Spin Hall Effect on Edge Magnetization and Electric Conductance of a 2D Semiconductor Strip

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The intrinsic spin Hall effect on spin accumulation and electric conductance in a diffusive regime of a 2D electron gas has been studied for a 2D strip of a finite width. It is shown that the spin polarization near the flanks of the strip, as well as the electric current in the longitudinal direction, exhibit damped oscillations as a function of the width and strength of the Dresselhaus spin-orbit interaction. Cubic terms of this interaction are crucial for spin accumulation near the edges. As expected, no effect on the spin accumulation and electric conductance have been found in case of Rashba spin-orbit interaction.

Spintronics is a fast developing area to use electron spin degrees of freedom in electronic devices [1]. One of its most challenging goals is to find a method for manipulating electron spins by electric fields. The spin-orbit interaction (SOI), which couples the electron momentum and spin, can be a mediator between the charge and spin degrees of freedom. Such a coupling gives rise to the so-called spin Hall effect (SHE) which attracted much interest recently. Because of SOI the spin flow can be induced perpendicular to the dc electric field, as has been predicted for systems containing spin-orbit impurity scatterers [2]. Later, similar phenomenon was predicted for noncentrosymmetric semiconductors with spin split electron and hole energy bands [3]. It was called the *intrinsic* spin Hall effect, in contrast to the *extrinsic* impurity induced effect, because in the former case it originates from the electronic band structure of a semiconductor sample. Since the spin current carries the spin polarization, one would expect a buildup of the spin density near the sample boundaries. In fact, this accumulated polarization is a first signature of SHE which has been detected experimentally, confirming thus the extrinsic SHE [4] in semiconductor films and intrinsic SHE in a 2D hole gas [5]. On the other hand, there was still no experimental evidence of intrinsic SHE in 2D electron gases. The possibility of such an effect in macroscopic samples with a finite elastic mean free path of electrons caused recently much debates. It has been shown analytically [6–11] and numerically [12] that in such systems SHE vanishes at arbitrary weak disorder in dc limit for isotropic as well as anisotropic [10] impurity scattering when SOI is represented by the so-called Rashba interaction [13]. As one can expect in this case, there is no spin accumulation at the sample boundaries, except for the pockets near the electric contacts [7]. At the same time, the Dresselhaus SOI [14], which dominates in symmetric quantum wells, gives a finite spin Hall conductivity [11]. The latter can be of the order of its universal value $e/8\pi\hbar$. The same has been shown for the cubic Rashba interaction in hole systems [12,15]. In this connection an important question is what

sort of the spin accumulation could Dresselhaus SOI induce near sample boundaries. Another problem which, as far as we know, was not discussed in literature, is how the *electric* current along the applied electric field will change under SHE. In the present work we will use the diffusion approximation for the electron transport to derive the drift-diffusion equations with corresponding boundary conditions for the spin and charge densities coupled to each other via SOI of general form. Then the spin density near the flanks of an infinite 2D strip and the correction to its longitudinal electric resistance will be calculated for Dresselhaus and Rashba SOI.

Let us consider two-dimensional electron gas (2DEG) confined in an infinite 2D strip. The boundaries of the strip are at $y = \pm d/2$. The electric field E drives the dc current in the x direction and induces the spin Hall current in the y direction. This current leads to spin polarization buildup near boundaries. Since $d \gg k_F^{-1}$, where k_F is the Fermi wave vector, this problem can be treated within the semiclassical approximation. Moreover, we will assume that d is much larger than the electron elastic mean free path l, so that the drift-diffusion equation can be applied for description of the spin and charge transport. Our goal is to derive this equation for SOI of general form

$$H_{so} = \boldsymbol{h_k} \cdot \boldsymbol{\sigma}, \tag{1}$$

where $\boldsymbol{\sigma} \equiv (\sigma^x, \sigma^y, \sigma^z)$ is the Pauli matrix vector, and the effective magnetic field $\boldsymbol{h_k} = -\boldsymbol{h_{-k}}$ is a function of the two-dimensional wave vector $\boldsymbol{k}$.

