

# A Fully Distributed Power Control Algorithm for Cellular Mobile Systems

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**Abstract**—Several distributed power control algorithms that can achieve carrier-to-interference ratio (CIR) balancing with probability one have been proposed recently for cellular mobile systems [5]–[8]. In these algorithms, only local information is used to adjust transmitting power. However, a normalization procedure is required in each iteration to determine transmitting power and, thus, these algorithms are not fully distributed. In this paper, we present a distributed power control algorithm which does not need the normalization procedure. We show that the proposed algorithm can achieve CIR balancing with probability one. Moreover, numerical results reveal our proposed scheme performs better than the algorithm presented in [7], which has the best performance among the distributed power control algorithms studied in [5]–[8]. The excellent performance and the fully distributed property make our proposed algorithm a good choice for cellular mobile systems.

## I. INTRODUCTION

**F**REQUENCY reuse is the core concept to increase system capacity of a cellular mobile system. Co-channel interference due to simultaneous use of the same channel sets a limit on minimum reuse distance. Therefore, reduction of co-channel interference is desirable in designing a high capacity cellular mobile system. An effective technique that can be used to reduce co-channel interference and allow as many receivers as possible to obtain satisfactory reception is to control transmitting power. Reception is said to be satisfactory if the carrier-to-interference ratio (CIR) is greater than the minimum CIR required or the system *protection ratio*.

Depending on the execution location, power control algorithms can be categorized as either centralized or distributed. An optimum centralized power control algorithm which can achieve the minimum outage probability was studied in [3]. It is assumed that all the active link gains are available and remain constant during execution of the algorithm. This assumption, of course, is not realistic because of the high computational complexity required for the algorithm.

The distributed power control algorithm proposed in [5] uses only local CIR information and utilizes an iterative scheme to control the transmitting power. This distributed algorithm was shown to be able to achieve CIR balancing with probability one and, thus, when combined with cell removal algorithm, can obtain a minimum outage probability. When CIR balancing is achieved, all admitted connections operate at the same CIR level. However, the convergence speed of the algorithm is not

satisfactory. If the allowed maximum number of iterations is not large enough, then the distributed algorithm may result in an outage probability much greater than the optimum value.

Some other distributed power control algorithms which can achieve CIR balancing with probability one can be found in [6]–[8]. Of particular interest is the algorithm presented in [7] which can achieve CIR balancing much faster than the algorithm proposed in [5]. This algorithm suggests to adjust transmitting power in proportion to the received interference power. The algorithms studied in [6] and [8] contain the algorithm in [7] as a special case, but do not improve its performance. Although only local information is used to adjust transmitting power for all the distributed power control algorithms investigated in [5]–[8], a normalization procedure is required in each iteration. That is, all base stations have to communicate with a central station to obtain a normalization factor to determine transmitting powers in the next iteration. Without the normalization procedure, transmitting powers may fall out of desired range.

In practical systems, it could be desirable to reduce transmitting powers as much as possible while maintaining the required quality of communication, especially for mobile terminals where transmitting power is provided by battery. To study this problem, one cannot neglect receiver noise. In [9], the maximum achievable CIR was derived under the constraint of maximum transmitting power. Also, a distributed constrained power control (DCPC) algorithm which achieves close to optimum performance was proposed. Even under the maximum power constraint, the algorithm achieves CIR balancing, if it is achievable, with the minimum power level.

In this paper we propose a fully distributed power control algorithm that does not need a normalization procedure. We prove that the proposed algorithm can achieve CIR balancing with probability one. Through extensive computer simulations, we found this algorithm gives excellent performance.

The rest of this paper is organized as follows. System model is described in Section II. The fully distributed power control algorithm and its several important properties are presented in Section III. In Section IV, we modify the proposed algorithm to satisfy the finite dynamic power range constraint that always exists in a real system. Numerical examples and discussions are provided in Section V. Conclusions are finally drawn in Section VI.

## II. SYSTEM MODEL

The cellular mobile system investigated in this paper consists of a finite number of cells. Cells using the same channel

Manuscript revised December 1995.

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Publisher Item Identifier S 0733-8716(96)01944-0.

are placed symmetrically in a hexagonal grid. Base stations use omnidirectional antennas and are located at the center of the cells. Mobiles are assumed to be uniformly distributed over the cell area. Each cell has  $M$  independent channel pairs, each consisting of independent uplink and downlink channels. The power control algorithm can be used for both uplink and downlink channels. However, for easy of description, we assume the algorithm is used for downlink channels.

