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The Phase of a Bose-Einstein Condensate by the Interference of Matter Waves

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We present a calculation which produces the additional phase of a matter wave by applying an adiabatically changing optical field to a double-well Bose-Einstein condensate. An examination of the additional quantum phase is proposed through the comparison of the free expansion interference patterns of the perturbed double-well condensate and the pure double-well condensate.

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I. INTRODUCTION

A physical quantity in quantum mechanics is given by the expectation of its corresponding observable. Usually the phase of the wave function will disappear due to the cancellation of the phase in the wave function and in its complex conjugate. Thus, direct information regarding the phase is not straightforwardly obtainable. However, the phase of a wave function is nontrivial and sometimes has important implications. For example, in the Aharonov-Bohm [1] effect, the wave function for the electron path across a region without a magnetic field will accumulate phase through the nonvanishing vector potential. The interference of wave functions from different paths manifests the flux quantization behavior. Another profound example is Berry's phase. A quantum system under an adiabatic loop change of parameter will get a global phase in the wave function in addition to the dynamical phase, and the former phase depends on the path taken. Since the appearance of Berry's classic paper [2], there are extensive theoretical and experimental investigations [3] on this subject. There are also other many interesting discussions on the phase problem of quantum mechanics [4].

Incidently, since the ingenious experimental realization of Bose-Einstein condensate (BEC) in dilute alkali gases in 1995 [5], BEC experiments are available worldwide now. The order parameter which describes the condensate can be well approximated by the nonlinear Gross-Pitaevskii equation. Because the order parameter is a measurable macroscopic matter wave, the equation provides a convenient and useful test case for the study of the phase in a matter wave. Ketterle *et al.* [6] constructed a double-well condensate by adding a far-blue detuned off-resonant laser light to the cigar-shaped trap. The contrast interference pattern from the free expansion of the two condensates of each trapping well shows that the matter waves are coherent throughout the spatial extent. A theoretical study [7] through the evolution of the Wigner function has been reported.

We present in this paper a way to produce an additional phase into the matter wave by adding the loop change of an additional optical dipole potential to the double-well trap. The *value* of the phase difference of a double-well BEC can be found through the interference pattern. Experimentally, the comparison of free expansion density patterns *with* and *without* the optical potential will provide the information for the phase of the matter wave. This is one of the realistic cases where the quantum mechanical phase can be studied in a quantum system.

The outline of the paper is as follows. In Sec. II, the numerical method for the nonlinear eigenvalue problem and the time-dependent integration are briefly discussed. The criterion of an adiabatic process is justified by numerical calculations with a double-well condensate. In Sec. III, we describe a model to create a global phase difference between the condensates in a double-well. We add a cyclic optical potential to the stationary double-well in order to explore the quantum mechanical phase; the results are presented. Finally in Sec. IV, we give a discussion.

II. NUMERICAL METHODS

Consider N dilute alkali atoms in a cylindrical harmonic trap. The Bose-Einstein condensate (BEC) can be described by the mean-field Gross-Pitaevskii equation (GPE):

$$\left(-\frac{\hbar^2}{2M}\nabla^2+\frac{1}{2}M(\omega_\perp^2x^2+\omega_\perp^2y^2+\omega^2z^2)+\frac{4\pi\hbar^2Na_s}{M}|\Psi|^2\right)\Psi(x,y,z)=\mu\Psi(x,y,z)\,, \quad (1)$$

where a_s is the s-wave scattering length. For the cigar-shaped trap, $\omega_\perp >> \omega$. The transverse motion can be approximated as frozen in the ground state of the 2-dimensional simple harmonic oscillator [8]. Define the transverse length scale $L_\perp = \sqrt{\hbar/M\omega_\perp}$, and integrate out the transverse part; the system is approximated by a one-dimensional nonlinear equation. Further, use the energy unit $\hbar\omega$, length unit L$(=\sqrt{\hbar/M\omega})$, and time unit $\tau = 2\pi/\omega$, and denote $g = 8N(a_s/L)(L/L_\perp)^2$, the GPE becomes :

$$\left(-\frac{1}{2}\frac{d^2}{dz^2}+\frac{1}{2}z^2+g|\Psi(z)|^2\right)\Psi(z)=\mu\Psi(z)\,. \quad (2)$$

