c}'|\boldsymbol{x}_j| \tag{8}$$

$$\text{s.t.} \quad y_j \geqslant \boldsymbol{x}_j'\boldsymbol{a} - |L^{-1}(h_j)|\boldsymbol{c}'|\boldsymbol{x}_j|,$$

$$y_j \leqslant \boldsymbol{x}_j'\boldsymbol{a} + |L^{-1}(h_j)|c'|x_j|, \quad j = 1, \ldots, m$$

$$\boldsymbol{c} \geqslant \boldsymbol{0}.$$

Solving the above mathematical programming model, we can calculate the uncertainty risk of the future return rate. Additionally, in order to obtain the relation risk of the security, the grey relational grade [6,7] is employed in this paper. Let two sequences $\boldsymbol{x}_i$ and $\boldsymbol{x}_j$ can be represented as

$$\boldsymbol{x}_i = (x_i(1), x_i(2), \ldots, x_i(k), \ldots, x_i(n))$$

and

$$\boldsymbol{x}_j = (x_j(1), x_j(2), \ldots, x_j(k), \ldots, x_j(n)).$$

Then, the grey relational coefficient can be obtained using the following formulation

$$\gamma(x_i(k), x_j(k)) = \frac{\min\limits_j \min\limits_k |x_i(k) - x_j(k)| + \zeta \min\limits_j \min\limits_k |x_i(k) - x_j(k)|}{|x_i(k) - x_j(k)| + \zeta \min\limits_j \min\limits_k |x_i(k) - x_j(k)|}, \tag{9}$$

where ζ is the grey relation recognition coefficient with numerical value between $[0, 1]$. The ζ can be adjusted for the requirement. In this paper, ζ is set at 0.1 to enlarge the scope of the grey relational coefficient. Finally, the grey relation grade can be expressed as follows

$$\gamma(\boldsymbol{x}_i, \boldsymbol{x}_j) = \frac{1}{n} \sum_{k=1}^{n} \gamma(x_i(k), x_j(k)). \tag{10}$$

After obtaining the results of the grey and possibilistic regression models, then, the proposed method can be formulated in the following mathematical programming equations

$$\max \sum_{i=1}^{n} \mu_i x_i \quad \text{(Excepted Return)} \tag{11}$$

$$\min \sum_{i=1}^{n} \left(x_i^+ - x_i^-\right) \cdot x_i \quad \text{(Uncertainty Risk)}$$

$$\min \sum_{i=1}^{n} \sum_{j=1}^{n} r_{ij} x_i x_j \quad \text{(Relation Risk)}$$

$$\sum_{i=1}^{n} x_i = 1$$

$$x_i \geqslant 0 \quad \forall i = 1, \ldots, n.$$

After solving the mathematical programming model, we can obtain the optimal portfolio selection alternative. However, it is clear that the above equations belong to the three-objective quadratic programming problem and it is hard to obtain the optimal portfolio selection using the conventional methods. In addition, the conventional method provides only one optimal portfolio selection rather than an efficient frontier set. Since the individual investor chooses the optimal portfolio selection based on his preference, the Pareto set should also be provided for various alternatives. In this paper, multi-objective evolutionary algorithms is employed to overcome the above problems.

3. Multi-objective evolutionary algorithms

Multi-objective evolutionary algorithms (MOEA) has been widely used since the 1990's to resolve the combinational problem in various domains such as scheduling [13], engineering [14] and finance [15]. The concept of MOEA is based on the method of genetic algorithms (GA). GA was pioneered in 1975 by Holland, and its concept is to mimic the natural evolution of a population by allowing solutions to reproduce, create new solutions, and compete for surviving in the next iteration [16–20]. Then, the fitness is improved over generations and the best solution is finally achieved.

The procedures of MOEA are similar to GA. The initial population, $P(0)$, is encoded randomly by strings. In each generation, t, the more fit elements are selected for the mating pool. Then, three basic genetic operators, reproduction, crossover, and mutation, are processed to generate new offspring. On the basis of the principle of survival of the fittest, the best chromosome of a candidate solution is obtained. The pseudo codes and the corresponding procedure graph of MOEA can be represented as shown in Figs. 2 and 3.