The interference caused by adjacent channels is assumed to be much smaller than co-channel interference and, thus, is neglected. Similarly, thermal noise is not considered. The set of cells using a certain channel at some given instant is called the co-channel set and the size of the co-channel set is denoted by  $N$ . All co-channels are assumed to be in use. For simplicity, the total interference power is modeled as the sum of the powers of all active interferers and the transmission quality is assumed to depend only on CIR. Furthermore, the cells of a co-channel set are numbered from 1 to  $N$  and the base station or mobile in cell  $i$  is referred to as base station  $i$  or mobile  $i$ , respectively. Under these assumptions, the CIR at mobile  $i$ , denoted by  $\text{CIR}_i$ , is given by

$$\begin{aligned} \text{CIR}_i &= \frac{R_i}{\sum_{j=1}^N R_j - R_i} = \frac{T_i L_{ii}}{\sum_{j=1}^N T_j L_{ij} - T_i L_{ii}} \\ &= \frac{T_i}{\sum_{j=1}^N T_j Z_{ij} - T_i} \end{aligned} \quad (1)$$

where  $R_j$  is the power received from the  $j$ th base station,  $T_j$  is the transmitting power used by base station  $j$ ,  $L_{ij}$  is the link gain between mobile  $i$  and base station  $j$  at some given moment, and  $Z_{ij} = \frac{L_{ij}}{L_{ii}}$  is the normalized link gain. It is clear that  $Z_{ii} = 1$  for all  $i$ ,  $1 \leq i \leq N$ . Moreover, the value of  $L_{ij}$  is assumed to be a constant. This assumption is reasonable if the power control algorithm can converge in a short period. The link gain  $L_{ij}$  is modeled as  $L_{ij} = \frac{A_{ij}}{d_{ij}^\nu}$ , where  $A_{ij}$  is the attenuation factor,  $d_{ij}$  is the distance between mobile  $i$  and base station  $j$ , and  $\nu$  is a constant that models the large scale propagation loss. The attenuation factor models power variation due to shadowing. We assume that  $A_{ij}$ ,  $1 \leq i, j \leq N$ , are independent, log-normal, identically distributed random variables with 0 dB expectation and  $\sigma$  dB log-variance. The parameter value of  $\sigma$  in the range of 4–10 dB and the propagation constant  $\nu$  in the range of 3–5 usually provide good models for urban propagation [14].

Let  $\overline{\text{CIR}}$  denote the system *protection ratio*. The outage probability, denoted by  $P_{\text{outage}}$ , is defined as

$$P_{\text{outage}} = \frac{1}{N} \sum_{j=1}^N P_r\{\text{CIR}_j < \overline{\text{CIR}}\}. \quad (2)$$

A CIR is said to be achievable in the co-channel set if there exists a power vector  $\mathbf{T} = [T_1, T_2, \dots, T_N]^T$  such that  $\text{CIR}_i \geq \overline{\text{CIR}}$  for all  $i$ ,  $1 \leq i \leq N$ . It was shown [5] that the maximum achievable CIR, denoted by  $\text{CIR}^*$ , is given by

$$\text{CIR}^* = \frac{1}{\lambda^* - 1} \quad (3)$$

where  $\lambda^*$  is the largest real eigenvalue of the positive link gain matrix  $\mathbf{Z} = [Z_{ij}]$ . Moreover, the power vector  $\mathbf{T}^*$  achieving

this maximum is the eigenvector of  $\mathbf{Z}$  corresponding to the eigenvalue  $\lambda^*$ . All mobiles experience the same  $\text{CIR}^*$  when  $\mathbf{T}^*$  is used.

### III. THE FULLY DISTRIBUTED POWER CONTROL ALGORITHM

In this section, we present and prove several properties of the proposed fully distributed power control (FDPC) algorithm. In the following description,  $\mathbf{1}$  represents the all-one vector,  $\mathbf{T}^0$  denotes the initial transmitting power vector,  $T_i^k$  denotes the transmitting power of base station  $i$  in the  $k$ th discrete time, and  $\text{CIR}_i^k$  denotes the CIR at mobile  $i$  in the  $k$ th discrete time.