We start from determining linear responses to the magnetic $\boldsymbol{B(r, t)}$ and electric $V(\boldsymbol{r}, t)$ potentials. The magnetic potentials are introduced in order to derive the diffusion equation and play an auxiliary role. The corresponding one-particle interaction with the spin density is defined as $H_{sp} = \boldsymbol{B(r, t)} \cdot \boldsymbol{\sigma}$. These potentials induce the spin and charge densities, $\boldsymbol{S(r, t)}$ and $n(\boldsymbol{r}, t)$, respectively. Because of SOI the charge and spin degrees of freedom are coupled, so that the electric potential can induce the spin density [16] and *vice versa*. Therefore, it is convenient to introduce

the four vector of densities $D_i(\boldsymbol{r}, t)$, such that $D_0(\boldsymbol{r}) = n(\boldsymbol{r}, t)$ and $D_{x,y,z}(\boldsymbol{r}, t) = S_{x,y,z}(\boldsymbol{r}, t)$. The corresponding four vector of potentials will be denoted as $\Phi_i(\boldsymbol{r}, t)$. Accordingly, the linear response equations can be written in the form

$$D_i(\boldsymbol{r}, t) = \int d^2 r' dt' \sum_j \Pi_{ij}(\boldsymbol{r}, \boldsymbol{r}', t - t')\Phi_j(\boldsymbol{r}', t')$$
$$+ D_i^0(\boldsymbol{r}, t). \qquad (2)$$

The response functions $\Pi_{ij}(\boldsymbol{r}, \boldsymbol{r}', t - t')$ can be expressed in a standard way [17] through the retarded and advanced Green functions $G^r(\boldsymbol{r}, \boldsymbol{r}', t)$ and $G^a(\boldsymbol{r}, \boldsymbol{r}', t)$. In the Fourier representation we get

$$\Pi_{ij}(\boldsymbol{r}, \boldsymbol{r}', \omega) = i\omega \int \frac{d\omega'}{2\pi} \frac{\partial n_F(\omega')}{\partial \omega'} \langle \mathrm{Tr}[G^a(\boldsymbol{r}', \boldsymbol{r}, \omega')$$
$$\times \Sigma_i G^r(\boldsymbol{r}, \boldsymbol{r}', \omega' + \omega)\Sigma_j]\rangle, \qquad (3)$$

where $\Sigma_0 = 1$, $\Sigma_i = \sigma_i$ at $i = x, y, z$, and $n_F(\omega)$ is the Fermi distribution function. The time Fourier components of densities $D_i^0(\boldsymbol{r}, t)$ at $\omega \ll E_F$ are defined as

$$D_i^0(\boldsymbol{r}, \omega) = i \int d^2 r' \sum_j \Phi_j(\boldsymbol{r}', \omega) \int \frac{d\omega'}{2\pi} n_F(\omega')$$
$$\times \langle \mathrm{Tr}[G^r(\boldsymbol{r}, \boldsymbol{r}', \omega')\Sigma_i G^r(\boldsymbol{r}', \boldsymbol{r}, \omega')\Sigma_j$$
$$- G^a(\boldsymbol{r}, \boldsymbol{r}', \omega')\Sigma_i G^a(\boldsymbol{r}', \boldsymbol{k}, \omega')\Sigma_j]\rangle. \qquad (4)$$

