Design of a New Fuzzy Suction Controller Using Fuzzy Modeling for Nonlinear Boundary Layer

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***Abstract*—There are two types of fuzzy modeling: 1) imitating an expert experiment or fulfilling an engineering knowledge, and 2) modeling a complex or unknown system. In this paper, based on the first type of fuzzy modeling, a new fuzzy suction controller (NFSC) is proposed using its linguistic rules to design nonlinear boundary layer. Two kinds of nonlinear boundary layers are discussed. The first kind is designed by three rules derived according to a new interpretation of the switching conditions for a suction controller such that the new controller reduces chattering and spends less energy than a suction controller does. A design procedure summarizes the NFSC design. The second kind of nonlinear boundary layer is the linguistic rules designed to have sliding sectors to control a mobile robot for trajectory tracking. The discussion emphasizes the advantage of nonlinear boundary layers, compared with traditional suction controllers usually using linear boundary. In addition, the proposed NFSC provides a flexible way to adjust the controller functions using linguistic rules based on the first type of fuzzy modeling.**

***Index Terms*—Fuzzy logic controllers, fuzzy modeling, motion control.**

I. INTRODUCTION

BASICALLY, there are two types of fuzzy modeling: 1) imitating an expert experiment or fulfilling an engineering knowledge, and 2) modeling a complex or unknown system. This paper focuses on applying the first type of fuzzy modeling to propose a new fuzzy suction controller (NFSC), which has nonlinear boundary layer for nonlinear systems. The proposed NFSC is dedicated to improve traditional suction controllers which used linear boundary layer and resulted in chattering due to the fact that the system states may not be sucked onto a switching line. This paper discusses the first type of fuzzy modeling for the improvement of a suction controller design.

The suction controller design is motivated by variable structure systems (VSS). After Utkin introduced variable structure systems in 1977 [1], VSS has become a popular tool for controller design in nonlinear systems. The basic principle involves switching a system between two distinctively different structures. Thus system uncertainty, parameter variation and disturbance can be handled within certain boundaries. Handling these boundaries is easier than handling their values using the traditional approaches. This is the main advantage of VSS.

VSS switches system structures such that their states hit the designed switching line (or surface) first, and then slide toward the desired states. This is a kind of trajectory of system states, called the sliding mode. A controller that fulfills the sliding mode is called a sliding-mode controller (SMC). Due to incomplete analysis, a SMC may produce chattering phenomena. Many analytical design methods were proposed to reduce the chattering effects [2]–[6]. Slotine [7] designed a boundary layer, formed by two boundary lines, to reduce chattering. When system states are outside the boundary layer, the controller output is either the upper bounded value (U^{+}) or the lower bounded value (U^{-}). Inside the boundary layer, the value of the controller output is proportional to the distance between the states and the switching line, which is a linear gain designed to suck the system states within the boundary layer and then toward the desired states. Thus the SMC is also called a suction controller.

However, it is not sufficient to use linear gain within a boundary layer for sucking system states on the switching line. Such insufficiency may result in system states moving away from the boundary layer. Thus, in [8], a sliding sector in the vicinity of the switching line was proposed. The sliding sector is the boundary layer of a suction controller designed with a nonlinear gain. However, the nonlinear gain is only designed to guarantee that system states will not go out of the boundary layer, i.e., the sliding sector. Controlling system states to slide on the switching line was neglected. The proposed sliding sector victimizes system performance for chattering reduction. In [9], chattering free and fast response smooth SMC was also proposed. However, the smooth SMC only smoothes the boundary lines at the boundary layer. In this paper, an innovative technique under a new interpretation of SMC stability is introduced to propose three rules for a NFSC design such that system stability is guaranteed and chattering is also reduced.

Although many researchers proposed various fuzzy sliding mode control (FSMC) methods [10], [11], and applied to different applications [12]–[14], the proposed NFSC designed for a nonlinear boundary layer adopts different approach. The NFSC also has the capability of smoothing nonlinear boundary layer. The smoothing functions had been proposed and demonstrated in [15], [16]. In [15], fuzzy rule based algorithms are used to smooth control input for the boundary manifolds of a VSS. In [16], three different fuzzy controllers were designed

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based on variable structure techniques. Fuzzy sliding mode methods usually use the distance between the system states and a switching line as the linguistic variable (the input of the FLC), but the NFSC uses the state variables as control inputs such that its output can be adjusted locally to suck system states on a switching line, and then to design a chattering-free SMC.

The fuzzy suction controller (FSC) was also proposed in [17], which is similar to the proposed NFSC. However, they have different objectives. The FSC is a fuzzy logic controller (FLC) that imitates a suction controller, but the NFSC is proposed using linguistic rules to produce better functions than a suction controller. In general, a suction controller is designed with zero output during system states on the designed switching line, but, on the switching line, the system dynamics are not zero or even far from zero. Based on a new interpretation of SMC stability, three rules, one for near a switching line and two for the both sides of a switching line, are proposed to design the linguistic rules for a NFSC controller output near the system dynamics. The NFSC rely on the first type of fuzzy modeling to design a controller that its output is just enough to suck system states and then prevent crossing over a switching line. The just enough controller output makes the NFSC spend lesser energy to maintain sliding conditions.

The NFSCs are also designed in this paper to control a mobile robot for following a trajectory, which is formed via fuzzy potential energy (FPE) proposed in [18]. This demonstration shows the fact that the first type of fuzzy modeling via designed linguistic rules can be flexible enough in constructing a variety of NFSCs for trajectories tracking.

This paper is organized as follows. In Section II, designing just enough controller output to hold switching conditions is formulated by a new SMC stability interpretation. A NFSC design is derived in Section III, and summarized by a procedure. A NFSC is designed for tracking in trajectory in Section IV. Conclusions are presented in Section V.

II. Problem Formulation

This paper is devoted to propose a NFSC controller based on a new point of view for reducing SMC chattering. In this section, the new point of view can be revealed by first describing the chattering phenomenon of SMC design.

The chattering phenomenon usually results from a SMC system in unstable situations that system states cross over a switching line. Therefore, sufficient conditions of SMC system stability are revealed to present how to guarantee system states sliding a switching line toward the desired state. It is well known that the sufficient conditions for SMC stability are the solutions to the following inequality:

$$S\dot{S} < -\eta \|S\| \tag{1}$$

where S is the designed switching line, and η is the convergence ratio of system states on the switching line S. When the system states are far from the switching line S, larger $\eta\|S\|$ will result in a faster forward move toward S. However, too large $\eta\|S\|$ may push the system states to cross S when they are near S. After the system states cross S, the SMC will have a reverse command to control the system states to go backward S, but crossing S may occur once again. This slightly unstable phenomenon of switching system states back and forth along the switching line S is called chattering. Therefore, maintaining systems states to stay on one side of the switching line as going toward the desired states can completely reduce chattering.

Consider a second-order nonlinear system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f(X) + u \end{aligned} \tag{2}$$

where $X^T = [x_1, x_2]$ are the state variables, and u is system input. Thus, the nonlinear system can be represented by

$$\dot{X} = F(X) + Bu \tag{3}$$

where $F^T(X) = [x_2, f(X)]$, and $B^T = [0, 1]$. A variable structure control law is of the form

$$u = -\alpha(S)\text{sgn}(S) \tag{4}$$

where S is a switching line, and $\alpha(S)$ is a gain function for controlling system states toward switching line S. Let

$$S = c_1x_1 + c_2x_2 = C^TX = 0 \quad \text{where } c_1 > 0, c_2 > 0 \text{ for SMC.} \tag{5}$$

The following Lemma shows the upper and lower bounds of $\alpha(S)$ to guarantee the system stability.