FDPC algorithm:

$$\mathbf{T}^0 = \mathbf{1}$$

and

$$T_i^{k+1} = \eta_i^k * T_i^k \quad (4)$$

where

$$\eta_i^k = \frac{\min(\text{CIR}_i^k, \Upsilon)}{\text{CIR}_i^k}, \quad 0 < \Upsilon < \infty. \quad (5)$$

Notice that there is one parameter  $\Upsilon$  in the above FDPC algorithm. As long as the value of  $\Upsilon$  is determined (*a priori*), base stations do not need to exchange information to perform the power control algorithm.

The basic idea of the above FDPC algorithm is to increase or decrease transmitting power (in dB) of base station  $i$  in proportion to the difference of CIR at mobile  $i$  and a constant. (The same idea but in linear scale was studied in [6].) Let  ${}_{\text{dB}}x = 10 \log_{10} x$ . If  $\eta_i^k = \frac{\Upsilon}{\text{CIR}_i^k}$ , then the above FDPC algorithm can be expressed as

$$\mathbf{T}^0 = \mathbf{1}$$

and

$${}_{\text{dB}}T_i^{k+1} - {}_{\text{dB}}T_i^k = -({}_{\text{dB}}\text{CIR}_i^k - {}_{\text{dB}}\Upsilon). \quad (6)$$

However, to avoid a normalization procedure to make the algorithm fully distributed, we choose  $\eta_i^k$  as the value expressed in (5). Clearly, when  $\Upsilon \rightarrow \infty$ , the FDPC algorithm becomes the fixed power control algorithm (i.e., without power control).

There are several nice properties, including CIR balancing, of the proposed FDPC algorithm. These properties are stated and proved below.

*Property 1:*  $T_i^{k+1} \leq T_i^k$  for all  $i$  and  $k$ .

*Proof:* Property 1 is obviously true because  $\eta_i^k \leq 1$ .  $\square$

*Property 2:* If  $\text{CIR}_i^k \leq \Upsilon$ , then  $\text{CIR}_i^{k+1} \geq \text{CIR}_i^k$ .

*Proof:* Suppose  $\text{CIR}_i^k \leq \Upsilon$ . As a result, we get  $\eta_i^k = 1$  and  $T_i^{k+1} = T_i^k$ . Since  $\sum_{j \neq i} Z_{ij} T_j^{k+1} \leq \sum_{j \neq i} Z_{ij} T_j^k$  (a consequence of Property 1), we have

$$\begin{aligned} \text{CIR}_i^{k+1} &= \frac{T_i^{k+1}}{\sum_{j \neq i} Z_{ij} T_j^{k+1}} = \frac{T_i^k}{\sum_{j \neq i} Z_{ij} T_j^{k+1}} \\ &\geq \frac{T_i^k}{\sum_{j \neq i} Z_{ij} T_j^k} = \text{CIR}_i^k. \end{aligned} \quad (7)$$

This completes the proof of Property 2.  $\square$

*Property 3:* If  $\text{CIR}_i^k \geq \Upsilon$ , then  $\text{CIR}_i^n \geq \Upsilon$  for all  $n \geq k$ .

*Proof:* To prove Property 3, it suffices to show that  $\text{CIR}_i^k \geq \Upsilon$  implies  $\text{CIR}_i^{k+1} \geq \Upsilon$ . It is clear that

$$\text{CIR}_i^{k+1} = \frac{T_i^{k+1}}{\sum_{j \neq i} Z_{ij} T_j^{k+1}} = \frac{\eta_i^k T_i^k}{\sum_{j \neq i} Z_{ij} T_j^{k+1}}. \quad (8)$$

From Property 1, we have

$$\text{CIR}_i^{k+1} = \frac{\eta_i^k T_i^k}{\sum_{j \neq i} Z_{ij} T_j^{k+1}} \geq \frac{\eta_i^k T_i^k}{\sum_{j \neq i} Z_{ij} T_j^k} = \eta_i^k * \text{CIR}_i^k. \quad (9)$$

Since  $\text{CIR}_i^k \geq \Upsilon$ ,  $\eta_i^k$  is given by

$$\eta_i^k = \frac{\Upsilon}{\text{CIR}_i^k} \quad (10)$$

which when substituted into (9) gives

$$\text{CIR}_i^{k+1} \geq \left( \frac{\Upsilon}{\text{CIR}_i^k} \right) \text{CIR}_i^k = \Upsilon. \quad (11)$$

This completes the proof of Property 3.  $\square$

*Property 4:* If  $\Upsilon \leq \min_i \{\text{CIR}_i^0\}$ , then  $T_i^{k+1} = \Upsilon * I_i^k$ , where  $I_i^k = \sum_{j \neq i} Z_{ij} T_j^k$ .

