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Short Communication

Technical note – On optimization approach for multidisk vertical allocation problems

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Abstract

Multidisk vertical allocation (MDVA) problems intend to find an allocation of relations to disks such that the expected query cost is minimized. Recently, Chang [European Journal of Operational Research 143 (2002) 210] modified Rotem et al.'s [IEEE Transactions on Knowledge and Data Engineering 5 (1993) 882] method for solving an MDVA problem using a smaller number of binary variables. Chang's method however is unable to treat MDVA problems with possible replication of relations. This paper proposes another method to solve MDVA problems, which is more effective than Rotem et al.'s and is able to treat replication problems insolvable by Chang's method.

Keywords: Data allocation; Linearization; Global optimization

1. Introduction

Early research by Rotem et al. [3] proposed a mathematical model for multidisk vertical allocation (MDVA) problems. Their model minimizes the expected query cost subjected to the constraints that all relations have to be allocated to a multidisk system with possible replication of relations. Many 0–1 variables are involved in Rotem et al.'s model, which cause a heavy computational burden. Recently, Chang [1] used a smaller number of binary variables to reformulate Rotem et al.'s model at the price of prohibiting the replication of relations. The prohibition means that each relation can only be assigned to a single disk. Since the replication of relations can improve the retrieval time on a distributed network system significantly [2], Chang's model may not generate the best solution of an MDVA problem.

This study proposes a novel model to improve both Rotem et al.'s model and Chang's model. By utilizing a linearization strategy, the proposed model not only reformulates Rotem et al.'s model using a

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smaller number of binary variables but keeps the possibility of replication of relations. The computation results of the numerical examples in Section 4 reveal that the proposed model is more efficient than Rotem et al.'s model and can lead to optimal solutions within less than three seconds.

The remainder of this paper is organized as follows. Section 2 discusses Rotem et al.'s and Chang's models. Section 3 presents the proposed method for MVDA problems with possible replication of relations. Section 4 contains numerical examples that demonstrate the advantages of the proposed model over the models of Rotem et al. and Chang.

2. Review of Rotem et al.'s and Chang's models

2.1. Rotem et al.'s model

Rotem et al. [3] developed an optimization model to find an allocation of relations to disks such that the expected query cost is minimized. Their model is based on database statistics of access patterns and sizes of relations and has to satisfy the constraint that each relation appears at least once in the system, as well as to guarantee that disk capacity limits are not exceeded. The notations used throughout this paper, referring to Rotem et al. [3], are described as below:

C_{IO}: I/O cost unit (in terms of I/O time per block of data);
C_k: capacity of disk k;
M: number of disks in the multidisk storage system;
N: number of relations in the database;
p_{ij}: the probability of a query $Q_{ij}\left(\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}p_{ij}=1\right)$;
R_i: relation i in the database;
S_i: size of the relation R_i (number of blocks);
T: value of the objective (cost) function;
x_{ik}: an indicator of relation R_i being allocated to disk k;
y_{ij}: an indicator of R_i and R_j not available on two different disks;
z_{ijk}: an indicator of R_i and R_j being placed on the disk k;
$S_{\max(i,j)}=\max(S_i,S_j)$, $S_{\min(i,j)}=\min(S_i,S_j)$;
$x_{ik}=1$ if R_i is allocated to disk k, otherwise $x_{ik}=0$;
$y_{ij}=1$ if R_i, R_j are not available on two different disks k, otherwise $y_{ij}=0$;
$z_{ijk}=1$ if R_i, R_j are stored on disk k, otherwise $z_{ijk}=0$.

An MDVA problem can be formulated as follows:

$$\min T=\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}p_{ij}C_{\mathrm{IO}}(S_{\max(i,j)}+y_{ij}S_{\min(i,j)}) \tag{2.1}$$

$$\text{s.t. }\sum_{i=1}^{N}S_ix_{ik}\leqslant C_k,\quad k=1,2,\ldots,M, \tag{2.2}$$

$$\sum_{k=1}^{M}x_{ik}\geqslant 1,\quad i=1,2,\ldots,N, \tag{2.3}$$

$$z_{ijk}\geqslant x_{ik}+x_{jk}-1,\quad i=1,\ldots,N-1,\ \ j=i+1,\ldots,N,\ \ k=1,\ldots,M, \tag{2.4}$$

$$y_{ij} > 1 + \sum_{k=1}^{M} (z_{ijk} - x_{ik} - x_{jk}), \quad i = 1, \ldots, N-1, \; j = i+1, \ldots, N, \tag{2.5}$$