The power of evolution algorithms lies in its simultaneously searching a population of points in parallel, not a single point. Therefore, evolution algorithms can find the approximate optimum quickly without falling into a local optimum. In the conventional mathematical programming techniques, these methods generally assume small and enumerable search spaces [21]. However, MOEA can handle various function problems such as discontinuous or con-

```
procedure
GA
begin
        t = 0
        initialize P(t)
        evaluate P(t)
        transform fitness vectors into a scalar
        while not satisfy stopping rule do
        begin
                t = t + 1
                select P(t) from P(t − 1)
                alter P(t)
                evaluate P(t)
                transform fitness vectors into a scalar
        end
end
```

Fig. 2. The pseudo code of MOEA.

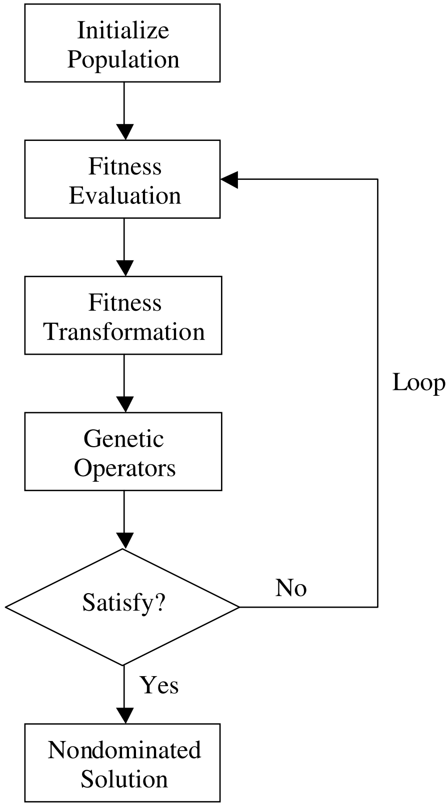

Fig. 3. The procedure graph of MOEA.

cave form and scaling problems [21–23]. In addition, we can obtain the Pareto optimal set rather than a special solution using the method of MOEA.

Next, we describe the three basic genetic operators used in MOEA as follows:

Crossover. The goal of crossover is to exchange information between two parent chromosomes in order to produce two new offspring for the next population. In this study, we use uniform crossover to generate the new offspring. The procedures of uniform crossover can be described as follows. Assume that two parents and a random template are selected by

$$\begin{aligned} \text{Template} &= \quad 0 \quad 1 \quad 0 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1 \\ \text{Parent}_1 &= \quad 1 \quad 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 1 \quad 1 \\ \text{Parent}_2 &= \quad 0 \quad 0 \quad 1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 0 \end{aligned}$$

then, two offspring will be generated as

$$\text{Offspring}_1 = \begin{matrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{matrix}$$
$$\text{Offspring}_2 = \begin{matrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{matrix}$$

Mutation. Mutation is a random process where one genotype is replaced by another to generate a new chromosome. Each genotype has the probability of mutation, P_m, changing from 0 to 1, and vice versa.

Selection. The selection operator selects chromosomes from the mating pool using the "survival of the fittest" concept, as in natural genetic systems. Thus, the best chromosomes receive more copies, while the worst die off. The probability of variable selection is proportional to its fitness value in the population, according to the formula given by

$$P(x_i) = \frac{f(x_i)}{\sum_{j=1}^{N} f(x_j)} \tag{12}$$

where $f(x_i)$ represents the fitness value of the ith chromosome, and N is the population size.

In addition, one of the crucial procedures of MOEA is to determine the fitness function. In this paper, the crowding distance [24,25] is used to sort the chromosomes and determine the Pareto set. In next section, we use a numerical example to illustrate the proposed method.

4. Numerical example

In this section, a numerical example is used to compare between the mean–variance approach and the proposed method. Let the sequent return rates of the six stocks from time $t-6$ to t can be represented as in Table 1. As mentioned previously, in order to obtain the optimal portfolio selection, a decision-maker should forecast the expected return in the $t+1$ period as accurately as possible.