*Proof:* Suppose  $\Upsilon \leq \min_i \{\text{CIR}_i^0\}$ . From Property 3, one gets  $\Upsilon \leq \min_i \{\text{CIR}_i^k\}$  for  $k \geq 1$ . Hence, for  $k \geq 0$ , we have  $\eta_i^k = \frac{\Upsilon}{\text{CIR}_i^k}$  and

$$T_i^{k+1} = \eta_i^k * T_i^k = \frac{\Upsilon}{\text{CIR}_i^k} * T_i^k = \Upsilon * I_i^k. \quad (12)$$

End of proof, Property 4.  $\square$

To prove Property 5, i.e., CIR balancing with probability one when  $\text{CIR}^* \geq \Upsilon$ , we need the following lemma.

*Lemma 1:* Let  $\mathbf{T}$  be a positive power vector, i.e., a vector with all entries greater than zero. Let  $\text{CIR}_j = \frac{T_j}{\sum_{i \neq j} Z_{ji} T_i}$ ,  $1 \leq j \leq N$ . If there exists an  $i$  such that  $\text{CIR}_i < \text{CIR}^*$ , then there exists a  $k$  such that  $\text{CIR}_k > \text{CIR}^*$ . Conversely, if there exists an  $i$  such that  $\text{CIR}_i > \text{CIR}^*$ , then there exists a  $k$  such that  $\text{CIR}_k < \text{CIR}^*$ .

*Proof:* For convenience, we shall use  $\text{CIR}_i(\mathbf{T})$  to represent the CIR at mobile  $i$  when the transmitting power vector is  $\mathbf{T}$ . Suppose  $\text{CIR}_i(\mathbf{T}) < \text{CIR}^*$  and there does not exist a  $k$  such that  $\text{CIR}_k > \text{CIR}^*$ . That is,  $\text{CIR}_j \leq \text{CIR}^*$  for all  $j$ ,  $1 \leq j \leq N$ .

Let  $\mathbf{e}_i = [x_1, x_2, \dots, x_N]^T$ , where

$$x_j = \begin{cases} 1, & \text{if } j = i \\ 0, & \text{if } j \neq i. \end{cases} \quad (13)$$

Set  $\mathbf{T}' = \mathbf{T} + \epsilon \mathbf{e}_i$ , where  $\epsilon > 0$ . For the transmitting power vector  $\mathbf{T}'$ , we have

$$\begin{aligned} \text{CIR}_i(\mathbf{T}') &= \frac{T_i + \epsilon}{\sum_{w \neq i} Z_{iw} T'_w} = \frac{T_i + \epsilon}{\sum_{w \neq i} Z_{iw} T_w} \\ &= \text{CIR}_i(\mathbf{T}) + \frac{\epsilon}{I_i} \end{aligned} \quad (14)$$

where  $I_i = \sum_{w \neq i} Z_{iw} T_w$ . Since  $\text{CIR}_i(\mathbf{T}) < \text{CIR}^*$ , one can choose  $\epsilon$  sufficiently small (e.g.,  $\epsilon = \frac{1}{2} I_i (\text{CIR}^* - \text{CIR}_i(\mathbf{T}))$ ) to satisfy  $\text{CIR}_i(\mathbf{T}') < \text{CIR}^*$ . Moreover, for other  $j \neq i$ , we have

$$\text{CIR}_j(\mathbf{T}') = \frac{T_j}{\sum_{w \neq j} Z_{jw} T'_w} = \frac{T_j}{\sum_{w \neq i, j} Z_{jw} T_w + Z_{ji} (T_i + \epsilon)} < \text{CIR}_j(\mathbf{T}), \quad (15)$$

which implies  $\text{CIR}_j(\mathbf{T}') < \text{CIR}^*$  because  $\text{CIR}_j(\mathbf{T}) \leq \text{CIR}^*$ . Consequently, we have  $\text{CIR}_m(\mathbf{T}') < \text{CIR}^*$  for all  $m$ ,  $1 \leq m \leq N$ . This contradicts with the fact [4]

$$\text{CIR}^* = \min_{\mathbf{T} > 0} \max_{1 \leq i \leq N} \{\text{CIR}_i\}. \quad (16)$$

The statement that  $\text{CIR}_i > \text{CIR}^*$  for some  $i$  implies  $\text{CIR}_k < \text{CIR}^*$  for some  $k$  can be similarly proved. This completes the proof of Lemma 1.