$$x_{ik}, y_{ij}, z_{ijk} \in \{0, 1\} \quad \text{for all } i, j, k.$$

The objective function represents the expected query cost. Constraint (2.2) enforces the total size of relations stored on a disk cannot exceed the disk's capacity. Constraint (2.3) assures that each relation is assigned to at least one disk; this constraint allows a relation being assigned to more disks. That is so called ''possible replication of relations''. As pointed by Loomis and Popek [2], placing multiple copies of data on a network can significantly improve the efficiency of queries. Constraints (2.4) and (2.5) guarantee correct values of y_{ij}. If $y_{ij} = 1$ for some pair i and j, then there does not exist a pair of disks such that R_i is on one of them and R_j is on the other. Otherwise, $y_{ij} = 0$. Although Rotem et al. [3] have proposed a promising solution for MDVA problems, their model contains too many 0–1 variables in constraints (2.4) and (2.5). Therefore, solving the problem by Rotem et al.'s model will be too time-consuming. For the MDVA problem with N relations and M disks, there are $MN + N(N-1)/2 + MN(N-1)/2$ 0–1 variables in their model.

2.2. Chang's model

Chang [1] proposed the following model for solving the MDVA problem without replication of relations:

$$\min T = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} p_{ij} C_{\mathrm{IO}} (S_{\max(i,j)} + y_{ij} S_{\min(i,j)})$$

$$\text{s.t.} \sum_{i=1}^{N} S_i x_{ik} \leqslant C_k, \quad k = 1, 2, \ldots, M,$$

$$\sum_{k=1}^{M} x_{ik} = 1, \quad i = 1, 2, \ldots, N, \tag{2.6}$$

$$y_{ij} \geqslant x_{ik} + x_{jk} - 1, \quad i = 1, \ldots, N-1, \; j = i+1, \ldots, N, \; k = 1, \ldots, M, \tag{2.7}$$

$$y_{ij} \geqslant 0, \quad i = 1, \ldots, N-1, \; j = i+1, \ldots, N, \tag{2.8}$$

where all variables are the same as defined in Rotem et al.'s model.

The equality constraint (2.6) restricts that each relation being assigned to one disk only. Constraints (2.7) and (2.8) guarantee that if $y_{ij} = 1$, R_i and R_j are on the same disk, and if $y_{ij} = 0$, R_i and R_j are on different disks. Comparing with Rotem et al.'s model, the benefit of Chang's model is that it only involves $MN + N(N-1)/2$ 0–1 variables. This benefit comes at a price of sacrificing the possibility of replication of relations. Since the constraint set in Chang's model is a subset of that in Rotem et al.'s model, the optimal solution of Chang's model may not be the optimum of Rotem et al.'s model. Later, we will illustrate it by examples.

3. Proposed method

Consider the following propositions:

Proposition 1. *Constraints* (2.4) *and* (2.5) *in Rotem et al.'s model can be replaced by the following expressions*:

$$\sum_{k=1}^{M} x_{ik}\left(x_{jk} - \sum_{k' \neq k} x_{jk'}\right) \leqslant y_{ij}, \quad i = 1, \ldots, N-1, \;\; j = i+1, \ldots, N, \tag{3.1}$$

where all variables are the same as defined in Rotem et al.'s model.

Proof

Case 1: For any k, if $x_{ik} = 1$, $x_{jk} = 1$, and $x_{jk'} = 0$ (for all $k' \neq k$), then $y_{ij} = 1$.

Case 2: For any k, if $x_{ik} = 1$, $x_{jk} = 1$, and $\sum_{k' \neq k} x_{jk'} \geqslant 1$, then $y_{ij} = 0$ or 1. Since the objective should be minimized, y_{ij} is then force to be 0.

Case 3: For any k, if $x_{ik} = 1$ and $x_{jk} = 0$, then $y_{ij} = 0$.

Therefore, the proposition is proved. □

Constraint (3.1) involves nonlinear term $x_{ik}(x_{jk} - \sum_{k' \neq k} x_{jk'})$ which requires to be linearized.