Now μ is the chemical potential in units of $\hbar\omega$ and is related to the number of atoms in the condensate. The normalization is

$$N\int_{-\infty}^{\infty}|\Psi(z)|^2dz=\int_{-\infty}^{\infty}\rho(z)dz=N\,. \quad (3)$$

In the following numerical calculations, we use 10000 sodium atoms in the trap with $\omega_\perp = 2\pi \times 250$ Hz, $\omega = 2\pi \times 20$ Hz, and $a_s = 3$ nm [9]. To solve the ground state of the nonlinear equation, we employ the Fourier-Grid-Hamiltonian method [10] iteratively until an accuracy criterion is satisfied. The coordinates range from -12 to 12, set to evenly spaced 401 grid points. The calculated chemical potential is $\mu = 20.78$ for the condensate

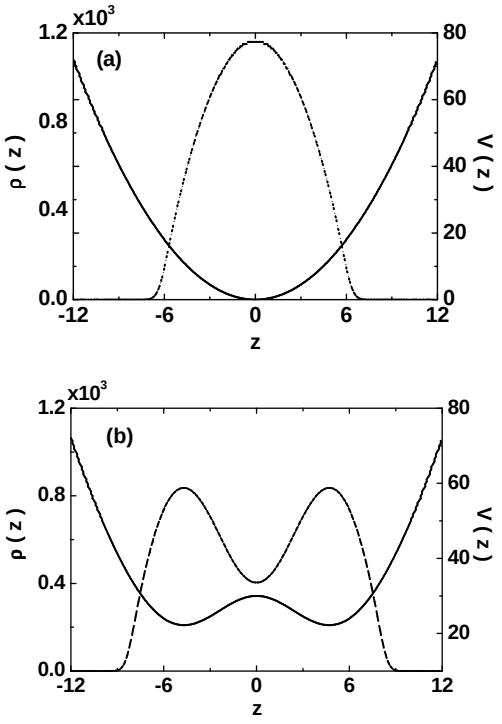

FIG. 1: Potential $V(z)$ (solid line) and density profile $\rho(z)$ (dotted line) of 10000 sodium atoms in a cigar-shaped harmonic trap of frequency $\omega = 2\pi \times 20$ Hz. (a) The condensate; (b) the condensate with a central barrier in the form of Eq. (4) at barrier height $U(t) = 30$.

with the described parameters. The density decays to 10^{-6} at a distance 8.6 from the trap center as shown in Fig. 1(a).

The double-well trap can be constructed by adding a blue-detuned far-off resonant laser to the trap center region. The perturbing potential is modelled as

$$V_b(z,t) = U(t)e^{-\frac{1}{2}(\frac{z}{\sigma})^2}\,. \tag{4}$$

The barrier height $U(t)$ ranges from 0 to 30 and is turned-on and -off linearly with time. The height is much smaller than the trap barrier, but large enough to affect the condensate behavior. In Fig. 1(b), we plot the condensate density profile for the double-well at $U(t) = 30$. At the maximum height of the perturbing potential, the density at the central barrier position is about one half of the peak density, so that the two separated condensates are correlated with each other. For a given initial BEC in a cigar-shaped trap, the condensate is subject to evolution in the trap with the added optical potential at the designed switching rate.

The initial value problem of the time-dependent Gross-Pitaevskii equation,

$$i\frac{\partial\Psi(z,t)}{\partial t} = \int dz'\langle z|\frac{\hat{p}^2}{2}|z'\rangle\Psi(z',t) + \{\frac{z^2}{2} + V_b(z,t) + g|\Psi|^2\}\Psi(z,t)\,, \tag{5}$$