The trace in Eqs. (3) and (4) runs through the spin variables, and the angular brackets denote the average over the random distribution of impurities. Within the semiclassical approximation the average of the product of Green functions can be calculated perturbatively. Ignoring the weak localization effects, the perturbation expansion consists of the so-called ladder series [17,18]. At small ω and large $|\boldsymbol{r} - \boldsymbol{r}'|$ they describe the particle and spin diffusion processes. The building blocks for the perturbation expansion are the average Green functions $\mathcal{G}^r$ and $\mathcal{G}^a$, together with the pair correlator of the impurity scattering potential $U_{sc}(\boldsymbol{r})$. A simple model of the short-range isotropic potential gives $\langle U_{sc}(\boldsymbol{r})U_{sc}(\boldsymbol{r}')\rangle = \Gamma\delta(\boldsymbol{r} - \boldsymbol{r}')/\pi N_0$, where N_0 is the electron density of states at the Fermi energy and $\Gamma = 1/2\tau$. Within the semiclassical approach the explicit behavior of the electron wave functions near the boundaries of the strip is not important. Therefore, the bulk expressions can be used for the average Green functions. Hence, in the plane wave representation

$$\mathcal{G}^r(\boldsymbol{k}, \omega) = [\mathcal{G}^a(\boldsymbol{k}, \omega)]^\dagger = (\omega - E_{\boldsymbol{k}} - \boldsymbol{h}_{\boldsymbol{k}} \cdot \boldsymbol{\sigma} + i\Gamma)^{-1}, \qquad (5)$$

where $E_{\boldsymbol{k}} = k^2/(2m^*) - E_F$. Since the integral in (4) rapidly converges at $|\boldsymbol{r} - \boldsymbol{r}'| \lesssim k_F^{-1}$, $D_i^0(\boldsymbol{r}, \omega)$ are given by the local values of potentials. From (4) and (5) one easily obtains the local equilibrium densities

$$D_i^0(\boldsymbol{r}, \omega) = -2N_0\Phi_i(\boldsymbol{r}, \omega). \qquad (6)$$

In their turn, the nonequilibrium spin and charge densities are represented by the first term in Eq. (2). Within the diffusion approximation this term is given by the gradient expansion of (3) [18]. Such an expansion is valid as far as spatial variations of $D_i(\boldsymbol{r}, \omega)$ are relatively small within the length of the order of the mean free path l. The length scale for spin density variations near the boundaries of the strip is given by v_F/h_{k_F}. Hence, the diffusion approximation can be employed only in the dirty limit $h_{k_F} \ll 1/\tau$. The diffusion equation is obtained after the ladder summation in the first term of Eq. (3) and multiplying this equation by the operator inverse to $\Pi_{ij}(\boldsymbol{r}, \boldsymbol{r}', \omega)$, as it has been previously done in [19,20]. After some algebraic manipulations one gets

$$\sum_j \mathcal{D}^{ij}(D_j - D_j^0) = -i\omega D_i, \qquad (7)$$

where the diffusion operator $\mathcal{D}^{ij}$ can be written as

$$\mathcal{D}^{ij} = \delta^{ij} D\nabla^2 - \Gamma^{ij} + R^{ijm}\nabla_m + M^{ij}. \qquad (8)$$

The first term represents the usual diffusion of the spin and charge densities, while the second one describes the D'akonov-Perel' [21] spin relaxation

$$\Gamma^{ij} = 4\tau\overline{h_{\boldsymbol{k}}^2[\delta^{ij} - n_{\boldsymbol{k}}^i n_{\boldsymbol{k}}^j]}, \qquad (9)$$

where $i, j \neq 0$, the overline denotes the average over the Fermi surface, and $\boldsymbol{n}_{\boldsymbol{k}} = \boldsymbol{h}_{\boldsymbol{k}}/h_{\boldsymbol{k}}$. The third term gives rise to precession of the inhomogeneous spin polarization in the effective field of SOI [19]

$$R^{ijm} = 4\tau\sum_l \varepsilon^{ijl}\overline{h_{\boldsymbol{k}}^l v_F^m}. \qquad (10)$$