*Property 5:* If  $\Upsilon \leq \text{CIR}^*$ , then  $\lim_{k \rightarrow \infty} \text{CIR}_i^k = \text{CIR}^*$  for all  $i$ .

*Proof:* Consider first the case  $\Upsilon < \text{CIR}^*$ . Suppose there is an  $i$  such that  $\text{CIR}_i^k \leq \Upsilon$  for all  $k$ . Since the co-channel set is finite, we know, according to Lemma 1, that there exists an  $m$  such that  $\text{CIR}_m^k > \text{CIR}^*$  for infinitely many  $k$ . As a result,  $\eta_m^k < \frac{\Upsilon}{\text{CIR}^*}$  for infinitely many  $k$  and we have

$$\begin{aligned} \text{CIR}_m^n &= \frac{T_m^n}{\sum_{w \neq m} Z_{mw} T_w^n} = \frac{\prod_{j=0}^{n-1} \eta_m^j}{\sum_{w \neq m} Z_{mw} T_w^n} \\ &\leq \frac{\prod_{j=0}^{n-1} \eta_m^j}{Z_{mi}} \rightarrow 0, \quad \text{as } n \rightarrow \infty \end{aligned} \quad (17)$$

a contradiction to  $\text{CIR}_m^k > \text{CIR}^*$  for infinitely many  $k$ . Therefore, there is a  $k_0$  such that  $\text{CIR}_j^k > \Upsilon$  for all  $k \geq k_0$ . Moreover, from iteration  $k_0 + 1$ , the FDPC algorithm becomes

$$T_i^k = \frac{\Upsilon}{\text{CIR}_i^{k-1}} \mathbf{T}_i^{k-1} = \Upsilon I_i^{k-1}, \quad k \geq k_0 + 1 \quad (18)$$

with initial transmitting power vector  $\mathbf{T}^{k_0}$ . In [7], it was proved that such an algorithm can achieve CIR balancing with probability one.

Consider now the case  $\Upsilon = \text{CIR}^*$ . Let  $\epsilon$  be an arbitrary small positive real number. Suppose there is an  $i$  such that  $\text{CIR}_i^k \leq \Upsilon - \epsilon$  for infinitely many  $k$ . According to Lemma 1, we know that there exists an  $m$  such that  $\text{CIR}_m^k > \Upsilon$  for infinitely many  $k$ . Consequently, we have

$$\begin{aligned} \text{CIR}_m^n &= \frac{T_m^n}{\sum_{w \neq m} Z_{mw} T_w^n} = \frac{\prod_{j=0}^{n-1} \eta_m^j}{\sum_{w \neq m} Z_{mw} T_w^n} \\ &\leq \frac{\prod_{j=0}^{n-1} \eta_m^j}{Z_{mi}} \rightarrow 0, \quad \text{as } n \rightarrow \infty \end{aligned} \quad (19)$$

which is again a contradiction. Therefore, we get  $\lim_{k \rightarrow \infty} \text{CIR}_i^k \geq \Upsilon$  for all  $i$ . Since  $\text{CIR}^* = \max_{\mathbf{T} > 0} \min_{1 \leq i \leq N} \{\text{CIR}_i\}$  [4], we conclude that  $\lim_{k \rightarrow \infty} \text{CIR}_i^k = \Upsilon$  for all  $i$ .

End of proof, Property 5.  $\square$

*Remarks:* Property 1 states that the transmitting power sequence for each base station is monotonic decreasing. Properties 2 and 3 suggest to choose  $\Upsilon$  to be greater than or equal to  $\overline{\text{CIR}}$ . In fact, from numerical results presented in Section V, we found that  $\Upsilon = \overline{\text{CIR}}$  is the best choice. Property 4 indicates that our proposed FDPC algorithm reduces to the distributed power control (DPC) algorithm proposed in [7] if  $\Upsilon$  is chosen to be an extremely small number. (Remember the DPC algorithm updates transmitting power according to  $T_i^{k+1} = \beta I_i^k$  where  $\beta > 0$  is the normalization factor.) Property 5 states that the proposed FDPC algorithm can achieve CIR balancing with probability one as long as  $\Upsilon \leq \text{CIR}^*$ .