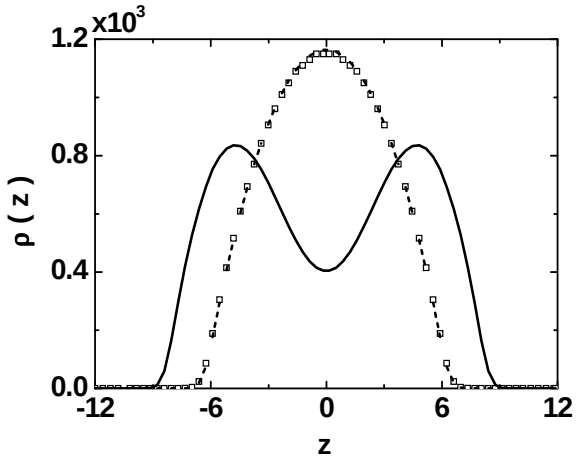

FIG. 2: Density profiles at the initial time (dotted line), the moment of maximum barrier height (solid line), and at the final moment of the perturbing potential (squared curve).

is again discretized by the Fourier-Grid-Hamiltonian method, and then integrated by error controlled routine [11]. Next, we will justify the fact that the adiabatic condition can be achieved by way of turning on the optical potential slowly enough. We take the longitudinal trap frequency (which is 20 Hz) as the intrinsic time scale (which is 50 ms). The adiabatic condition criterion is satisfied if the characteristic time of the additional potential is much larger than 50 ms; otherwise, it will be a fast process. For the adiabatic case, we turn on and off the optical dipole barrier within a time interval of 60.3 s at time step 50.3 μs. The turned-on and -off time interval for the fast process is reduced to 0.5 ms and at time step 50.3 μs.

In Fig. 2 we plot the density profiles of the adiabatic process at the initial time, the moment of maximum barrier height and the final time. The density profile at the moment of maximum barrier height is identical to that of Fig. 1(b). At the other time, we double checked and found that the density calculated by solving the stationary Gross-Pitaevskii equation with additional optical potential, and calculated by the corresponding time-evolving Eq. (5) are identical with each other. This shows that the system changes adiabatically and always stays at the new ground state from time to time. On the other hand, the density profile of the fast process is always identical with the initial one and is not shown here. The results imply that the modelled adiabatic and fast conditions are satisfactory.

As described by the adiabatic theorem in Ref. [12], consider a quantum system that starts from the ground state, when the system changes adiabatically, the system will stay at the ground state of the time-changing hamiltonian due to the slow process. Since our additional potential $V_b(z,t)$ undergoes a cycle of change in time, the hamiltonian returns to the originally one and the density returns to the original ground state. On the other hand, for the fast process, the system still stays in the initial state while an additional barrier rapidly undergoes a cycle of change. We found that the basic principle of the adiabatic

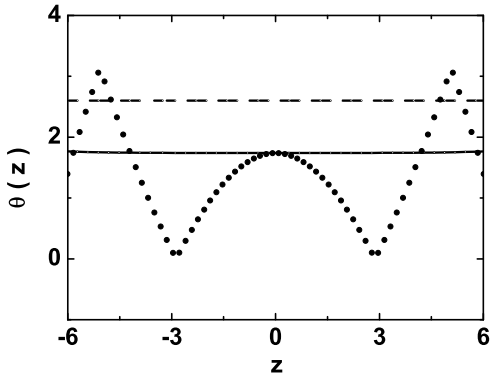

FIG. 3: The phase functions of the order parameter after a loop of perturbation for adiabatic (dashed line), nonadiabatic (dotted line) cases, respectively; also shown is the additional phase (solid line) of the adiabatic process.

theorem is still valid with the nonlinear GPE, in this case.

During the time-dependent process, if the adiabatic condition is satisfied, or the process is fast, the accumulated phase will be global after a cycle of parameter change. Otherwise, it will produce a position dependent phase function (that is, a local phase). In Fig. 3, we plot the phase with respect to the initial state for the adiabatic, and the nonadiabatic processes at the moment the optical barrier vanishes. The switching of the adiabatic and fast cases are described above. For the nonadiabatic case, the turned-on and -off time interval is changed to 50.3 ms. Thus, the pulse duration ratio of adiabatic, nonadiabatic and the fast case is $1.2 \times 10^5 : 100 : 1$. We see that well-defined global phases are shown for the adiabatic cases, while the phase function for the nonadiabatic case is local. Furthermore, in the adiabatic process, the additional global phase of the matter wave is equal to the difference of the total phase and the dynamic phase $\int \mu(t)dt$ over the loop. Also shown in Fig. 3 is the additional phase for the adiabatic process with a perturbing potential having the form of Eq. (4).