The nondiagonal elements of the form $\mathcal{D}^{i0}$ appear due to spin-orbit mixing of spin and charge degrees of freedom. They are collected in M^{ij}. For Rashba SOI M^{i0} have been calculated in [7,8]. In general case

$$M^{i0} = \overline{\frac{h_{\boldsymbol{k}}^3}{\Gamma^2} \frac{\partial n_{\boldsymbol{k}}^i}{\partial \boldsymbol{k}}} \cdot \boldsymbol{\nabla}. \qquad (11)$$

When a time independent homogeneous electric field is applied to the system one has $\Phi_0 = eEx$ and $D_0^0 = -2N_0eEx$. At the same time, $\Phi_i = 0$ and, hence, $D_i^0 = 0$ at $i = x, y, z$. Because of charge neutrality the induced charge density $eD_0 = 0$. It should be noted that in the system under consideration the charge neutrality cannot be fulfilled precisely. The spin polarization accumulated at the strip boundaries gives rise to charge accumulation via the M^{0i} terms in (7) and (8). The screening effect will, however, strongly reduce this additional charge, because the screening length of 2DEG is much less than the typical length scale of spin density variations. We will ignore such a small correction and set $D_0 = 0$ in (7). In this way one

arrives to the closed diffusion equation for three components of the spin density. This equation coincides with the usual equation describing diffusive propagation of the spin density [19], for exception of the additional term $-M^{i0}D_0^0 = 2N_0eE\overline{h_{\boldsymbol{k}}^3 \nabla_{\boldsymbol{k}}^x n_{\boldsymbol{k}}^i}/\Gamma^2$ due to the external electric field. Its origin becomes more clear in an infinite system where the spin density is constant in space and only Γ^{ij} and M^{ij} are retained in (7) and (8). Hence, the corresponding solution of (8) at $\omega = 0$ is $S_i \equiv S_i^{\mathrm{b}}$, with

$$S_i^{\mathrm{b}} \equiv D_i^0/2 = \frac{N_0eE}{\Gamma^2}\sum_j(\Gamma^{-1})^{ij}\overline{h_{\boldsymbol{k}}^3 \frac{\partial n_{\boldsymbol{k}}^j}{\partial k_x}}, \quad (12)$$

where $(\Gamma^{-1})^{ij}$ is the matrix inverse to (9). Such a phenomenon of spin orientation by the electric field was predicted in Ref. [16] and recently observed in [22]. In the special case of Rashba SOI $\boldsymbol{h_k} = \alpha(k)(\boldsymbol{k} \times \boldsymbol{z})$ it is easily to get from (12) the result of Ref. [16] $S_y^{\mathrm{b}} = -N_0eE\alpha\tau$.

In addition to the diffusion equation one needs the boundary conditions. These conditions are that the three components of the spin flux I_x^y, I_y^y, I_z^y flowing in the y direction turn to 0 at $y = \pm d/2$. The linear response theory, similar to (2), gives

$$I_i^l(\boldsymbol{r}, t) = \int d^2r'dt'\sum_j \Xi_{ij}^l(\boldsymbol{r}, \boldsymbol{r}', t - t')\Phi_j(\boldsymbol{r}', t'), \quad (13)$$

where the response function Ξ is given by

$$\Xi_{ij}^l(\boldsymbol{r}, \boldsymbol{r}', \omega) = i\omega\int\frac{d\omega'}{2\pi}\frac{\partial n_F(\omega')}{\partial\omega'}\langle\mathrm{Tr}[G^a(\boldsymbol{r}', \boldsymbol{r}, \omega')$$
$$\times J_i^l G^r(\boldsymbol{r}, \boldsymbol{r}', \omega' + \omega)\Sigma_j]\rangle, \quad (14)$$

with the one-particle spin-current operator defined by $J_i^l = (\sigma^i v_l + v_l\sigma^i)/4$ and the particle velocity

$$v_l = \frac{k_l}{m^*} + \frac{\partial}{\partial k_l}(\boldsymbol{h_k} \cdot \boldsymbol{\sigma}). \quad (15)$$