Lemma 1: The nonlinear system (3) and the variable structure control law (4) satisfy the switching condition, if

$$\alpha(S) > \frac{C^TF(X)}{C^TB} \quad \text{when } S > 0 \tag{6}$$

and

$$\alpha(S) > -\frac{C^TF(X)}{C^TB} \quad \text{when } S < 0. \tag{7}$$

Proof: Since $S = C^TX$, thus

$$\begin{aligned} S\dot{S} &= SC^T\dot{X} = SC^T(F(X) + Bu) \\ &= SC^T(F(X) - B\alpha(S)\text{sgn}(S)) \\ &= S(C^TF(X) - C^TB\alpha\text{sgn}(S)). \end{aligned} \tag{8}$$

For $S > 0$, (8) becomes

$$S\dot{S} = \|S\|(C^TF(X) - C^TB\alpha(S)).$$

Thus, if

$$\alpha(S) > \frac{C^TF(X)}{C^TB}.$$

Then, $S\dot{S} < -\|S\|$, since $C^TB = c_2 > 0$.

Next, when $S < 0$, (8) can be rewritten as

$$S\dot{S} = -\|S\|(C^TF(X) + C^TB\alpha(S)).$$

Therefore, if

$$\alpha(S) > -\frac{C^TF(X)}{C^TB}.$$

Then, $S\dot{S} < -\|S\|$, since $C^TB = c_2 > 0$. □

The interpretation of this Lemma is that the variable structure control law is designed according to $(C^T F(X)/C^T B)$ by selecting upper bounded as $S > 0$, and lower bounded as $S < 0$. This fact is also illustrated in Fig. 1. In Fig. 1, u_1 is the bang–bang controller, which uses only two values, $+\alpha_0$ and $-\alpha_0$, to envelop the system dynamics $(C^T F(X)\text{sgn}(S)/C^T B)$. As $S > 0$, the upper boundary $+\alpha_0$ pushes the system dynamics toward $S = 0$, while the lower boundary $-\alpha_0$ pulls it toward $S = 0$ as $S < 0$. However, the bang–bang controller usually results in system states moving backward and forward from the switching line S, i.e., chattering. A suction controller u_2 is thus designed with a boundary layer as shown in Fig. 1. In Fig. 1, the boundary layer ϕ reduces controller output to prevent chattering.

As shown in Fig. 1, the output of the suction controller is

$$u_2 = -\alpha_0 S/\phi \tag{9}$$

as the system states lie inside the designed boundary $\|S\| < \phi$. In this situation, u_2 is dependent on S, the distance between the system states and the switching line, to push or pull the system states toward the switching line $S = 0$. It is unable to reduce chattering by using S as the controller input because the system dynamics $(C^T F(X)\text{sgn}(S)/C^T B)$ depends on its system states, not S. As a result, the suction controller input must replace S with X such that its output can be adequately designed to envelop its system dynamics for completely sucking the system states onto a switching line.

This paper proposes an NFSC that uses system states X as the input. It is difficult to use traditional control algorithms for completely enveloping system dynamics. So, based on the first type of fuzzy modeling, the NFSC is designed to be just enough to envelop system dynamics for spending less energy and then reduce chattering. In addition, the traditional suction controller usually designs its output with zero as shown in Fig. 1. The proposed NFSC is convenient for designing a controller whose output is not zero during system states on a switching line such that the controller output approaches $(C^T F(X)\text{sgn}(S)/C^T B)$, and then reduces the possibility of pushing or pulling system state to cross over the switching line, that is, reduce chattering.

III. NFSC

An NFSC is an extension of FLC, and its idea is motivated by the FSC in [17]. Therefore, this section will begin with FLC definitions and some results from [17]. To guarantee the constructed NFSC that satisfies Lemma 1, a design procedure is derived. In addition, two illustrative examples are also included to demonstrate the NFSC design.

Consider a simple FLC as shown in Fig. 2. In the FLC, e_e and e_c are inputs, T is actual output, G_e and G_c are normalizing scaling factors, G_u is denormalizing scaling factor, E_e and E_c are the normalized inputs, and u is normalized output. Note that the inputs, e_e and e_c, relate to the normalized inputs, E_e and E_c, with

$$E_e = e_e G_e \tag{10}$$

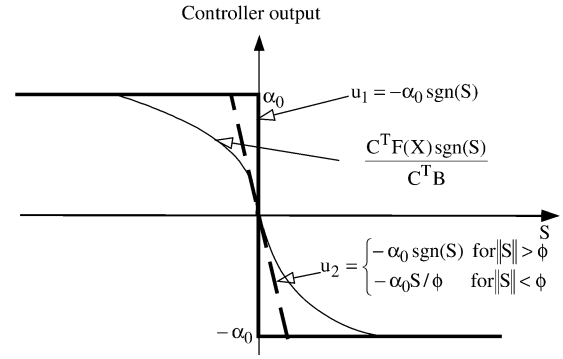

Fig. 1. Design of variable structure control lows.

and

$$E_c = e_c G_c \tag{11}$$

where e_e and e_c are the FLC inputs, and G_e and G_c are the normalizing scaling factors.

Let E_e and E_c be represented by term sets Ω_{Ee} and Ω_{Ec}, respectively, where Ω_{Ee} and Ω_{Ec} are fuzzy sets $\{\tilde{A}_{-q}, \ldots, \tilde{A}_{-1}, \tilde{A}_0, \tilde{A}_1, \ldots, \tilde{A}_q\}$ and $\{\tilde{B}_{-q}, \ldots, \tilde{B}_{-1}, \tilde{B}_0, \ldots, \tilde{B}_q\}$, respectively. Furthermore, the fuzzy subsets $\tilde{A}_i$ and $\tilde{B}_i$ are usually characterized by membership functions $\mu_{\tilde{A}_i}$ and $\mu_{\tilde{B}_i}$, respectively. An example of the triangular-shaped membership functions is shown in Fig. 3, where the number of term sets Ω_{Ee} is q $(q \geq 1)$ for positive E_e, q for negative E_e and one for near zero E_e. Therefore, the total number of membership functions for E_e is

$$N = 2q + 1. \tag{12}$$

Similarly, the number of term sets Ω_{Ec} is $2q + 1$.

Let the central value of the fuzzy set $\tilde{A}_i$ be denoted by A_i. In general, $A_{-q} = -1, A_0 = 0$, and $A_q = 1$(see Fig. 3), and the space between two adjacent central values is equal. Hence

$$A_i - A_{i-1} = 1/q$$

and

$$A_i = i/q. \tag{13}$$

Suppose that the following linguistic rule is applied:

$$\text{IF } E_e \; is \; \tilde{A}_j \text{ and } E_c \; is \; \tilde{B}_l, \text{ THEN } U \text{ is } U_k \tag{14}$$

where $\tilde{A}_j$ and $\tilde{B}_l$ are the fuzzy subsets of E_e and E_c, respectively, and U_k is the defuzzifier value for $\tilde{U}_k$. As (13)

$$U_q = 1 \quad U_{-q} = -1, \text{ and } U_k = k/q. \tag{15}$$

Note that the number of membership functions for $\tilde{U}_k$ is $2q + 1$. From the defuzzification, the normalized output of the FLC, u, is given by

$$u = \frac{\sum_{j,l,k} \mu_{j,l} U_k}{\sum_{j,l} \mu_{j,l}} \tag{16}$$

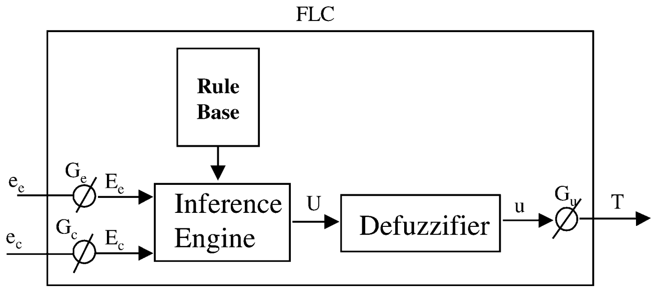

Fig. 2. Simple FLC.

where $\mu_{j,l} = \min(\mu_{\tilde{A}_j}(E_e), \mu_{\tilde{B}_l}(E_c))$, and U_k denotes the defuzzifier value, for j, l, and $k = -q, \ldots, -1, 0, 1, \ldots, q$.

In [17], a FSC is a simple FLC as shown in Fig. 2 with the linguistic rules designed as a suction controller, shown in Fig. 4. In Fig. 4, the control actions of diagonal terms in the rule table are U_0's, arranged like a switching line. Not only U_0's acts like a switching line, but the FSC can also be shown to act like a suction controller and has a switching line as shown in the following Lemma 2.