Table 1
The sequences of the six stocks

Period	$t-6$	$t-5$	$t-4$	$t-3$	$t-2$	$t-1$	t
Stock 1	0.07	0.06	0.10	0.08	0.09	0.12	0.14
Stock 2	0.03	0.05	0.11	0.05	0.13	0.14	0.09
Stock 3	0.07	0.11	0.07	0.07	0.05	0.10	0.09
Stock 4	0.06	0.12	0.16	0.08	0.05	0.10	0.12
Stock 5	0.06	0.10	0.09	0.06	0.15	0.07	0.13
Stock 6	0.04	0.01	0.07	0.10	0.11	0.07	0.12

Table 2
Arithmetic mean of the excepted return

Stock	1	2	3	4	5	6
Forecast value	0.09	0.09	0.08	0.10	0.09	0.07

Table 3
Variance-covariance matrix of the excepted return

	Stock 1	Stock 2	Stock 3	Stock 4	Stock 5	Stock 6
Stock 1	0.00027	0.00045	0	0.00004	−0.00036	−0.00008
Stock 2		0.00179	0.00062	−0.00028	−0.00080	−0.00002
Stock 3			0.00112	−0.00008	−0.00040	0.00032
Stock 4				0.000307	−0.000270	−0.00024
Stock 5					0.001707	−0.00016
Stock 6						0.00076

Using the conventional arithmetic mean, we can obtain the further return rates and the variance-covariance matrix of the six stocks as shown in Tables 2 and 3.

Then, we can use the weighted sum method and assume the weights are equal to resolve the mean–variance model to obtain the conventional optimal portfolio selection as shown in Table 4.

Now, we illustrate the proposed method as follows. First, according to the information in Table 1, we can use the grey prediction method, shown in Eqs. (2) and (3), to calculate the future return rate of the six stocks in the $t + 1$ as shown in Table 5.

Next, we can obtain the possibilistic interval (PI) of each stock in the $t + 1$ period using the possibilistic regression model (i.e. Eq. (8)) and also derive the uncertainty risk as shown in Table 6. In order to obtain the relation risk, we can calculate the grey relation matrix using Eqs. (9) and (10) and the corresponding results can be shown as in Table 7.

Table 4
Optimal portfolio selection using the conventional method

Stock	1	2	3	4	5	6	Return Rate	Portfoliorisk
Portfolio	0	0	0	1	0	0	0.10	0.0003

Table 5
The future return rate using the grey prediction model

Stock	1	2	3	4	5	6
Forecast value	0.16	0.13	0.08	0.08	0.12	0.14

Table 6
The possibilistic interval and the uncertainty risk

Stock	1	2	3	4	5	6
PI	(0.10, 0.18)	(0.055, 0.195)	(0.02, 0.14)	(−0.005, 0.205)	(0.019, 0.263)	(0.09, 0.19)
Uncertainty risk	0.08	0.14	0.12	0.231	0.244	0.10

Table 7
The grey relation matrix

$\gamma(\boldsymbol{x}_i, \boldsymbol{x}_j)$	Stock 1	Stock 2	Stock 3	Stock 4	Stock 5	Stock 6
Stock 1	1	0.289	0.471	0.567	0.561	0.438
Stock 2		1	0.341	0.360	0.399	0.408
Stock 3			1	0.557	0.563	0.335
Stock 4				1	0.483	0.379
Stock 5					1	0.396
Stock 6						1

Now, we can formulate the multi-objective mathematical programming based on the above information as the following equations:

$$
\begin{aligned}
&\max 0.16x_1 + 0.13x_2 + \cdots + 0.14x_6\\
&\min 0.8x_1 + 0.14x_2 + \cdots + 0.1x_6\\
&\min x_1^2 + 0.289x_1x_2 + \cdots + x_6^2\\
&\quad x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 1\\
&\quad x_i \geqslant 0 \quad \forall i = 1, \ldots, 6.
\end{aligned}
$$

In order to deal with this three-objective quadratic programming problem, multi-objective evolutionary algorithms is employed in this paper and the corresponding parameter value can also be shown as in Table 8.

