T-S Fuzzy Controllers for Nonlinear Interconnected Systems With Multiple Time Delays

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***Abstract*—This paper investigates the effectiveness of a passive tuned mass damper (TMD) and fuzzy controller in reducing the structural responses subject to the external force. In general, TMD is good for linear systems. We proposed here an approach of Takagi–Sugeno (T-S) fuzzy controller to deal with the nonlinear system. To overcome the effect of modeling error between nonlinear multiple time-delay systems and T-S fuzzy models, a robustness design of fuzzy control via model-based approach is proposed in this paper. A stability criterion in terms of Lyapunov's direct method is derived to guarantee the stability of nonlinear multiple time-delay interconnected systems. Based on the decentralized control scheme and this criterion, a set of model-based fuzzy controllers is then synthesized via the technique of parallel distributed compensation (PDC) to stabilize the nonlinear multiple time-delay interconnected system and the H^{∞} control performance is achieved at the same time. Finally, the proposed methodology is illustrated by an example of a nonlinear TMD system.**

***Index Terms*—Fuzzy control, modeling error, multiple time delay, Takagi–Sugeno (T-S) fuzzy model.**

I. Introduction

THE control methods so far have been broadly classified into passive control and active control. One may dissipates the energy in localized elements in the passive control methods and they are already finding applications in many design practice such as noise control and structural control. The active control methods, on the other hand, reduce system response or control undesired disturbance by applying counteracting control forces externally or by creating reactive internal forces in systems. In recent years, there are several important works proposed in active control methods and their applications (see, e.g., [1]–[7] and the references therein).

The well known installation of passive control in structural systems is the use of passive tuned mass dampers (TMDs), also known as dynamic absorbers, which was first proposed by Frahm in 1909 [8]. Since then, many studies have been done to investigate the control effectiveness of passive TMDs [9]–[12]. These articles show that the TMDs are suitable for a linear resonant system. However, nonlinearities are not negligible in some cases so that using TMD is not appropriate [13].

During the recent years, a number of research activities have been concerned with the topic of stability analysis and stabilization of interconnected systems, also called large-scale systems or composite systems [14]. In practices, due to the information transmission between subsystems, time delays naturally exist in interconnected systems. The existence of time delays is frequently a source of instability and encountered in various engineering systems [15], [16]. Hence, the problem of stability analysis of time-delay systems has been one of the main concerns of researchers wishing to inspect the properties of such systems and there have been several research efforts [16]–[20] on this issue.

Recently, fuzzy control has been successfully applied to control design of nonlinear systems (see [16], [20]–[31]). In most of these papers, a so-called Takagi–Sugeno (T-S) fuzzy model was employed to approximate a nonlinear plant, and then a fuzzy controller was designed to stabilize the T-S fuzzy model. In [30], Tseng and Chen proposed an H infinity decentralized fuzzy control scheme to solve the model reference tracking control problem of nonlinear interconnected systems. Based on LMI optimization techniques, a simple and systematic algorithm is developed to solve the fuzzy tracking control problem. Also, other existing techniques of decentralized control have been addressed to deal with interconnected systems or time-delay problems such as Shoulie *et al.* [32]; Souza and Li [17]–[19], [33]. All of them, however, neglect the modeling error between nonlinear system and fuzzy model. Existence of modeling error may be a potential source of instability for control designs that have been based on the assumption that the fuzzy model exactly matches the plant [34]. Recently, Cao and Frank [34], Kiriakidis [35], Chen *et al.* [36], and Cao and Lin [37] have proposed novel approaches to overcome the influence of modeling error in the field of model-based fuzzy control for nonlinear systems.

However, a literature search indicates that the effect of modeling error for nonlinear multiple time-delay interconnected systems has not been discussed yet. Hence, a robustness design of fuzzy control for nonlinear interconnected systems with multiple time delays needs further study. Therefore, this paper may be viewed as a generalization of Tseng *et al.* [29], [30] to the robustness design of fuzzy control via model-based approach for nonlinear multiple time-delay interconnected systems.

In summary, the purpose of this paper is to derive a stability criterion for model-based fuzzy controller to guarantee the stability of nonlinear interconnected systems with multiple time

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delays. Accordingly, the T-S fuzzy model is employed to approximate each nonlinear system. The control design is carried out based on the fuzzy model via the parallel distributed compensation (PDC) scheme. The idea is that a linear feedback control is designed for each local linear model. The resulting overall fuzzy controller, which is nonlinear in general, is a fuzzy blending of each individual linear controller.