Lemma 2: Let an FLC shown in Fig. 2 have the linguistic rules as shown in Fig. 4. Then, the following equation holds:

$$u = F_{FLC}(\bar{e}) = 0 \quad \text{when } G_c e_c + G_e e_e = 0 \tag{17}$$

where e_c and e_e are the FLC inputs, $\bar{e} = [e_c e_e]^T$, G_c and G_e are the normalizing scaling factors of the FLC, u is the FLC output, and $F_{FLC}(\bar{e})$ is the function of the FLC.

Proof: See [17].

Note that the normalizing scaling factors G_c and G_e are used not only to normalize the input scale, but to also determine the slope of a switching line in the FSC. In [17], we also derived that the slope of a switching line could be adjusted to get fast time response.

Similar to the traditional suction controller, the circled U_3 and U_{-3} shown in Fig. 4 construct two boundary lines. Outside the boundary lines, the control actions are U_3 or U_{-3}. Inside the boundary lines the control actions are similar to the output of a suction controller whose output is proportional to the distance between the state position and the switching line. The previous paper was devoted to proposing a FSC for imitating a suction controller.

The previous FSC was designed to make a FLC just like a suction controller. In this paper, a NFSC is proposed to improve the function of traditional suction controllers. The improvement is based on (6) and (7) in Lemma 1 to design linguistic rules. Equations (6) and (7) divide the controller design into two situations: $S > 0$, and $S < 0$, respectively. However, the controller design also includes the situation near $S = 0$ which also need another rule. Hence, the linguistic rules are separated into three groups. The first group is the upper part of the diagonal terms in the rule table. The second group is the lower part of the diagonal terms. And the third group is the diagonal terms in the rule table. Consider (14) in the first group of linguistic rules

$$j > l \tag{18}$$

where j and l are the indexes of fuzzy sets $\tilde{A}_j$ and $\tilde{B}_l$, respectively. From (13), we have

$$A_j > B_l \tag{19}$$

where A_j and B_l are the center values of $\tilde{A}_j$ and $\tilde{B}_l$, respectively. Consequently, the first group of linguistic rules is designed as $S > 0$ because of $E_e + E_c > 0$. Similarly, The second group of linguistic rules is designed as $S < 0$. In addition, the third group is designed for near the switching line S.

The linguistic rules of an NFSC are designed using (14), which manipulates the NFSC on a normal space $E_e \times E_c \to u$, relating to a real space $e_e \times e_c \to uG_u$. Therefore, the linguistic rule design must consider scaling factors G_e, G_c, and G_u such that the NFSC results in proper magnitude output for the controlled system. Equation (14) takes the control action U_k with respect to E_e and E_c. When E_e and E_c are under the cover range of $\tilde{A}_j$ and $\tilde{B}_l$, respectively, i.e., $E_e \in [A_{j-1}, A_{j+1}]$ and $E_c \in [B_{l-1}, B_{l+1}]$, where A_{j-1} and A_{j+1} are the center values of $\tilde{A}_{j-1}$ and $\tilde{A}_{j+1}$, respectively, while B_{l-1} and B_{l+1} are those of $\tilde{B}_{l-1}$ and $\tilde{B}_{l+1}$, respectively. Translated into the real space from (10) and (11), the design of linguistic rules become to take U_k/G_u when $e_e \in [A_{j-1}/G_e, A_{j+1}/G_e]$ and $e_c \in [B_{l-1}/G_c, B_{l+1}/G_c]$. Note that since the term $(C^T F(X)/C^T B)$ in (6) and (7) must be normalized for producing control actions. Let V_N be a normalizing value, defined as follows:

$$V_N = \max\left(\sup\left(\frac{C^T F(X)}{C^T B}\right), \left|\inf\left(\frac{C^T F(X)}{C^T B}\right)\right|\right) \tag{20}$$

where

$$V_k^{\sup} = \sup_{\substack{A_{j-1}/G_e \le e_e \le A_{j+1}/G_e \\ B_{l-1}/G_c \le e_c \le B_{l+1}/G_c}} \frac{C^T F(e_e, e_c)}{C^T B} \tag{21}$$

and

$$V_k^{\inf} = \inf_{\substack{A_{j-1}/G_e \le e_e \le A_{j+1}/G_e \\ B_{l-1}/G_c \le e_c \le B_{l+1}/G_c}} \frac{C^T F(e_e, e_c)}{C^T B} \tag{22}$$

are the functions of the upper and lower portions, respectively. Note that $F(e_e, e_c)$ is obtained from $F(x_1, x_2)$ in (3) via translating variables $\{x_1, x_2\}$ into $\{e_e, e_c\}$. In addition, the denormalizing scaling factor G_u in a NFSC is designed to be equal to the normalizing value V_N such that the scale of a NFSC output is consistent with that of $(C^T F(X)/C^T B)$.

To summarize, the three groups of linguistic rules are then designed using the following three rules:

$$\text{Rule (1) : Select } U_k \textit{ by } U_{k-1} < \frac{V_k^{\sup}}{V_N} < U_k \quad \textit{as } j + l > 0 \tag{23}$$

$$\text{Rule (2) : Select } U_k \textit{ by } U_k < \frac{V_k^{\inf}}{V_N} < U_{k+1} \quad \text{as } j + l < 0 \tag{24}$$

$$\text{Rule (3) : Select } U_k \textit{ by } U_{k-1} < \frac{V_k^{\sup} + V_k^{\inf}}{2V_N} < U_k \quad \text{as } j + l = 0 \tag{25}$$

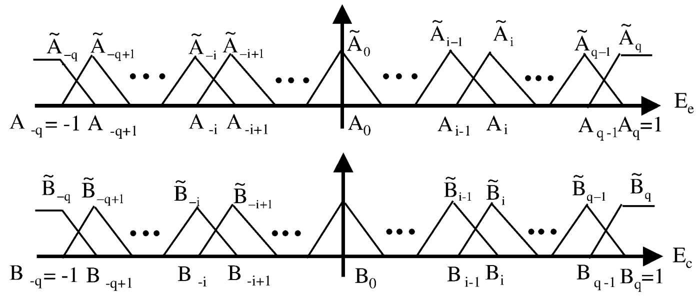

Fig. 3. Triangle-shaped membership functions.

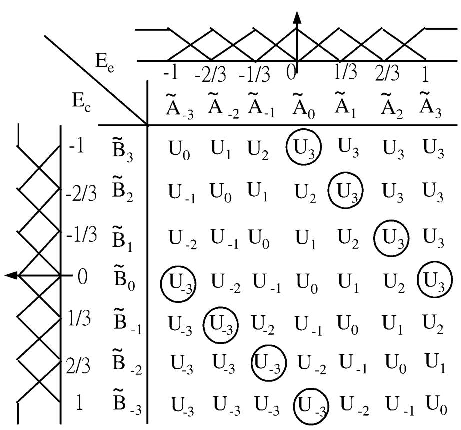

Fig. 4. Linguistic rules designed as a suction controller.

where j and l are the indexes of fuzzy sets $\tilde{A}_j$ and $\tilde{B}_l$, respectively. Rule (1) is designed for $S > 0$ because $\tilde{A}_j$ and $\tilde{B}_l$ represent the system states satisfying $S > 0$ as $j + l > 0$. Similarly, Rules (2) and (3) are designed for $S < 0$ and $S = 0$, respectively.

The proposed NFSC is summarized using a design procedure for holding the switching conditions and completely reducing chattering. Two illustrative examples demonstrate the NFSC design. The first example is simple enough to show the design process of NFSC, and the second example, which is so easy to have chattering by using traditional SMC, will be controlled by NFSC to have nearly chattering-free effect. The design procedure can be summarized as follows.

1) Choose normalizing scaling factors G_e and G_c by the desired response of rise time.

   The normalizing scaling factors have two functions: normalizing input space and deciding the slope of a switching line in the NFSC. We should follow these to choose adequate normalizing scaling factors. Note that, according to [17], the slope of a switching line in a FSC can be adjusted to achieve fast time response. Although a switching line becomes a switching curve in the NFSC, it is also dominated by the normalizing scaling factors. Therefore, the normalizing scaling factors of a NFSC can improve system performance as well.