Using MOEA, we can obtain the efficient frontier set and the 55 portfolio alternatives as shown in Table 9 or Appendix A.

Table 8
Parameter setting in MOEA

Parameter	Value
Chromosome	Binary
Population size	100
Number of generations	2000
Selection strategy	Tournament
Crossover type	Uniform
Crossover probability	0.8
Mutation probability	0.02

Table 9
Portfolio alternatives of the efficient frontier set

Stock	1	2	3	4	5	6	Return rate	Uncertainty risk	Relation risk
Alternative 1	0.279570	0.116325	0.077224	0.099707	0.203324	0.223851	0.1297	0.1429	0.3762
Alternative 2	0.310850	0.115347	0.077224	0.100684	0.203324	0.192571	0.1303	0.1424	0.3821
Alternative 3	0.371457	0.116325	0.030303	0.099707	0.187683	0.194526	0.1347	0.1379	0.4060
Alternative 4	0.371457	0.124145	0.022483	0.100684	0.187683	0.193548	0.1350	0.1382	0.4063
Alternative 5	0.373412	0.108504	0.053763	0.085044	0.125122	0.254154	0.1356	0.1271	0.4073
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
Alternative 52	0.621701	0.124145	0.022483	0.037146	0	0.194526	0.1476	0.0978	0.5537
Alternative 53	0.623656	0.124145	0.022483	0.037146	0	0.192571	0.1477	0.0978	0.5550
Alternative 54	0.624633	0.107527	0.022483	0.037146	0	0.208211	0.1478	0.0971	0.5595
Alternative 55	0.623656	0.108504	0.022483	0.022483	0	0.222874	0.1487	0.0953	0.5618

On the basis of Table 9, a decision-maker can determine the optimal portfolio alternative based on his preference. Next, we provide the discussion about our numerical example in next section.

5. Discussions

Mean–variance is widely used in the finance area to deal with the portfolio selection problem. However, the conventional method usually fails under the small sample situation. We can describe the shortcomings of the conventional method from its purpose and its theory, respectively, as follows.

The purpose of the mean–variance approach is to determine the $t + 1$ period optimal investing rate to each security based on the sequent return rate. The key is to forecast the $t + 1$ period return rate as accurately as possible. However, it is clear that the arithmetic mean only reflects the average states of the past return rate instead of forecasting. Although many regression-based methods have been proposed to overcome the problem, these methods must obey the assumption of the large sample theory and cannot be used in the small sample situation theoretically. In this paper, we propose the grey and possibilistic regression models to deal with the previously mentioned problem completely.

In order to highlight the shortcoming of the conventional method and to compare it to the proposed method, a numerical example is used. We can depict the sequence of the Stock 4 to describe the irrational results using the arithmetic mean as shown in Fig. 4.

First, it is clear that the sequence shows the dramatically decreasing trend when the sequence rises to the peak. Second, the possibilistic interval is very large in Stock 4. This characteristic shows the large uncertainty risk in Stock

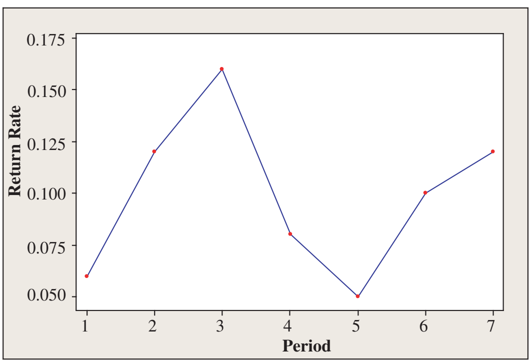

Fig. 4. The sequent graph of the Stock 4.