III. THE METHOD OF PRODUCING A PHASE DIFFERENCE BETWEEN CONDENSATES

In the above calculation, we have settled down numerically the condition for the adiabatic process and obtained the global phase. However, the phase so obtained is unmeasurable. We propose, in the following, a mechanism to produce a measurable phase for verification. First we start with a double-well condensate, which is built by adding a stationary optical barrier to the central region of a cigar-shaped trap as described above. Here we model the central barrier as

$$V_b = U_0 e^{-\frac{1}{2}(\frac{z}{\sigma})^2}, \tag{6}$$

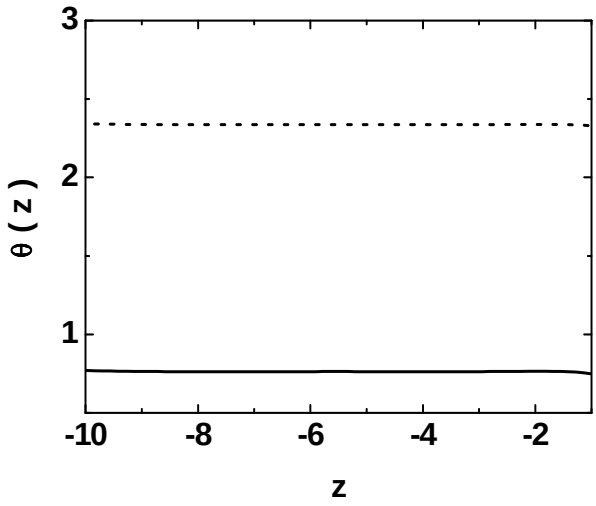

FIG. 4: The phase functions at the end of adiabatically added perturbing potential to the left well. Since the central barrier of the double-well trap is much higher than the left-well perturbation, the additional phase (solid line) appears only in the left-well order parameter. The dashed line is the corresponding total phase.

with $U_0 = 50$, $\sigma = 18$. The higher barrier is to reduce the effect of the upcoming perturbation in the left well on the right- hand side condensate, so that an additional phase will appear in the left well condensate only when we add a perturbing optical potential to the left well. The exact values of the related parameters are actually not critical. In the next step, we add a time-dependent smaller Gaussian potential barrier only to the left well. The form is just like V_b above but with potential maximum equal to 4, $\sigma = 0.5$, and duration 8.04 s. After the adiabatic evolution of the system, the calculated additional phase is 0.763 rad. as shown in Fig. 4. Since the central barrier is much higher than the barrier added to the left well, the additional phase will appear only in the order parameter of the left side condensate of the double well.

Next, to measure this additional phase, we remove the trap potential and barrier potential after the disturbance to the left well. The condensates start to expand freely. Before the time of overlap, two individual condensates are shown in Fig. 5. When the two condensates overlap with each other, the interference pattern of matter waves shows up. It has previously been discussed that the interference fringe period is the de Broglie wavelength and is proportional to the temporal distance from the moment of potential relaxation [7]. This is a common phenomenon for waves and was reported for double-well condensates [6]. Especially in the case that there is a further difference of the additional phase in the right and left condensates, the pattern is different from that of purely double-well BEC free expansion. In Fig. 5 we depict the free expansion interference patterns of a pure double-well BEC and those with an additional phase. At $t = 30.2$ ms, two separate condensates are shown in both cases. At $t = 65.3$ ms, both cases show interferences pattern but they are displaced from each other due to the additional phase. At later time, $t = 80.4$