2) Determine denormalizing scaling factor G_u.

   G_u is equal to V_N, where V_N is the bounded value of $(C^T F(X)/C^T B)$ as shown in (20).

3) Design linguistic rules from Rules (1) to (3).

   Eventually, this step builds a linguistic rule table. The selection of q is totally by experience and is a tradeoff between computation time and system performance.

4) Simulate and test the system whether satisfy design requirement, otherwise go back step 1) to redesign the NFSC.

Example 1: Consider a nonlinear system

$$\ddot{x} = \dot{x} + x^3 + u \tag{26}$$

where x is the state variable, and u is the input of the system. Let

$$x_1 = x \quad \text{and} \quad x_2 = \dot{x}.$$

Then, the nonlinear system can be represented by

$$\dot{X} = F(X) + Bu \tag{27}$$

where $\dot{X}^T = [\dot{x}_1, \dot{x}_2], F^T(X) = [x_2, x_2 + x_1^3]$, and $B^T = [0, 1]$.

A NFSC is designed as follows.

1) Choose $G_e = 1.5$ and $G_c = 1.5$.

   Hence, with respect to the variables x_1 and $x_2, (C^T F(X)/C^T B)$ is shown in Fig. 5.

2) Let $G_u = 3.375$ because $V_N = 3.375$.

3) From Rules (1) to (3), the linguistic rules of the NFSC are shown in Fig. 6. Note that the total number of fuzzy sets are 21, i.e., $q = 10$. Fig. 7 is the output of the NFSC. In addition, Fig. 8 shows that $(\alpha(S) - (C^T F(X)/C^T B))\text{sgn}(S)$ is always a positive value as the state variables leave from the switching line. It implies that $\alpha(s)$ is larger than $(C^T F(X)/C^T B)$ during $S > 0$; but during $S < 0, \alpha(S)$ is smaller. Both provide the controller output to push or pull the state variables toward the switching curve such that the system stability is held by (1).

4) Fig. 9 shows the state trajectories of the system controlled by the NFSC under various initial states. It is obvious that those state trajectories go along the switching curve and then toward the original state regardless with their initial states. In Fig. 9, the circled line is the switching curve.

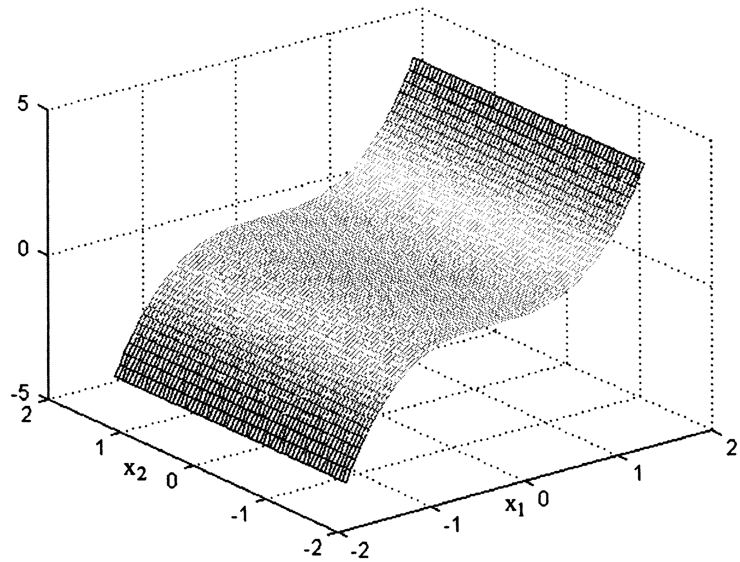

Fig. 5. Plot of $(C^T F(X)/C^T B)$ with respect to x_1 and x_2.

E_c \ E_e	$\tilde{A}_{-10}$	$\tilde{A}_{-9}$	$\tilde{A}_{-8}$	$\tilde{A}_{-7}$	$\tilde{A}_{-6}$	$\tilde{A}_{-5}$	$\tilde{A}_{-4}$	$\tilde{A}_{-3}$	$\tilde{A}_{-2}$	$\tilde{A}_{-1}$	$\tilde{A}_0$	$\tilde{A}_1$	$\tilde{A}_2$	$\tilde{A}_3$	$\tilde{A}_4$	$\tilde{A}_5$	$\tilde{A}_6$	$\tilde{A}_7$	$\tilde{A}_8$	$\tilde{A}_9$	$\tilde{A}_{10}$
$\tilde{B}_{10}$	U_{-7}	U_{-3}	U_{-2}	U_{-1}	U_0	U_0	U_0	U_0	U_0	U_1	U_1	U_1	U_1	U_1	U_1	U_2	U_3	U_4	U_6	U_8	U_{10}
$\tilde{B}_9$	U_{-10}	U_{-5}	U_{-2}	U_{-1}	U_0	U_0	U_0	U_0	U_0	U_1	U_1	U_1	U_1	U_1	U_1	U_2	U_3	U_4	U_6	U_8	U_{10}
$\tilde{B}_8$	U_{-10}	U_{-8}	U_{-4}	U_{-1}	U_0	U_0	U_0	U_0	U_0	U_1	U_1	U_1	U_1	U_1	U_1	U_2	U_3	U_4	U_6	U_8	U_{10}
$\tilde{B}_7$	U_{-10}	U_{-8}	U_{-6}	U_{-2}	U_0	U_0	U_0	U_0	U_0	U_1	U_1	U_1	U_1	U_1	U_1	U_2	U_3	U_4	U_6	U_8	U_{10}
$\tilde{B}_6$	U_{-10}	U_{-8}	U_{-6}	U_{-4}	U_{-1}	U_0	U_0	U_0	U_0	U_1	U_1	U_1	U_1	U_1	U_1	U_2	U_3	U_4	U_6	U_8	U_{10}
$\tilde{B}_5$	U_{-10}	U_{-8}	U_{-6}	U_{-4}	U_{-3}	U_{-1}	U_0	U_0	U_0	U_1	U_1	U_1	U_1	U_1	U_1	U_2	U_3	U_4	U_6	U_8	U_{10}
$\tilde{B}_4$	U_{-10}	U_{-8}	U_{-6}	U_{-4}	U_{-3}	U_{-2}	U_0	U_0	U_0	U_1	U_1	U_1	U_1	U_1	U_1	U_2	U_3	U_4	U_6	U_8	U_{10}
$\tilde{B}_3$	U_{-10}	U_{-8}	U_{-6}	U_{-4}	U_{-3}	U_{-2}	U_{-1}	U_0	U_0	U_1	U_1	U_1	U_1	U_1	U_1	U_2	U_3	U_4	U_6	U_8	U_{10}
$\tilde{B}_2$	U_{-10}	U_{-8}	U_{-6}	U_{-4}	U_{-3}	U_{-2}	U_{-1}	U_{-1}	U_0	U_1	U_1	U_1	U_1	U_1	U_1	U_2	U_3	U_4	U_6	U_8	U_{10}
$\tilde{B}_1$	U_{-10}	U_{-8}	U_{-6}	U_{-4}	U_{-3}	U_{-2}	U_{-1}	U_{-1}	U_{-1}	U_0	U_1	U_1	U_1	U_1	U_1	U_2	U_3	U_4	U_6	U_8	U_{10}
$\tilde{B}_0$	U_{-10}	U_{-8}	U_{-6}	U_{-4}	U_{-3}	U_{-2}	U_{-1}	U_{-1}	U_{-1}	U_{-1}	U_1	U_1	U_1	U_1	U_1	U_2	U_3	U_4	U_6	U_8	U_{10}
$\tilde{B}_{-1}$	U_{-10}	U_{-8}	U_{-6}	U_{-4}	U_{-3}	U_{-2}	U_{-1}	U_{-1}	U_{-1}	U_{-1}	U_{-1}	U_1	U_1	U_1	U_1	U_2	U_3	U_4	U_6	U_8	U_{10}
$\tilde{B}_{-2}$	U_{-10}	U_{-8}	U_{-6}	U_{-4}	U_{-3}	U_{-2}	U_{-1}	U_{-1}	U_{-1}	U_{-1}	U_{-1}	U_0	U_1	U_1	U_1	U_2	U_3	U_4	U_6	U_8	U_{10}
$\tilde{B}_{-3}$	U_{-10}	U_{-8}	U_{-6}	U_{-4}	U_{-3}	U_{-2}	U_{-1}	U_{-1}	U_{-1}	U_{-1}	U_{-1}	U_0	U_0	U_1	U_1	U_2	U_3	U_4	U_6	U_8	U_{10}
$\tilde{B}_{-4}$	U_{-10}	U_{-8}	U_{-6}	U_{-4}	U_{-3}	U_{-2}	U_{-1}	U_{-1}	U_{-1}	U_{-1}	U_{-1}	U_0	U_0	U_0	U_1	U_2	U_3	U_4	U_6	U_8	U_{10}
$\tilde{B}_{-5}$	U_{-10}	U_{-8}	U_{-6}	U_{-4}	U_{-3}	U_{-2}	U_{-1}	U_{-1}	U_{-1}	U_{-1}	U_{-1}	U_0	U_0	U_0	U_0	U_2	U_3	U_4	U_6	U_8	U_{10}
$\tilde{B}_{-6}$	U_{-10}	U_{-8}	U_{-6}	U_{-4}	U_{-3}	U_{-2}	U_{-1}	U_{-1}	U_{-1}	U_{-1}	U_{-1}	U_0	U_0	U_0	U_0	U_0	U_2	U_4	U_6	U_8	U_{10}
$\tilde{B}_{-7}$	U_{-10}	U_{-8}	U_{-6}	U_{-4}	U_{-3}	U_{-2}	U_{-1}	U_{-1}	U_{-1}	U_{-1}	U_{-1}	U_0	U_0	U_0	U_0	U_0	U_0	U_3	U_6	U_8	U_{10}
$\tilde{B}_{-8}$	U_{-10}	U_{-8}	U_{-6}	U_{-4}	U_{-3}	U_{-2}	U_{-1}	U_{-1}	U_{-1}	U_{-1}	U_{-1}	U_0	U_0	U_0	U_0	U_0	U_0	U_1	U_5	U_8	U_{10}
$\tilde{B}_{-9}$	U_{-10}	U_{-8}	U_{-6}	U_{-4}	U_{-3}	U_{-2}	U_{-1}	U_{-1}	U_{-1}	U_{-1}	U_{-1}	U_0	U_0	U_0	U_0	U_0	U_0	U_1	U_2	U_6	U_{10}
$\tilde{B}_{-10}$	U_{-10}	U_{-8}	U_{-6}	U_{-4}	U_{-3}	U_{-2}	U_{-1}	U_{-1}	U_{-1}	U_{-1}	U_{-1}	U_0	U_0	U_0	U_0	U_0	U_0	U_1	U_2	U_3	U_8