Table 10
Portfolio alternatives using MOEA

Stock	1	2	3	4	5	6	Return rate	Uncertainty risk	Relation risk
Alternative 1	0.27957	0.116325	0.077224	0.099707	0.203324	0.223851	0.1297	0.1429	0.3762
Alternative 2	0.31085	0.115347	0.077224	0.100684	0.203324	0.192571	0.1303	0.1424	0.3821
Alternative 3	0.371457	0.116325	0.030303	0.099707	0.187683	0.194526	0.1347	0.1379	0.406
Alternative 4	0.371457	0.124145	0.022483	0.100684	0.187683	0.193548	0.135	0.1382	0.4063
Alternative 5	0.373412	0.108504	0.053763	0.085044	0.125122	0.254154	0.1356	0.1271	0.4073
Alternative 6	0.31085	0.233627	0.049853	0.085044	0.00391	0.316716	0.1357	0.1158	0.4086
Alternative 7	0.373412	0.108504	0.053763	0.069404	0.140762	0.254154	0.1362	0.1273	0.4088
Alternative 8	0.357771	0.233627	0.053763	0.085044	0	0.269795	0.1365	0.1144	0.4115
Alternative 9	0.365591	0.233627	0.053763	0.085044	0	0.261975	0.1366	0.1142	0.4126
Alternative 10	0.373412	0.233627	0.053763	0.085044	0	0.254154	0.1368	0.1141	0.414
Alternative 11	0.343109	0.108504	0.053763	0.052786	0.125122	0.316716	0.1369	0.1235	0.4123
Alternative 12	0.373412	0.249267	0.053763	0.069404	0	0.254154	0.1376	0.1127	0.4165
Alternative 13	0.342131	0.233627	0.053763	0.053763	0	0.316716	0.1381	0.1106	0.4201
Alternative 14	0.342131	0.233627	0.049853	0.053763	0.00391	0.316716	0.1382	0.1111	0.4201
Alternative 15	0.373412	0.077224	0.116325	0.022483	0	0.410557	0.1384	0.1009	0.4663
Alternative 16	0.342131	0.233627	0.038123	0.053763	0.01564	0.316716	0.1387	0.1126	0.4202
Alternative 17	0.342131	0.249267	0.049853	0.038123	0.00391	0.316716	0.139	0.1097	0.4236
Alternative 18	0.343109	0.107527	0.053763	0.053763	0	0.441838	0.1393	0.1056	0.4685
Alternative 19	0.374389	0.108504	0.022483	0.052786	0.125122	0.316716	0.1394	0.1222	0.4285
Alternative 20	0.342131	0.108504	0.049853	0.053763	0.00391	0.441838	0.1395	0.1061	0.468
Alternative 21	0.342131	0.233627	0.049853	0.022483	0.00391	0.347996	0.1401	0.107	0.4336
Alternative 22	0.357771	0.233627	0.053763	0.022483	0	0.332356	0.1402	0.1062	0.4336
Alternative 23	0.373412	0.233627	0.049853	0.022483	0.00391	0.316716	0.1407	0.1064	0.4344
Alternative 24	0.373412	0.077224	0.053763	0.022483	0.062561	0.410557	0.1409	0.1087	0.4653
Alternative 25	0.623656	0.108504	0.147605	0.022483	0	0.097752	0.1412	0.0978	0.5483
Alternative 26	0.373412	0.12219	0.022483	0.037146	0.062561	0.382209	0.1414	0.1117	0.4517
Alternative 27	0.373412	0.100684	0.053763	0.022483	0.00782	0.441838	0.1417	0.1017	0.4824
Alternative 28	0.373412	0.108504	0.053763	0.022483	0	0.441838	0.1418	0.1009	0.4828

(*continued on next page*)

Table 10 (*continued*)