Unfortunately, the FDPC algorithm also possesses an unpleasant feature. According to Property 5, if  $\text{CIR}^* > \Upsilon$ , then CIR balancing can be achieved with probability one, which implies there exists a  $k_0$  such that  $\text{CIR}_i^k > \Upsilon$  for all  $i$  and  $k \geq k_0$ . Consequently,  $T_i^n$  approaches zero as  $n$  increases. This feature may cause problem to a real system in which thermal noise cannot be omitted. In the next section, we suggest two approaches to deal with this problem.

#### IV. FDPC WITH FINITE POWER RANGE CONSTRAINT

Suppose the transmitting power of each base station is restricted to be within  $[\delta, 1]$ . That is, the dynamic range is  $-10 \log_{10} \delta$  dB. For example, the dynamic range is 10 dB if  $\delta = 0.1$  or 20 dB if  $\delta = 0.01$ . Let  $L$  denote the number of iterations. The following two schemes, called FDPC-I and FDPC-II, guarantee the transmitting power of each base station falls in the range  $[\delta, 1]$ .

Scheme FDPC-I:

$$\mathbf{T}^0 = 1$$

and

$$T_i^{k+1} = \max(\eta_i^k * T_i^k, \delta) \quad (20)$$

where

$$\eta_i^k = \frac{\min(\text{CIR}_i^k, \Upsilon)}{\text{CIR}_i^k}, \quad 0 < \Upsilon < \infty.$$

Scheme FDPC-II:

$$\mathbf{T}^0 = 1$$

and

$$T_i^{k+1} = \eta_i^k * T_i^k \quad (21)$$

where

$$\eta_i^k = \max\left(\frac{\min(\text{CIR}_i^k, \Upsilon)}{\text{CIR}_i^k}, \eta_{\min}\right), \quad 0 < \Upsilon < \infty \quad (22)$$

and  $\eta_{\min} = \delta^{1/L}$ .

Scheme FDPC-I simply forces the transmitting power to be  $\delta$  if the value calculated from (4) is smaller than  $\delta$ . Scheme FDPC-II limits the maximum decrement of transmitting power in each iteration to satisfy the dynamic range constraint. Notice that Properties 1–3 still hold for both FDPC-I and FDPC-II

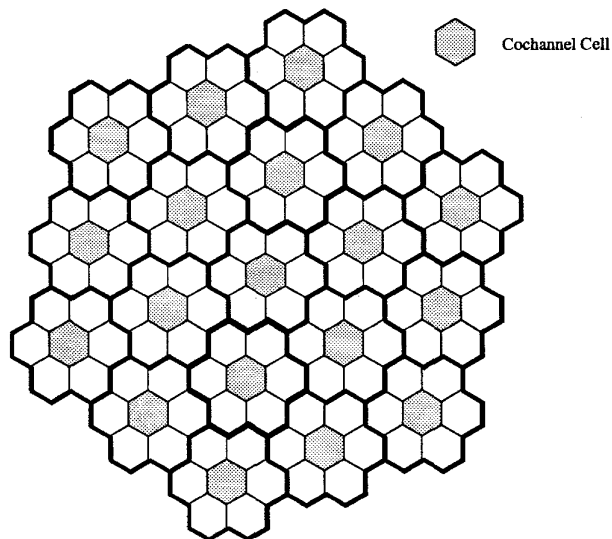


Fig. 1. 19 co-channel cells with seven-cell cluster.

schemes. The performance of FDPC-I and FDPC-II schemes are studied in the following section.

#### V. NUMERICAL EXAMPLES AND DISCUSSIONS

In this section, we study the outage probability of the proposed FDPC algorithm with numerical examples. A removal algorithm which step by step removes one cell, if necessary, until the CIR at each active mobile is greater than or equal to  $\overline{\text{CIR}}$  is adopted. We assume the FDPC algorithm is performed for  $L$  iterations. For the purpose of removing cells when necessary,  $\text{CIR}^0$  is reported to a Mobile Telephone Switching Office (MTSO). After  $L$  iterations, base station  $i$  sends a message to the MTSO if  $\text{CIR}_i^L < \overline{\text{CIR}}$ . The MTSO notifies cell  $i$  to cease transmission if  $\text{CIR}_i^0 = \min_j \{\text{CIR}_j^0\}$ . The FDPC algorithm is reinitialized after a removal.