Fig. 6. Linguistic rules of the first NFSC.

At last, the spending energy of the NFSC is also compared with a FSC [1] and the traditional suction controller whose boundary is 0.5. Let an energy function be

$$\int |u| dt. \tag{28}$$

Then Table I is obtained to compare the average expended energy of three controllers due to miscellaneous initial states. It is shown that the NFSC spends lesser energy than that in FSC and suction controller.

Example 2: Consider a nonlinear system

$$\ddot{x} + 2\dot{x}^2 \cos 3x = u.$$

Note that the system is very easy to occur chattering by using a SMC [19]. Our NFSC is designed as follows.

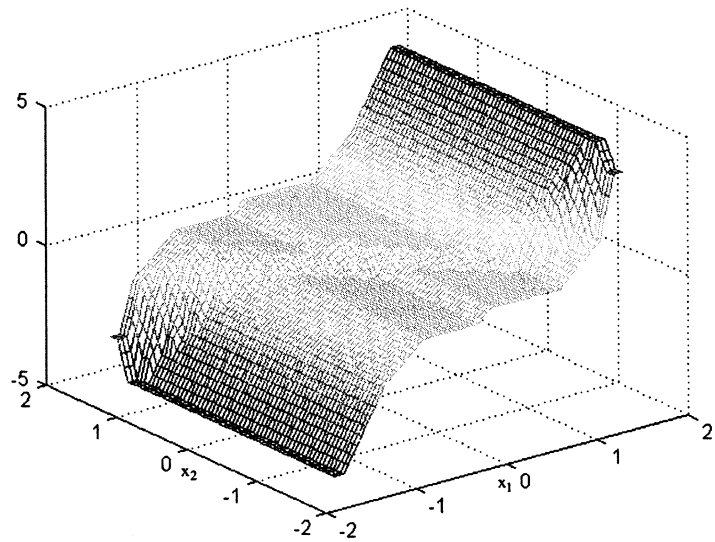

Fig. 7. NFSC output with the designed linguistic rules.

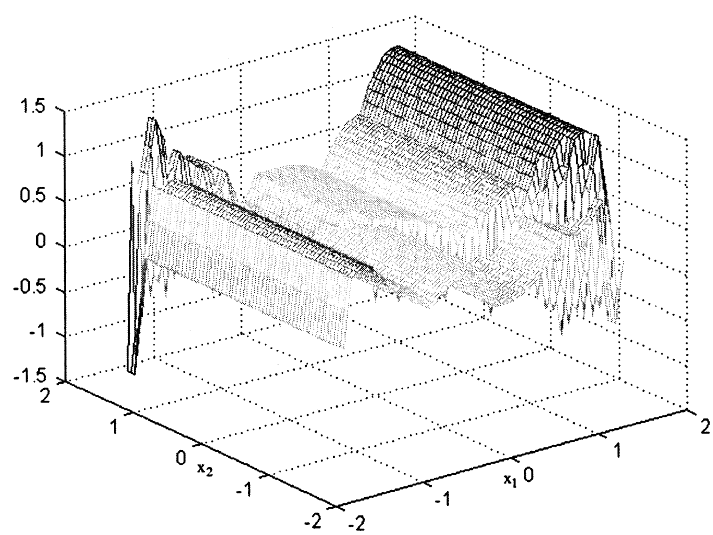

Fig. 8. Plot of $(\alpha(S) - (C^T F(X)/C^T B))\mathrm{sgn}(S)$ with respect to x_1 and x_2.

1) Choose $G_e = 1.5$ and $G_c = 1.5$.
2) Let $G_u = 17.875$ because $V_N = 17.875$.
3) From Rules (1) to (3), the linguistic rules of the NFSC are shown in Fig. 10.
4) Fig. 11 shows the state trajectories of the system controlled by the second NFSC and a suction controller, respectively, under various initial states. It is obvious that the second NFSC do not occur chattering, but the suction controller does.

Table II shows that the spending energy of the NFSC is reduced vastly.

IV. DESIGN OF NFSCs FOR FPE GRADIENT TRACKING

In this section, we design NFSCs to control a mobile robot for trajectory tracking. In particular, the tracked trajectories are the gradient trajectories, which are formed by fuzzy potential energy (FPE) proposed in [18]. This section demonstrates that the NFSC's can control the mobile robot to follow gradient trajectory toward the goal point without colliding with the obstacle.