Stock	1	2	3	4	5	6	Return rate	Uncertainty risk	Relation risk
Alternative 29	0.373412	0.108504	0.018573	0.053763	0.00391	0.441838	0.142	0.1048	0.4848
Alternative 30	0.373412	0.077224	0.053763	0.022483	0	0.473118	0.1421	0.0996	0.5027
Alternative 31	0.373412	0.124145	0.022483	0.037146	0	0.442815	0.1426	0.1028	0.4855
Alternative 32	0.373412	0.12219	0.022483	0.037146	0	0.44477	0.1427	0.1027	0.4866
Alternative 33	0.373412	0.124145	0.016618	0.038123	0.005865	0.441838	0.1428	0.1037	0.4852
Alternative 34	0.55914	0.092864	0.085044	0.037146	0	0.225806	0.1429	0.0991	0.5092
Alternative 35	0.622678	0.053763	0.092864	0.037146	0	0.193548	0.1441	0.0964	0.5582
Alternative 36	0.498534	0.100684	0.049853	0.022483	0.01173	0.316716	0.1444	0.0997	0.4937
Alternative 37	0.621701	0.124145	0.069404	0.037146	0.01564	0.131965	0.1445	0.101	0.543
Alternative 38	0.561095	0.124145	0.022483	0.068426	0	0.223851	0.1445	0.1032	0.5121
Alternative 39	0.55914	0.116325	0.030303	0.037146	0.062561	0.194526	0.1447	0.108	0.5033
Alternative 40	0.621701	0.053763	0.077224	0.037146	0	0.210166	0.145	0.0961	0.5602
Alternative 41	0.623656	0.053763	0.077224	0.037146	0	0.208211	0.1451	0.0961	0.5615
Alternative 42	0.623656	0.092864	0.069404	0.037146	0	0.176931	0.1452	0.0975	0.5518
Alternative 43	0.621701	0.059629	0.069404	0.037146	0.00391	0.208211	0.1454	0.0968	0.5589
Alternative 44	0.623656	0.108504	0.053763	0.022483	0.062561	0.129032	0.1456	0.1049	0.5463
Alternative 45	0.621701	0.115347	0.030303	0.037146	0.065494	0.13001	0.1459	0.1071	0.5456
Alternative 46	0.622678	0.092864	0.053763	0.037146	0	0.193548	0.1461	0.0972	0.5541
Alternative 47	0.624633	0.100684	0.030303	0.037146	0.062561	0.144673	0.1462	0.106	0.5499
Alternative 48	0.621701	0.124145	0.022483	0.037146	0.064516	0.13001	0.1463	0.1071	0.5457
Alternative 49	0.623656	0.124145	0.022483	0.037146	0.062561	0.13001	0.1464	0.1068	0.5473
Alternative 50	0.623656	0.092864	0.022483	0.037146	0.062561	0.16129	0.1467	0.1056	0.5521
Alternative 51	0.621701	0.116325	0.030303	0.037146	0	0.194526	0.1472	0.0977	0.5533
Alternative 52	0.621701	0.124145	0.022483	0.037146	0	0.194526	0.1476	0.0978	0.5537
Alternative 53	0.623656	0.124145	0.022483	0.037146	0	0.192571	0.1477	0.0978	0.555
Alternative 54	0.624633	0.107527	0.022483	0.037146	0	0.208211	0.1478	0.0971	0.5595
Alternative 55	0.623656	0.108504	0.022483	0.022483	0	0.222874	0.1487	0.0953	0.5618

4. To summarize the above finding, it is risky to invest too much money in Stock 4 over the next period. On the other hand, the proposed method can accurately reflect this characteristic of Stock 4. On the basis of Table 9 or Appendix A, we can conclude that the portfolio selection of Stock 4 should not exceed 10 percent.

In addition, the proposed method can provide the more flexible portfolio alternatives. A decision-maker can select his optimal alternative based on the results of the Pareto set. For example, a risk averse may choose the alternative 1 to obtain the excepted return rate 0.1297. However, a risk lover may choose the alternative 55 to obtain the excepted return rate 0.1487 but a higher risk than a risk averse.

6. Conclusions

Portfolio selection problem has been a popular issue in the finance area since the 1950's. However, the conventional mean–variance method can not provide the satisfied solution under the small sample situation. In this paper, we propose a hybrid method which incorporates the grey and possibisitic regression models to deal with this situation. In order to resolve the three-objective quadric programming, MOEA is employed here. In addition, a numerical example is illustrated to show the procedures of the proposed method. On the basis of the numerical results, the proposed method can provide the more flexible and accurate results.