Numerical examples were obtained for a system containing 19 co-channel cells that uses a seven-cell cluster (see Fig. 1). All the values were accomplished by means of Monte Carlo simulation for 500 independent configurations. In our study, we use  $\sigma = 6$  and  $\nu = 4$ . Outage probability is measured over all 19 co-channels.

The average number of iterations required to reach a certain percentage of  $\text{CIR}^*$  for the FDPC algorithm is shown in Table I. In this table, DPC denotes the distributed power control algorithm presented in [7], which as shown in Property 4 can be considered as a special case of our proposed FDPC algorithm. It can be seen from the entries of Table I that the performance of the DPC algorithm can be improved. Table II gives the average number of iterations required to achieve CIR at every mobile station greater than  $\overline{\text{CIR}}$  for three different values of  $\overline{\text{CIR}}$ . From the values listed in Table II, we find that  $\Upsilon = \overline{\text{CIR}}$  gives the best performance. A "\*" in Table II means  $\Upsilon > \text{CIR}_i^0$  for all  $i$  and  $\text{CIR}_j^0 < \overline{\text{CIR}}$  for some  $j$  happened at least once in our simulations. When it happens,  $\text{CIR}_j^k < \overline{\text{CIR}}$  for all  $k$  and, thus, the average number of iterations to achieve  $\text{CIR}_i^k > \overline{\text{CIR}}$  for all  $i$  becomes infinity. Notice that the FDPC algorithm can achieve  $\text{CIR}_i^k > \overline{\text{CIR}}$  very fast (in less than

TABLE I  
THE AVERAGE NUMBER OF ITERATIONS REQUIRED TO REACH A CERTAIN PERCENTAGE OF CIR\* FOR THE DPC AND FDPC ALGORITHMS

	$\Upsilon = 10$ dB			$\Upsilon = 5$ dB			$\Upsilon = 1$ dB		
	90%	95%	98%	90%	95%	98%	90%	95%	98%
DPC	8.282	12.830	19.964	8.836	13.472	20.724	8.830	13.474	20.736
FDPC	4.784	8.328	14.570	7.106	11.538	18.760	8.320	12.978	20.348

TABLE II  
THE AVERAGE NUMBER OF ITERATIONS REQUIRED TO ACHIEVE CIR AT EVERY MOBILE STATION GREATER THAN CIR FOR THREE DIFFERENT VALUES OF CIR

CIR	$\Upsilon$									
	17 dB	15 dB	12 dB	10 dB	8 dB	5 dB	1 dB	0.1 dB		
5 dB	*	*	*	*	0.798	0.734	1.730	2.006		
10 dB	*	*	1.496	1.272	2.389	3.880	5.068	5.273		
15 dB	2.384	1.674	2.594	3.204	3.716	4.186	4.338	4.352		

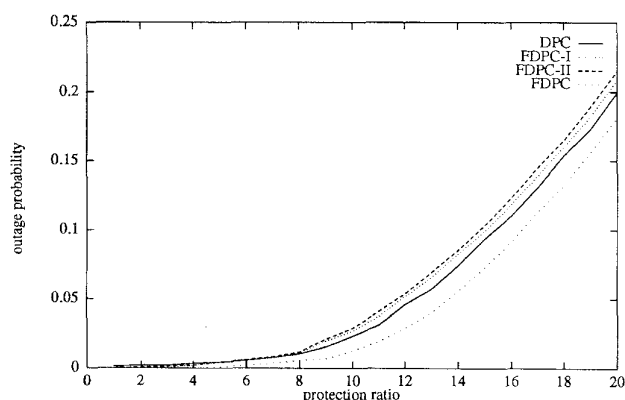


Fig. 2. Outage probability as a function of protection ratio for various algorithms. The dynamic power range is 10 dB for both FDPC-I and FDPC-II schemes.

three iterations if  $\Upsilon = \overline{\text{CIR}}$ . The entries of Tables I and II are obtained without dynamic power range constraint (i.e.,  $\delta = 0$ ). Also, no cell removal is conducted in generating the tables.