Consider the holonomic mechanical system of a mobile robot, the general dynamic model is described by

$$M(Z)\ddot{Z} + f(Z, \dot{Z}) = u \tag{29}$$

where $Z \in R^2$ describes the robot configuration (here, $Z = [x, y]^T$), $M(Z) \in R^{2\times 2}$ is the positive definite inertia or mass matrix, $f(Z, \dot{Z}) \in R^2$ comprises centripetal, coriolis, gravitational effect, and additive disturbances, and $u \in R^2$ are mechanical control force supplied by ideal actuators. The terms $M(Z)$ and $f(Z, \dot{Z})$ are unknown. However, we assumed that the Euclidean norm, denoted by $\|\cdot\|$, of the term $f(Z, \dot{Z})$ is uniformly bounded over the range of operating conditions Ω by a known scalar constant

$$\|f(Z, \dot{Z})\| \le f_u \qquad \forall (Z, \dot{Z}) \in \Omega \tag{30}$$

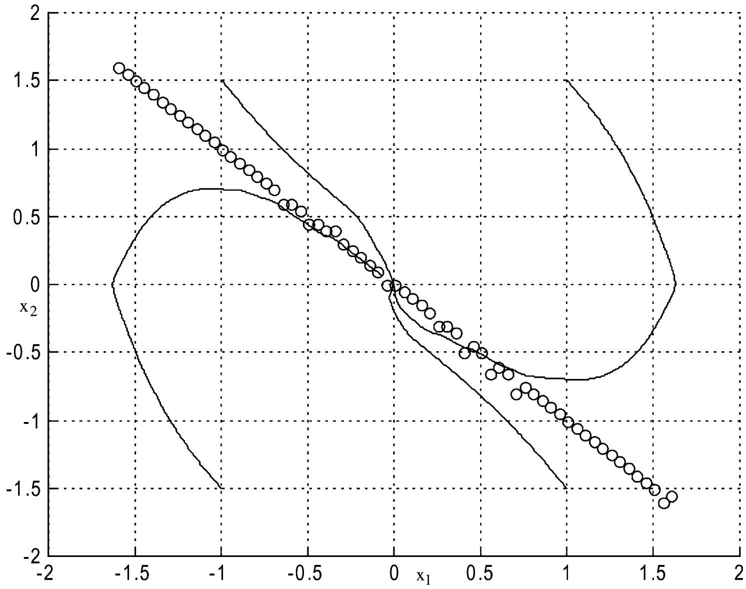

Fig. 9. Switching curve (circled line) and State trajectories (solid line) on phase plane.

$E_c \backslash E_e$	$\tilde{A}_{-10}$	$\tilde{A}_{-9}$	$\tilde{A}_{-8}$	$\tilde{A}_{-7}$	$\tilde{A}_{-6}$	$\tilde{A}_{-5}$	$\tilde{A}_{-4}$	$\tilde{A}_{-3}$	$\tilde{A}_{-2}$	$\tilde{A}_{-1}$	$\tilde{A}_{0}$	$\tilde{A}_{1}$	$\tilde{A}_{2}$	$\tilde{A}_{3}$	$\tilde{A}_{4}$	$\tilde{A}_{5}$	$\tilde{A}_{6}$	$\tilde{A}_{7}$	$\tilde{A}_{8}$	$\tilde{A}_{9}$	$\tilde{A}_{10}$
$\tilde{B}_{10}$	U_0	U_0	U_0	U_5	U_{10}	U_{10}	U_{10}	U_{10}	U_8	U_2	U_{-1}	U_2	U_8	U_{10}	U_{10}	U_{10}	U_{10}	U_5	U_0	U_1	U_6
$\tilde{B}_{9}$	U_{-6}	U_{-3}	U_0	U_4	U_8	U_{10}	U_{10}	U_{10}	U_7	U_2	U_{-1}	U_2	U_7	U_{10}	U_{10}	U_{10}	U_8	U_4	U_0	U_1	U_5
$\tilde{B}_{8}$	U_{-5}	U_{-6}	U_{-2}	U_4	U_7	U_8	U_8	U_8	U_6	U_2	U_0	U_2	U_6	U_8	U_8	U_8	U_7	U_4	U_1	U_1	U_5
$\tilde{B}_{7}$	U_{-4}	U_{-5}	U_{-5}	U_0	U_6	U_7	U_7	U_7	U_5	U_2	U_0	U_2	U_5	U_7	U_7	U_7	U_6	U_4	U_1	U_1	U_4
$\tilde{B}_{6}$	U_{-4}	U_{-4}	U_{-4}	U_{-4}	U_2	U_6	U_6	U_6	U_5	U_2	U_1	U_2	U_5	U_6	U_6	U_6	U_5	U_3	U_1	U_2	U_4
$\tilde{B}_{5}$	U_{-3}	U_{-3}	U_{-3}	U_{-3}	U_{-2}	U_3	U_5	U_5	U_4	U_2	U_1	U_2	U_4	U_5	U_5	U_5	U_4	U_3	U_1	U_2	U_3
$\tilde{B}_{4}$	U_{-2}	U_{-3}	U_{-3}	U_{-2}	U_{-2}	U_{-1}	U_2	U_4	U_3	U_2	U_1	U_2	U_3	U_4	U_4	U_4	U_4	U_3	U_2	U_2	U_3
$\tilde{B}_{3}$	U_{-2}	U_{-2}	U_{-2}	U_{-2}	U_{-2}	U_{-1}	U_{-1}	U_1	U_3	U_2	U_2	U_2	U_3	U_3	U_3	U_3	U_3	U_2	U_2	U_2	U_2
$\tilde{B}_{2}$	U_{-2}	U_{-2}	U_{-2}	U_{-2}	U_{-1}	U_{-1}	U_{-1}	U_{-1}	U_1	U_2	U_2	U_2	U_2	U_3	U_3	U_3	U_2	U_2	U_2	U_2	U_2
$\tilde{B}_{1}$	U_{-1}	U_{-1}	U_{-1}	U_{-1}	U_{-1}	U_{-1}	U_{-1}	U_{-1}	U_{-1}	U_1	U_2	U_2	U_2	U_2	U_2	U_2	U_2	U_2	U_2	U_2	U_2
$\tilde{B}_{0}$	U_{-2}	U_{-2}	U_{-2}	U_{-2}	U_{-2}	U_{-2}	U_{-2}	U_{-2}	U_{-2}	U_{-2}	U_0	U_2	U_2	U_2	U_2	U_2	U_2	U_2	U_2	U_2	U_2
$\tilde{B}_{-1}$	U_{-2}	U_{-2}	U_{-2}	U_{-2}	U_{-2}	U_{-2}	U_{-2}	U_{-2}	U_{-2}	U_{-2}	U_{-2}	U_0	U_2	U_2	U_2	U_2	U_2	U_2	U_2	U_2	U_2
$\tilde{B}_{-2}$	U_{-2}	U_{-3}	U_{-3}	U_{-3}	U_{-2}	U_{-2}	U_{-2}	U_{-2}	U_{-2}	U_{-3}	U_{-3}	U_{-3}	U_0	U_2	U_2	U_2	U_2	U_1	U_1	U_1	U_1
$\tilde{B}_{-3}$	U_{-3}	U_{-3}	U_{-3}	U_{-3}	U_{-3}	U_{-2}	U_{-2}	U_{-2}	U_{-3}	U_{-3}	U_{-3}	U_{-3}	U_{-3}	U_0	U_2	U_2	U_2	U_1	U_1	U_1	U_1
$\tilde{B}_{-4}$	U_{-4}	U_{-4}	U_{-4}	U_{-4}	U_{-3}	U_{-2}	U_{-2}	U_{-2}	U_{-3}	U_{-4}	U_{-4}	U_{-4}	U_{-3}	U_{-2}	U_1	U_3	U_2	U_1	U_1	U_1	U_1
$\tilde{B}_{-5}$	U_{-5}	U_{-5}	U_{-5}	U_{-5}	U_{-4}	U_{-2}	U_{-1}	U_{-3}	U_{-4}	U_{-5}	U_{-5}	U_{-5}	U_{-4}	U_{-3}	U_{-1}	U_1	U_3	U_1	U_0	U_1	U_2
$\tilde{B}_{-6}$	U_{-6}	U_{-6}	U_{-6}	U_{-6}	U_{-4}	U_{-2}	U_{-1}	U_{-3}	U_{-5}	U_{-6}	U_{-6}	U_{-6}	U_{-5}	U_{-3}	U_{-1}	U_{-2}	U_0	U_1	U_0	U_0	U_2
$\tilde{B}_{-7}$	U_{-7}	U_{-7}	U_{-7}	U_{-7}	U_{-5}	U_{-2}	U_{-1}	U_{-3}	U_{-6}	U_{-7}	U_{-7}	U_{-7}	U_{-6}	U_{-3}	U_{-1}	U_{-2}	U_{-5}	U_{-2}	U_{-1}	U_0	U_2
$\tilde{B}_{-8}$	U_{-8}	U_{-8}	U_{-8}	U_{-8}	U_{-6}	U_{-2}	U_0	U_{-3}	U_{-7}	U_{-8}	U_{-8}	U_{-8}	U_{-7}	U_{-3}	U_0	U_{-2}	U_{-6}	U_{-8}	U_{-4}	U_{-1}	U_2
$\tilde{B}_{-9}$	U_{-9}	U_{-10}	U_{-10}	U_{-10}	U_{-6}	U_{-2}	U_0	U_{-3}	U_{-8}	U_{-10}	U_{-10}	U_{-10}	U_{-8}	U_{-3}	U_0	U_{-2}	U_{-6}	U_{-10}	U_{-10}	U_{-5}	U_3
$\tilde{B}_{-10}$	U_{-10}	U_{-10}	U_{-10}	U_{-10}	U_{-7}	U_{-2}	U_1	U_{-4}	U_{-9}	U_{-10}	U_{-10}	U_{-10}	U_{-9}	U_{-4}	U_1	U_{-2}	U_{-7}	U_{-10}	U_{-10}	U_{-1}	U_{-3}