Taking into account (7) and (6), we obtain from (13) and (14)

$$I_i^y(\boldsymbol{r}) = -D\frac{\partial S_i}{\partial y} - \frac{1}{2}R^{ijy}(S_j - S_j^{\mathrm{b}}) + \delta_{iz}I_{sH}. \quad (16)$$

The first two terms represent the diffusion spin current and the current associated with the spin precession. The third term is the uniform spin Hall current polarized along the z axis. It is given by

$$I_{sH} = -\frac{1}{2}R^{zjy}S_j^{\mathrm{b}} + eE\frac{N_0}{\Gamma^2}\overline{v_F^y\left(\frac{\partial\boldsymbol{h_k}}{\partial k^x} \times \boldsymbol{h_k}\right)_z}. \quad (17)$$

From (10) and (12) it is easy to see that for Rashba SOI both terms in (17) cancel each other making $I_{sH} = 0$, in accordance with [6–12]. Therefore, in case of the strip the solution of the diffusion equation satisfying the boundary condition is $S_j = \delta_{jy}S_y^{\mathrm{b}}$. Hence, the spin density is uniform and does not accumulate near boundaries. It should be

noted that such accumulation can, however, take place in the ballistic regime of electron scattering [23]. At the same time, as shown in Ref. [11], even in the diffusive regime $I_{sH} \neq 0$ for the Dresselhaus SOI. This inevitably leads to the spin accumulation. Taking Dresselhaus SOI in the form

$$h_{\boldsymbol{k}}^x = \beta k_x(k_y^2 - \kappa^2); \qquad h_{\boldsymbol{k}}^y = -\beta k_y(k_x^2 - \kappa^2), \quad (18)$$

one can see that the bulk spin polarization (12) has a nonzero S_x^{b} component, $R^{zxy} \neq 0$, while $R^{zyy} = 0$. Hence, the solution of the diffusion Eq. (7) with the boundary condition $I_x^y(\pm d/2) = I_z^y(\pm d/2) = 0$ is $S_x, S_z \neq 0$ $S_y = 0$. Let us define $\Delta S_i(y) = S_i(y) - S_i^{\mathrm{b}}$. The dependence of $\Delta S_i(\pm d/2)$ from the strip width, as well as an example of ΔS_z coordinate dependence, are shown in Fig. 1. The damped oscillation in the d-dependence of the spin accumulation on the flanks of the strip can be seen for the S_z polarization. Similar oscillations take place also in the coordinate dependence. The length scale of these oscillations is determined by the spin precession in the effective spin-orbit field.

The arbitrary units have been used in Fig. 1. For a numerical evaluation let us take $E = 10^4$ V/m, $\sqrt{\overline{h_{\boldsymbol{k}_F}^2}}\tau/\hbar = 0.1$, and $\kappa/k_F = 0.8$ for a GaAs quantum
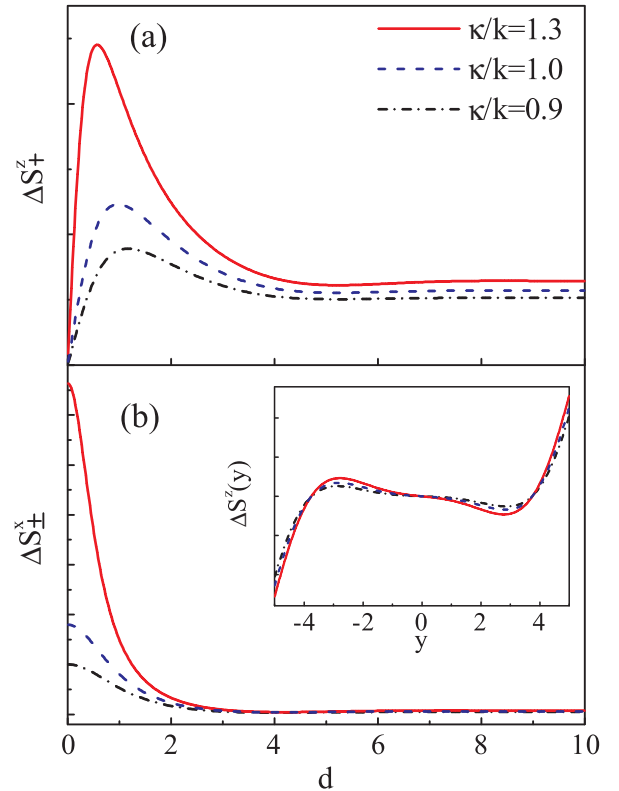