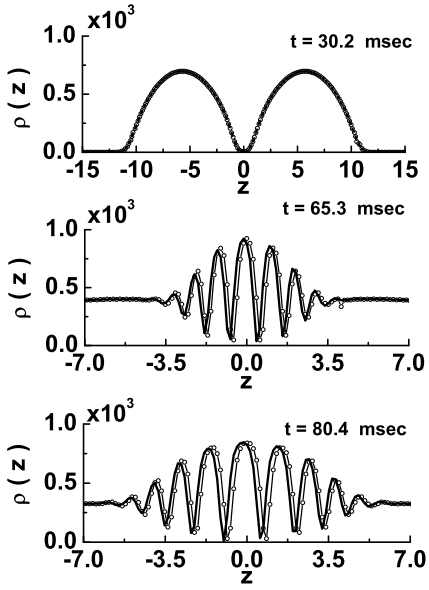

FIG. 5: Free expansion interference patterns for the pure double-well BEC (dotted line) and the two condensates with a difference of the additional phase (solid line).

ms, the fringe period is larger than that of $t = 65.3$ ms.

Since the interference pattern of the pure double-well condensates was known [6], the measurement of similar interference of double-well condensates with an additional phase would be feasible. Comparison of the two interference patterns will provide an experimental result of the additional quantum phase.

IV. DISCUSSIONS

Through the time-dependent integration of the nonlinear GPE, we investigate the adiabatic condition to obtain the global additional quantum phase for matter waves in a double-well condensate under a perturbed optical potential cycle. We then propose a way to measure it. Next, our result for the obtained quantum phase can be estimated below. Consider the simple case of two colliding sinusoidal waves *with* and *without* a relative phase difference ϕ, the two interference patterns will show displaced patterns and the phase ϕ can be read out from the interference pattern. The method was used to show that the coherence of two condensates extend in the double-well trap [6]. Similarly, from the displacement of interference patterns described in Fig. 5, we can estimate their phase difference through $\phi = 2\pi\Delta z/\lambda$, where Δz is the displacement of corresponding interference peaks in pure double-well condensate and BEC with the additional quantum phase, and λ is the wavelength of interference pattern. The estimated value is about 0.78 rad. The estimation fits quite well to the calculated additional phase obtained from the numerical integration of the

time-dependent GPE. Finally, the parameters we used in our modelling are not critical, with some changes to fit the experimental environments they will give the same qualitative conclusion. The proposed way would provide a method to measure the phase in quantum mechanics.

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References

[1] Y. Aharonov and D. Bohm, Phys. Rev. **115**, 485 (1959).
[2] M. V. Berry, Proc. R. Soc. London, Ser. A **392**, 45 (1984).
[3] A. Shapere and F. Wilczek eds., *Geometric Phases in Physics,* (World Scientific, Singapore, 1989).
[4] P. W. Anderson, in *The Lesson of Quantum Theory*, J. D. Boer, E. Dal, O. Ulfbecks, Eds. (Elsevier, Amsterdam, 1986); A. J. Leggett and F. Sols, Found. Phys. **21**, 353 (1991); P. R. Berman ed., *Atom Interferometry* (Academic Press, New York, 1997).
[5] M. H. Anderson *et al.*, Science **269**, 198 (1995); C. C. Bradley *et al.*, Phys. Rev. Lett. **75**, 1687 (1995); K. B. Davis *et al.*, Phys. Rev. Lett. **75**, 3969 (1995).
[6] M. R. Andrews, C. G. Townsend, H.-J. Miesner, D. S. Durfee, D. M. Kurn, and W. Ketterle, Science **275**, 637 (1997).
[7] H. Wallis, A. Röhrl, M. Naraschewski, and A. Schenzle Phys. Rev. A **55**, 2109 (1997).
[8] Y. S. Kivshar, T. J. Alexander, and S. K. Turutsyn, Phys. Lett. A **278**, 225 (2001).
[9] C. C. Bradley, C. A. Sackett, J. J. Toellett, and R. G. Hulet, Phys. Rev. Lett. **75**, 1687 (1995).
[10] Shih-I Chu, Chem. Phys. Lett. **167**, 155 (1990); C. C. Marston and G. G. Balint-Kurti, J. Chem. Phys. **91**, 3571 (1989).
[11] NAG Fortran Library Mark 17, NAG Ltd. (Oxford, UK, 1995).
[12] D. J. Griffiths, *Introduction to Quantum Mechanics* (Prentice Hall, New Jersey, 1995), Chapter 10.