Fig. 10. Linguistic rules of the second NFSC.

and that the positive definite symmetric inertia/mass matrix $M(Z)$ is a uniformly bounded function of the configuration vector $Z \in R^2$ satisfying

$$m_\ell \|w\|^2 \leq w^T M(Z) w \leq m_u \|w\|^2 \qquad \forall w \in R^2 \tag{31}$$

where m_ℓ and m_u are positive constants. Moreover, we assumed that the system states $(Z, \dot{Z})$ are available for feedback.

In [18], FPE gradient is used to guide a mobile robot. Its gradient directions ε_x and ε_y are thus regarded as the desired velocity when the NFSC's control a mobile robot. In other words, NFSC's are designed to orient the robot velocity vector collinear to the gradient directions. However, the gradient directions are virtually vectors and do not directly relate to the velocity of a mobile robot. There is no exact dynamic behavior between the mobile robot's velocity and the FPE gradient, which is the design base for the NFSCs' linguistic rules using Rules (1)–(3). Therefore, the NFSCs' design for the mobile robot guided by FPE's needs other consideration that is different from the previous discussion.

First of all, we consider the structure of the designed NFSC. To conveniently use the desired velocity, the gradient direc-

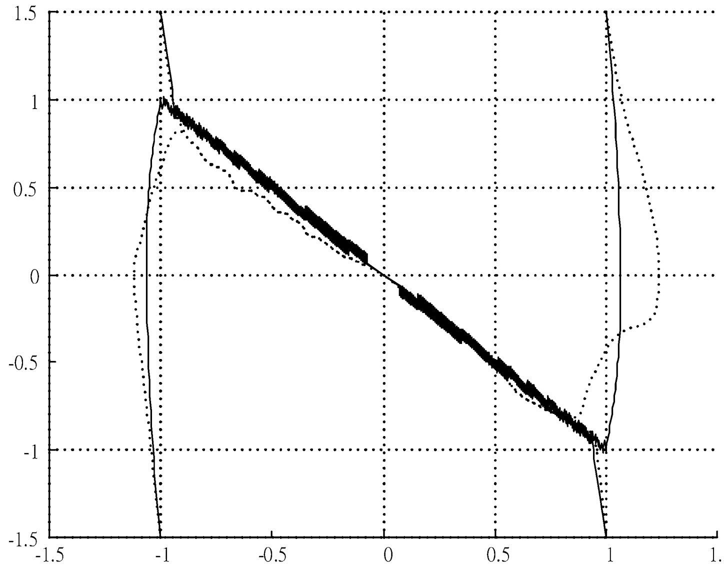

Fig. 11. State trajectories on phase plane, where dotted lines and solid lines are the state trajectories controlled by the second NFSC and a suction controller, respectively.

TABLE I
COMPARISON AMONG THE FIRST NFSC, FSC, AND A SUCTION CONTROLLER

	∫|u|dt
NFSC	1387.4
FSC	1668.5
Suction Controller	1576.7

TABLE II
COMPARISON AMONG THE SECOND NFSC, FSC, AND A SUCTION CONTROLLER

	∫|u|dt
NFSC	549.7
FSC	4997.5
Suction Controller	4781.6

tions are normalized by $\varepsilon/||\varepsilon||$, where ε representing the gradient at point $z = [x, y]^T$ is $[\varepsilon_x, \varepsilon_y]^T$, and $||\varepsilon||$ is the norm of $\varepsilon \cdot \varepsilon_x/||\varepsilon||, \varepsilon_y/||\varepsilon||, \dot{x}$ and $\dot{y}$ are the inputs of NFSC's. Note that $\dot{x}$ and $\dot{y}$ are the velocity of the controlled mobile robot. There are two NFSC's for controlling $\dot{x}$ and $\dot{y}$ to be consistent with $\varepsilon_x/||\varepsilon||$ and $\varepsilon_y/||\varepsilon||$, respectively.

The main problem in the NFSCs' design is its linguistic rules. Because the linguistic rule design cannot adopt Rules (1)–(3), we make the NFSC's imitate a SMC with sliding sectors [9], and design its linguistic rules as shown in Fig. 12. As shown in Fig. 12, the control actions ZO's of the diagonal terms form a switching line as a SMC, and the control actions on the vicinity of the switching line are increasing from U_1 to U_3 and decreasing from U_{-1} to U_{-3}, respectively, to form sliding sectors. From Lemma 2, the NFSC also has an exact switching line. Let $\dot{x}$ and $\varepsilon_x/||\varepsilon||$ be the inputs of the NFSC. The switching line is then

$$G_{e,1}\dot{x} + G_{c,1}\varepsilon_x/||\varepsilon|| = 0 \quad (32)$$

E_c \ E_e	$\tilde{A}_{-3}$	$\tilde{A}_{-2}$	$\tilde{A}_{-1}$	$\tilde{A}_0$	$\tilde{A}_1$	$\tilde{A}_2$	$\tilde{A}_3$
$\tilde{B}_3$	U_0	U_1	U_2	U_3	U_3	U_3	U_3
$\tilde{B}_2$	U_{-1}	U_0	U_1	U_2	U_3	U_3	U_3
$\tilde{B}_1$	U_{-2}	U_{-1}	U_0	U_1	U_3	U_3	U_3
$\tilde{B}_0$	U_{-3}	U_{-2}	U_{-1}	U_0	U_1	U_2	U_3
$\tilde{B}_{-1}$	U_{-3}	U_{-3}	U_{-3}	U_{-1}	U_0	U_1	U_2
$\tilde{B}_{-2}$	U_{-3}	U_{-3}	U_{-3}	U_{-2}	U_{-1}	U_0	U_1
$\tilde{B}_{-3}$	U_{-3}	U_{-3}	U_{-3}	U_{-3}	U_{-2}	U_{-1}	U_0

Fig. 12. Linguistic rules designed with sliding sectors.

Where $G_{e,1}$ and $G_{c,1}$ are normalizing scaling factors. Similarly, a switching line formed by the other NFSC is

$$G_{e,2}\dot{y} + G_{c,2}\varepsilon_y/||\varepsilon|| = 0 \quad (33)$$

In all, the design of linguistic rules make two NFSC's to have two switching lines, as shown in (32) and (33).

Equations (32) and (33) form a switching manifold, which is presented as follows:

$$\begin{aligned} S &= \begin{bmatrix} G_{e,1} & 0 \\ 0 & G_{e,2} \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} G_{c,1} & 0 \\ 0 & G_{c,2} \end{bmatrix} \begin{bmatrix} \frac{\varepsilon_x}{||\varepsilon||} \\ \frac{\varepsilon_y}{||\varepsilon||} \end{bmatrix} \\ &= G_e\dot{Z} + G_c\frac{\varepsilon}{||\varepsilon||} = 0 \end{aligned} \quad (34)$$

where $S = [S_1, S_2]^T, G_{e,1}, G_{c,1}, G_{e,2}$, and $G_{c,2}$ are the scaling factors, and ε is the gradient of FPE at the point $Z = (x, y)$.