FIG. 1 (color online). Spin densities $\Delta S_i(\pm d/2) \equiv \Delta S_{\pm}^i$ for $i = x, z$ on the boundaries of the strip, as functions of its width d, for $\kappa/k = 0.9$, 1.0, and 1.3, respectively. The inset shows the dependence of $\Delta S_z(y)$ on the transverse coordinate y. Lengths are measured in units of $l_{so} = v_F^2\hbar/2(\overline{v_{Fy}h_{ky}})$.

well of the width $w = 100$ Å doped with 1.5×10^{15} m^{-2} electrons. We thus obtain $|\Delta S_z(\pm d/2)| \simeq 5 \times 10^{11}$ m^{-2}. The corresponding volume density $\Delta S_z/w \simeq 5 \times 10^{19}$ m^{-3}, which is within the sensitivity range of the Faraday rotation method [4].

It should be noted that in the considered here "dirty" limit $\sqrt{\overline{h_{k_F}^2}}\,\tau/\hbar \ll 1$ the spin Hall current is suppressed by the impurity scattering. As shown in [11,12] for Dresselhaus and cubic Rashba SOI, this current decreases as $\overline{h_{k_F}^2}\,\tau^2/\hbar^2$ down from its highest universal value. At the same time, an analysis of the diffusion equation shows that the accumulated at the flanks of the strip spin density decreases slower, as $\sqrt{\overline{h_{k_F}^2}}\,\tau/\hbar$. This explains why for the considered above realistic numerical parameters, even in the dirty case, the noticeable spin polarization can be accumulated near the boundary.

Usually, the spin Hall effect is associated with the spin polarization flow, or the spin density accumulation on the sample edges, in response to the electric field. On the other hand, this effect can show up in the *electric* conductance as well. To see such an effect we take 0-projection of (13), which by definition is the electric current. The current flows along the x axis. The corresponding response function Ξ_{0j}^x is given by (14) with $J_0 = v_x$. Using Eqs. (14), (15), and (7), and expressing Φ_i from (6) one gets the electric current density

$$I^x = \sigma E + A \frac{\partial S_z}{\partial y}, \tag{19}$$

where σ is the Drude conductivity and

$$A = e\frac{1}{2\Gamma^2}\left[2\overline{v_F^y\left(\frac{\partial \boldsymbol{h_k}}{\partial k^x}\times \boldsymbol{h_k}\right)_z} + \overline{v_F^x\left(\frac{\partial \boldsymbol{h_k}}{\partial k^y}\times \boldsymbol{h_k}\right)_z}\right]. \tag{20}$$

The total current is obtained by integrating (19) over y. Therefore, the spin Hall correction to the strip conductance

$$\Delta G = \frac{A}{E}[S_z(d/2) - S_z(-d/2)] = \frac{2A}{E}S_z(d/2). \tag{21}$$

Hence, the dependence of ΔG on the strip width coincides with that of the spin density shown in Fig. 1(a).

In conclusion, we employed the diffusion approximation to study the spin Hall effect in an infinite 2D strip. In case of the Dresselhaus spin-orbit interaction this effect leads to spin accumulation near the flanks of the strip, as well as to a correction to the longitudinal electric conductance. Both the spin accumulation and the conductance exhibit damped oscillations as a function of the strip width.

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