Proposition 2 [5]. *A product term $z = uf(x)$ is equivalent to the following linear inequalities*:

(i) $M(u-1) + f(x) \leqslant z \leqslant M(1-u) + f(x)$,
(ii) $-Mu \leqslant z \leqslant Mu$,

$u \in \{0, 1\}$, *z is an unrestricted in sign variable*, *and $M = \max f(x)$ is a large constant.*

Proof. If $u = 1$ then $z = f(x)$, and if $u = 0$ then $z = 0$. □

Following Proposition 2, a product term $x_{ik}(x_{jk} - \sum_{k' \neq k} x_{jk'})$ can be linearized as follows by introducing the continuous variable z'_{ijk}:

$$M(x_{ik} - 1) + x_{jk} - \sum_{k' \neq k} x_{jk'} \leqslant z'_{ijk} \leqslant M(1 - x_{ik}) + x_{jk} - \sum_{k' \neq k} x_{jk'}, \quad i = 1, \ldots, N-1, \;\; j = i+1, \ldots, N,$$
$$k = 1, \ldots, M, \tag{3.2}$$

$$-Mx_{ik} \leqslant z'_{ijk} \leqslant Mx_{ik}, \quad i = 1, \ldots, N-1, \;\; j = i+1, \ldots, N, \;\; k = 1, \ldots, M, \tag{3.3}$$

where z'_{ijk} is continuous variable (for $i = 1, \ldots, N-1$, $j = i+1, \ldots, N$, $k = 1, \ldots, M$), and the other variables are the same as defined in Rotem et al.'s model.

It is clear that z'_{ijk} is equivalent to $x_{ik}(x_{jk} - \sum_{k' \neq k} x_{jk'})$ following Proposition 2. Replacing the product term $z'_{ijk} = x_{ik}(x_{jk} - \sum_{k' \neq k} x_{jk'})$ by (3.2) and (3.3), Rotem et al.'s model can be reformulated as a mixed 0–1 integer program below:

$$\min T = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} p_{ij} C_{\mathrm{IO}}(S_{\max(i,j)} + y_{ij} S_{\min(i,j)})$$

$$\text{s.t.} \sum_{i=1}^{N} S_i x_{ik} \leqslant C_k, \quad k = 1, 2, \ldots, M,$$

$$\sum_{k=1}^{M} x_{ik} \geqslant 1, \quad i = 1, 2, \ldots, N,$$

$$
\begin{aligned}
& M(x_{ik}-1)+x_{jk}-\sum_{k'\neq k} x_{jk'} \leqslant z'_{ijk} \leqslant M(1-x_{ik})+x_{jk}-\sum_{k'\neq k} x_{jk'},\\
& \quad i=1,\ldots,N-1,\ \ j=i+1,\ldots,N,\ \ k=1,\ldots,M,\\
& -Mx_{ik} \leqslant z'_{ijk} \leqslant Mx_{ik}, \quad i=1,\ldots,N-1,\ \ j=i+1,\ldots,N,\ \ k=1,\ldots,M,\\
& \sum_{k=1}^{M} z'_{ijk} \leqslant y_{ij}, \quad i=1,\ldots,N-1,\ \ j=i+1,\ldots,N,
\end{aligned} \tag{3.4}
$$

where z'_{ijk} is continuous variable (for $i=1,\ldots,N-1$, $j=i+1,\ldots,N$, $k=1,\ldots,M$), and the other variables are the same as defined in Rotem et al.'s model.

Since the binary variable z_{ijk} in Rotem et al.'s model can be replaced by the continuous variable z'_{ijk} in the proposed model, the required number of 0–1 variables used in an MDVA problem is reduced from $MN+N(N-1)/2+MN(N-1)/2$ to $MN+N(N-1)/2$.

4. Numerical examples

Example 1. The problem represented in Fig. 1 shows a database of four relations as vertices of the graph where each edge represents two-way join query likelihood. Suppose that all applications in the example run only two-way join queries and the entire database must be allocated to two disks.

This example with the disk capacity $C_1=C_2=1$ can be formulated by the proposed method as the following program:

$$
\begin{aligned}
\min\ & 0.5(0.5+0.3y_{12})+0.1(0.5+0.45y_{13})+0.1(0.5+0.4y_{14})+0.1(0.45+0.3y_{23})+0.1(0.4+0.3y_{24})\\
& \quad +0.1(0.45+0.4y_{34})\\
\text{s.t. } & 0.5x_{11}+0.3x_{21}+0.45x_{31}+0.4x_{41} \leqslant 1,\\
& 0.5x_{12}+0.3x_{22}+0.45x_{32}+0.4x_{42} \leqslant 1,\\
& x_{11}+x_{12} \geqslant 1, \quad x_{21}+x_{22} \geqslant 1, \quad x_{31}+x_{32} \geqslant 1, \quad x_{41}+x_{42} \geqslant 1,\\
& z'_{121}+z'_{122} \geqslant 1, \quad z'_{131}+z'_{132} \geqslant 1, \quad z'_{141}+z'_{142} \geqslant 1,\\
& z'_{231}+z'_{232} \geqslant 1, \quad z'_{241}+z'_{242} \geqslant 1, \quad z'_{341}+z'_{342} \geqslant 1,\\
& 2(x_{11}-1)+x_{21}-x_{22} \leqslant z'_{121} \leqslant 2(1-x_{11})+x_{21}-x_{22},\\
& 2(x_{12}-1)+x_{22}-x_{21} \leqslant z'_{122} \leqslant 2(1-x_{12})+x_{22}-x_{21},\\
& 2(x_{11}-1)+x_{31}-x_{32} \leqslant z'_{131} \leqslant 2(1-x_{11})+x_{31}-x_{32},
\end{aligned}
$$

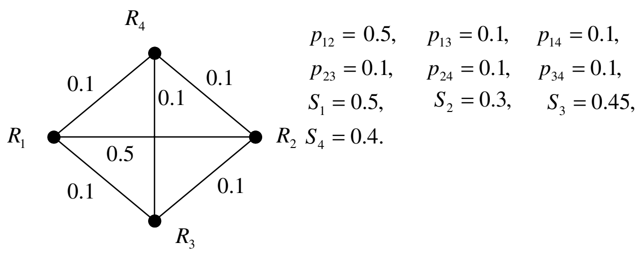

Fig. 1. Examples of two disks and four relations [1].

$$
\begin{aligned}
&2(x_{12}-1)+x_{32}-x_{31}\leqslant z'_{132}\leqslant 2(1-x_{12})+x_{32}-x_{31},\\
&2(x_{11}-1)+x_{41}-x_{42}\leqslant z'_{141}\leqslant 2(1-x_{11})+x_{41}-x_{42},\\
&2(x_{12}-1)+x_{42}-x_{41}\leqslant z'_{142}\leqslant 2(1-x_{12})+x_{42}-x_{41},\\
&2(x_{21}-1)+x_{31}-x_{32}\leqslant z'_{231}\leqslant 2(1-x_{21})+x_{31}-x_{32},\\
&2(x_{22}-1)+x_{32}-x_{31}\leqslant z'_{232}\leqslant 2(1-x_{22})+x_{32}-x_{31},\\
&2(x_{21}-1)+x_{41}-x_{42}\leqslant z'_{241}\leqslant 2(1-x_{21})+x_{41}-x_{42},\\
&2(x_{22}-1)+x_{42}-x_{41}\leqslant z'_{242}\leqslant 2(1-x_{22})+x_{42}-x_{41},\\
&2(x_{31}-1)+x_{41}-x_{42}\leqslant z'_{341}\leqslant 2(1-x_{31})+x_{41}-x_{42},\\
&2(x_{32}-1)+x_{42}-x_{41}\leqslant z'_{342}\leqslant 2(1-x_{32})+x_{42}-x_{41},\\
&-2x_{11}\leqslant z'_{121}\leqslant 2x_{11},\qquad -2x_{12}\leqslant z'_{122}\leqslant 2x_{12},\\
&-2x_{11}\leqslant z'_{131}\leqslant 2x_{11},\qquad -2x_{12}\leqslant z'_{132}\leqslant 2x_{12},\\
&-2x_{11}\leqslant z'_{141}\leqslant 2x_{11},\qquad -2x_{12}\leqslant z'_{142}\leqslant 2x_{12},\\
&-2x_{21}\leqslant z'_{231}\leqslant 2x_{21},\qquad -2x_{22}\leqslant z'_{232}\leqslant 2x_{22},\\
&-2x_{21}\leqslant z'_{241}\leqslant 2x_{21},\qquad -2x_{22}\leqslant z'_{242}\leqslant 2x_{22},\\
&-2x_{31}\leqslant z'_{341}\leqslant 2x_{31},\qquad -2x_{32}\leqslant z'_{342}\leqslant 2x_{32},
\end{aligned}
$$

where $x_{ik}, x_{jk}, y_{ij} \in \{0,1\}$ and z'_{ijk} is continuous variable (for $i = 1,2,3,\ j = i+1,\ldots,4,\ k = 1,2$).