Let

$$G_{e,u} = \max(G_{e,1}, G_{e,2}) \quad (35)$$

and $|S| = [|S_1|, |S_2|]^T$ be regarded as the distance between the system states and the switching lines. When the system states are controlled to follow the switching lines

$$|S| = G_e \dot{Z} + G_c \frac{\varepsilon}{\|\varepsilon\|} = 0.$$

That is

$$\dot{Z} = -G_e^{-1} G_c \frac{\varepsilon}{\|\varepsilon\|}.$$

The system states, the velocity of the mobile robot $(\dot{x}, \dot{y})$ track the gradient directions of the FPEs.

Next, we will describe how the NFSCs can control the system state vector Z to converge on the switching manifold. This is given in the following theorem.

Theorem 1: Consider the dynamical system (29), if the control law is

$$u = -G_u \frac{s}{\|S\|}$$

and

$$G_u \geq \left(\xi + m_u G_{e,u}^{-1} \max \|F\|\right) \tag{36}$$

the system state Z then converges on the switching manifold (34), where ξ is a positive constant, $F = -G_e M^{-1} f + G_c (d/dt)(\varepsilon/\|\varepsilon\|)$, and m_u is the upper boundary of $M(Z)$ as shown in (31).

Proof: Define the Lyapunov function candidate

$$V = \frac{1}{2} S^T S$$

then

$$\dot{V} = S^T \left(G_e \ddot{z} + G_c \frac{d}{dt} \frac{\varepsilon}{\|\varepsilon\|} \right). \tag{37}$$

Manipulating (29) and then substitute it into (37), we have

$$\dot{V} = S^T G_e \left(M^{-1} u - M^{-1} f + G_e^{-1} G_c \frac{d}{dt} \frac{\varepsilon}{\|\varepsilon\|} \right). \tag{38}$$

Substituting (36) into (38) and after some manipulations using (35), we have

$$\begin{aligned}
\dot{V} &= -S^T G_e M^{-1} G_u \frac{S}{\|S\|} + S^T F \\
&\leq \frac{-S^T G_e M^{-1} S}{\|S\|} (\xi + m_u G_{e,u}^{-1} \max \|F\|) + S^T F \\
&\leq -G_{e,u} m_u^{-1} \xi \|S\| - G_{e,u} m_u^{-1} m_u G_{e,u}^{-1} \|S\| \max \|F\| \\
&\quad + \|S\| \|F\| \\
&\leq -G_{e,u} m_u^{-1} \xi \|S\| - \|S\| \max \|F\| + \|S\| \|F\| \\
&\leq -G_{e,u} m_u^{-1} \xi \|S\|.
\end{aligned}$$

□

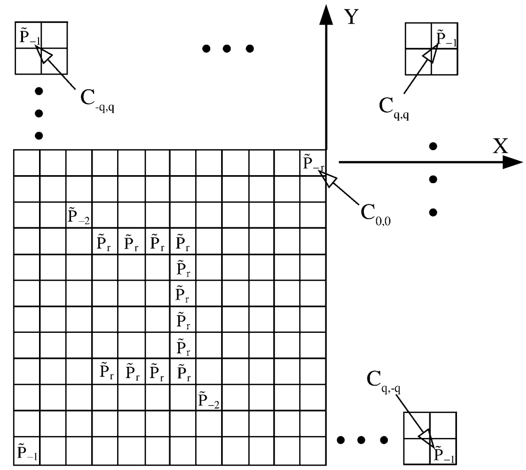

Fig. 13. FPE layout for preventing robots from entering the trap of a U-shaped obstacle.

As a result, when system state is on the switching manifold

$$\dot{X} = -\frac{G_{c,1}}{G_{e,1}} \frac{\varepsilon_x}{\|\varepsilon\|}$$

and

$$\dot{Y} = -\frac{G_{c,2}}{G_{e,2}} \frac{\varepsilon_y}{\|\varepsilon\|}. \tag{39}$$

That is, velocity of a mobile robot follows gradient trajectories of FPEs.

Assume that the controlled robot is a point robot whos the body mass is 2 kg and has a viscous friction of 0.5 kg/m/sec on the ground. Thus, the dynamic equations are

$$\begin{cases} 2\ddot{X} + 0.5\dot{X} = u_x \\ 2\ddot{Y} + 0.5\dot{Y} = u_y \end{cases}$$

where (X, Y) denotes the position of the body, and u_x and u_y are the force which acts on the body at the direction of X and Y, respectively.

There are two switching lines designed in two NFSC's. The switching lines are dominated by the scaling factors as shown in (34). Choosing $G_{e,1} = G_{e,2} = 6$ and $G_{c,1} = G_{c,2} = 3$ forms the switching lines in the NFSCs. From (36), we can select $G_{u,1} = G_{u,2} = 7$ to finish the NFSCs' design such that the system states can be controlled to converge on the switching manifold.

The designed NFSCs are applied to control the mobile robot to follow the gradient trajectories of FPEs for robot navigation. We only use the gradient trajectories of FPE's to demonstrate the ability of NFSC tracking trajectory here. The detail design ideas of FPE are presented in [18]. According to [18], the FPE's are arranged as shown in Fig. 13 when a mobile robot is out of the trap of the U-shape obstacle in order to prevent it to enter into the trap and then to arrive at the goal point, $C_{0,0}$. Fig. 14 shows that both the designed NFSCs and FSC's, regardless of the initial position, control the mobile robot to arrive at the goal point successfully, but FSCs result in larger damping as the trajectories near the U-shape obstacle. This results present that the

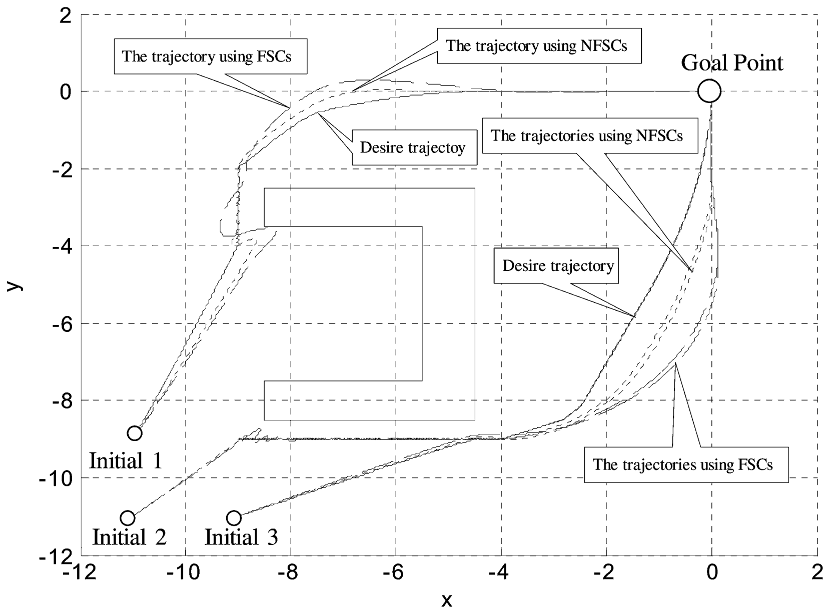

Fig. 14. Three trajectories as mobile robot starts outside U-shaped obstaclen (Solid lines are desired trajectories, dotted lines are the motion trajectories using FSCs, and dashed lines are those using NFSCs).

NFSCs can control a mobile robot to follow the gradient trajectories of the FPEs for obstacle avoidance very well.

V. Conclusion

This paper proposes the NFSC with nonlinear boundary layer for spending less energy and reducing chattering. There are two kinds of nonlinear boundary layers discussed in this paper. The first kind proposes three rules for designing linguistic rules not only to hold system stability but also make the NFSC have two special properties. The first property is that the NFSC output on a switching line does not need to be zero. The second property is that an NFSC provides a way to spend less energy for sucking system states onto a switching line. In comparison with the previous FSC and the traditional SMC, the proposed NFSC not only has the second property as shown in Tables I and II but also reduces chattering more effectively. The proposed NFSC improves the functions of both suction controllers and FSC. The second kind of nonlinear boundary layer proposes the NFSCs' linguistic rules designed as sliding sectors to control a mobile robot for trajectory tracking. Simulation results showed that, regardless of its initial positions, the mobile robot was controlled to follow the gradient trajectories of FPEs, to stay away from the obstacle and to arrive at the goal point.

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