Appendix A. The full portfolio alternatives can be shown as in Table 10.

References

[1] H. Markowitz, Portfolio selection, Journal of Finance 7 (1) (1952) 77–91.
[2] H. Markowitz, Portfolio Selection: Efficient Diversification of Investments, Wiley, New York, 1959.
[3] H. Markowitz, Mean–Variance Analysis in Portfolio Choice and Capital Market, Basil Blackwell, New York, 1987.
[4] E.J. Elton, M.J. Gruber, Modern Portfolio Theory and Investment Analysis, Wiley, New York, 1995.
[5] E.J. Elton, M.J. Gruber, T.J. Urich, Are Betas Best?, Journal of Finance 33 (5) (1978) 1357–1384.
[6] J. Deng, Control problems of Grey System, Systems and Control Letter 1 (5) (1982) 288–294.
[7] J. Deng, Introduction to Grey System Theory, The Journal of Grey System 1 (1) (1989) 1–24.
[8] J. Deng, Properties of multivariable Grey Model GM (1,1), The Journal of Grey System 1 (1) (1989) 25–42.

[9] Y.F. Wang, Predicting stock price using fuzzy Grey Prediction System, Expert Systems with Applications 22 (1) (2002) 33–39.
[10] C.C. Wong, C.C. Chen, A simulated annealing approach to switching Grey Prediction Fuzzy Control System design, International Journal of Systems Science 29 (6) (1998) 341–352.
[11] H. Tanaka, P. Guo, Possibilistic Data Analysis for Operations Research, Physica-Verlag, New York, 2001.
[12] D. Dubois, H. Prade, Possibility Theory, Plenum Press, New York, 1988.
[13] J. Murata, H. Ishibuchi, H. Tanaka, Mutli-Objective Genetic Algorithm and Its Application to Flowshop Scheduling, Computers and Industrial Engineering 30 (4) (1996) 957–968.
[14] C.M. Fonseca, P.J. Fleming, Multiobjective optimization and multiple constraint handling with evolutionary algorithms-part II: application example, IEEE Transactions on Systems, Man, and Cybernetics, Part A: Systems and Humans 28 (1) (1998) 38–47.
[15] S. Mardle, S. Pascoe, M. Tamiz, An investigation of genetic algorithms for the optimization of multiobjective fisheries bioeconomic models, International Transactions of Operations Research 7 (1) (2000) 33–49.
[16] J.M. Holland, Adaptation in Natural and Artificial Systems, The University of Michigan Press, Ann Arbor, 1975.
[17] D.E. Goldberg, Genetic Algorithms in Search, Optimization and Machine Learning, Addison-Wesley, Reading, MA, 1989.
[18] L. Davis, Handbook of Genetic Algorithms, Van Nostrand Reinhold, 1991.
[19] J.R. Koza, Genetic Programming, The MIT Press, Cambridge, MA, 1992.
[20] Z. Michalewicz, Genetic Algorithms + Data Structures = Evolution Programs, Springer-Verlag, 1992.
[21] C.A. Coello Coello, A.V.V. David, B.L. Gary, Evolutionary Algorithms for Solving Multi-Objective Problems, Kluwer Academic/Plenum, New York, 2002.
[22] K. Deb, Multi-Objective Optimization using Evolutionary Algorithms, John Wiley & Sons, New York, 2001.
[23] M. Gen, R. Cheng, Genetic Algorithms and Engineering Optimization, John Wiley & Sons, New York, 2000.
[24] K. Deb, A. Pratap, S. Agarwal, T. Meyarivan, A fast and elitist multiobjective genetic algorithm: NSGA-II, IEEE Transactions on Evolutionary Computation 6 (2) (2002) 182–197.
[25] M.T. Jensen, Reducing the run-time complexity of multiobjective EAs: The NSGA-II and other algorithms, IEEE Transactions on Evolutionary Computation 7 (5) (2003) 503–515.