Solving the program using LINDO [4] in the same computer, we can have the results in Table 1. Table 1 shows that (i) For the case $C_1 = 1.5$ and $C_2 = 1$, the objective value found by Chang [1] is 0.55 which is worse than the objective value found by Rotem et al. [3] and the proposed method. This is owing to the fact that the constraint set in Chang's model is more restrictive than that in Rotem et al.'s model. (ii) The proposed model contains less 0–1 variables than Rotem et al.'s model for solving the same problems to get a global optimum. The optimal allocations of this example with different disk capacities are depicted in Fig. 2.

Example 2. An example of a relational database adapted from Rotem et al. [3] is illustrated in Fig. 3. Suppose that the database with 19 relations must be allocated to three disks each of capacity 256 Mb, and all applications run only two-way join queries.

Table 1
Computational results of three models with different disk capacities

Disk capacity	Rotem et al.'s model		Chang's model		Proposed model	
	0–1 variables	Objective value	0–1 variables	Objective value	0–1 variables	Objective value
$C_1 = C_2 = 1$	26	0.55	14	0.55	14	0.55
$C_1 = 1.5,\ C_2 = 1$	26	0.51	14	0.55 (no replication)	14	0.51

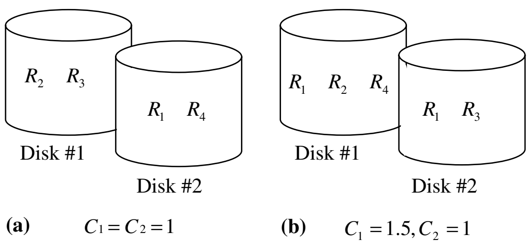

Fig. 2. Optimal allocations with replication of relations.

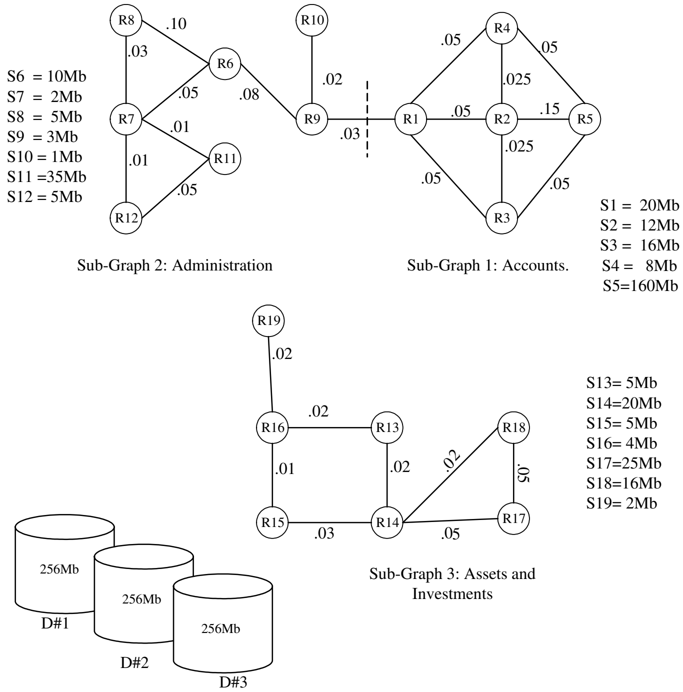

Fig. 3. A weighted graph of a relational database [3].

Table 2
Computational comparison of two models

Sub-graph	$M \times N$	Rotem et al.'s model		Proposed model	
		Number of 0–1 variables	Computing time (ss ms)	Number of 0–1 variables	Computing time (ss ms)
1	3×5	55	0.21	25	0.2
2	3×7	105	24.72	42	1.65
3	3×7	105	31.09	42	2.64

For solving the problem by Rotem et al.'s and the proposed models, the problem has to be decomposed into three sub-problems such that each sub-problem can be solved on the commercially available software LINDO [4]. Solving the sub-problems by LINDO [4] on the same computer, we can find that the proposed model is more computational efficient than Rotem et al.'s model. The computational results are shown in Table 2.

5. Conclusions

An optimal data allocation could lead to significant decrease in communication costs and/or unreasonable response times in satisfying user queries. Although Rotem et al.'s model can obtain the optimal allocation of relations to a multidisk database system, their model contains too many 0–1 variables which will increase the computational burden in the solution process. Chang's model can only treat an MDVA problem without the replication of relations. Chang's model, therefore, may not save retrieval time by keeping copies of some of the relations local to various disks. The proposed method not only uses fewer additional 0–1 variables than Rotem et al.'s model to formulate the same problem but also is more general than Chang's method. The computational results show that the proposed method can efficiently solve an MDVA problem with possible replication of relations which cannot be treated by Change's method.

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