Shown in Figs. 2–5 are the outage probabilities as a function of protection ratio for different dynamic power ranges. The number of iterations  $L$  is chosen to be eight, a reasonable value for typical vehicle speeds [5]. The curves labeled FDPC are for the proposed algorithm without dynamic power range constraint. It can be seen that Scheme FDPC-I has a smaller outage probability than Scheme FDPC-II does. When the dynamic power range is greater than or equal to 15 dB, both FDPC-I and FDPC-II schemes outperform the DPC algorithm, as demonstrated in Figs. 3 and 4. Also, for a dynamic power range of 20 dB, the performance of the FDPC-I scheme is very close to that of the proposed FDPC algorithm without dynamic power range constraint.

## VI. CONCLUSION

We have in this paper proposed and proved some nice properties of a fully distributed power control algorithm for cellular mobile systems. Elimination of a normalization procedure renders fully distributed power control of our proposed algorithm possible. Normalization could be helpful if one intends to reduce transmitting powers. However, it causes transmission and real-time processing of data between base

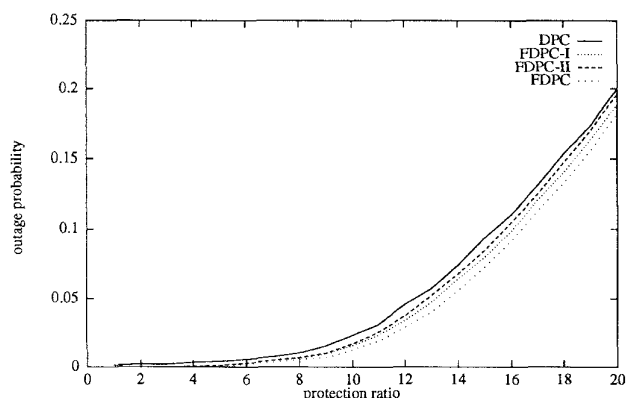


Fig. 3. Outage probability as a function of protection ratio, for various algorithms. The dynamic power range is 15 dB for both FDPC-I and FDPC-II schemes.

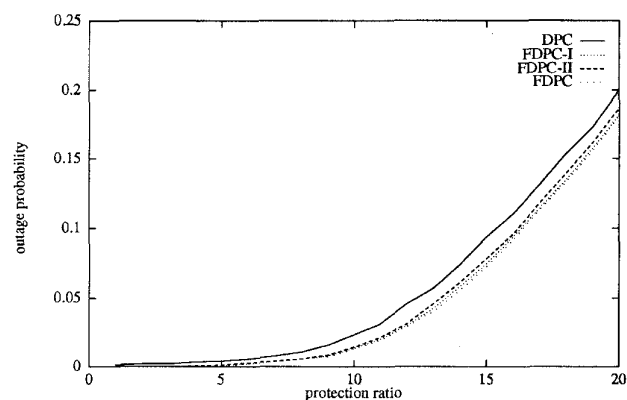


Fig. 4. Outage probability as a function of protection ratio for various algorithms. The dynamic power range is 20 dB for both FDPC-I and FDPC-II schemes.

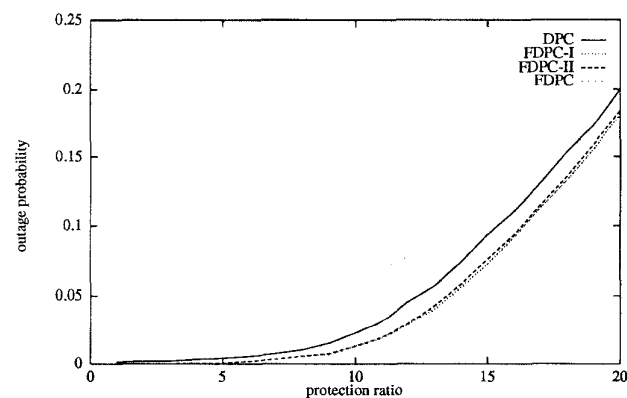


Fig. 5. Outage probability as a function of protection ratio for various algorithms. The dynamic power range is 25 dB for both FDPC-I and FDPC-II schemes.

stations and the central station which obviously increases system complexity and, therefore, may not be necessary. Through extensive computer simulations, we found the proposed FDPC algorithm gives excellent performance. The effect of finite power range to our proposed FDPC algorithm is

also investigated. Numerical results show that for  $L = 8$ , the degradation in terms of outage probability is negligible as long as the dynamic range is greater than 20 dB, which is practical in most real world systems. The excellent performance and the fully distributed property make the proposed FDPC algorithm a good choice for cellular mobile systems.

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