This paper is organized as follows. The system description is presented and the T-S fuzzy model is briefly reviewed in Section II. The concept of PDC is in Section III. In Section IV, the H^∞ control performance is described to attenuate the influence of the external disturbance. In Section V, a stability criterion is derived to guarantee the stability of nonlinear interconnected systems with multiple time delays. In Section VI, the fuzzy control methodology proposed in this study is utilized to stabilize the nonlinear interconnected TMD system and a numerical example of nonlinear TMD systems is given to illustrate the results.

II. System Description

Consider a nonlinear multiple time-delay interconnected system N composed of J subsystems N_j, $j = 1, \cdots 2, J$. The jth subsystem N_j is described as follows:

$$\dot{x}_j(t) = f_j\left(x_j(t), u_j(t)\right) + \sum_{k=1}^{D_j} g_{kj}\left(x_j(t-\tau_{kj})\right) + \sum_{\substack{n=1\\ n\neq j}}^{J} b_{nj}\left(x_n(t)\right) + \phi_j(t) \quad (1)$$

where $f_j(\cdot)$ and $g_{kj}(\cdot)$ are the nonlinear vector-valued function, τ_{kj} (the kth time delay) $k = 1, 2, \cdots, D_j$ are positive real numbers, and b_{nj} is the nonlinear interconnection between the nth and jth subsystems; $x_j(t) = [x_{1j}(t), x_{2j}(t), \cdots, x_{gj}(t)]^T$ is the state vector; $u_j(t) = [u_{1j}(t), u_{2j}(t), \cdots, u_{mj}(t)]^T$ is the input vector; $\phi_j(t) = [\phi_{1j}(t), \phi_{2j}(t), \cdots, \phi_{gj}(t)]^T$ denotes the unknown disturbances with a known upper bound $\phi_{upj}(t) \geq \|\phi_j(t)\|$.

A fuzzy dynamical model had been developed primarily from the pioneering work of Takagi and Sugeno [38] to represent local linear input/output relations of nonlinear systems. This dynamical model is described by fuzzy IF-THEN rules and it is employed here to handle the control design problem of the nonlinear interconnected system N. The ith rule of this fuzzy model for the nonlinear interconnected subsystem N_j is proposed as the following form:

$$\begin{aligned} &\text{Rule } i: \text{IF } x_{1j}(t) \text{ is } M_{i1j} \text{ and} \cdots \text{and } x_{gj}(t) \text{ is } M_{igj} \\ &\text{THEN } \dot{x}_j(t) = A_{ij}x_j(t) + \sum_{k=1}^{D_j} A_{ikj}x_j(t-\tau_{kj}) \\ &\qquad + \sum_{\substack{n=1\\ n\neq j}}^{J} \hat{A}_{inj}x_n(t) + B_{ij}u_j(t) + \phi_j(t) \end{aligned} \quad (2)$$

where $i = 1, 2, \cdots, r_j$ and r_j is the number of IF-THEN rules; A_{ij}, A_{ikj}, $\hat{A}_{inj}$ and B_{ij} are constant matrices with appropriate dimensions; M_{ipj} $(p = 1, 2, \cdots, g)$ are the fuzzy sets, and $x_{1j}(t) \sim x_{gj}(t)$ are the premise variables. The final state of this fuzzy dynamic model is inferred as follows:

$$\begin{aligned} \dot{x}_j(t) &= \frac{1}{\sum_{i=1}^{r_j} w_{ij}(t)} \cdot \sum_{i=1}^{r_j} w_{ij}(t) \\ &\quad \times \left[A_{ij}x_j(t) + \sum_{k=1}^{D_j} A_{ikj}x_j(t-\tau_{kj}) + \sum_{\substack{n=1\\ n\neq j}}^{J} \hat{A}_{inj}x_n(t) + B_{ij}u_j(t) + \phi_j(t) \right] \\ &= \sum_{i=1}^{r_j} h_{ij}(t) \left(A_{ij}x_j(t) + \sum_{k=1}^{D_j} A_{ikj}x_j(t-\tau_{kj}) + \sum_{\substack{n=1\\ n\neq j}}^{J} \hat{A}_{inj}x_n(t) + B_{ij}u_j(t) \right) \\ &\quad + \phi_j(t) \end{aligned} \quad (3)$$

with

$$w_{ij}(t) \equiv \prod_{p=1}^{g} M_{ipj}\left(x_{pj}(t)\right), \quad h_{ij}(t) \equiv \frac{w_{ij}(t)}{\sum_{i=1}^{r_j} w_{ij}(t)} \quad (4)$$

in which $M_{ipj}(x_{pj}(t))$ is the grade of membership of $x_{pj}(t)$ in M_{ipj}. In this paper, it is assumed that $w_{ij}(t) \geq 0$

$$i = 1, 2, \cdots, r_j;\ j = 1, 2, \cdots, J \text{ and } \sum_{i=1}^{r_j} w_{ij}(t) > 0 \text{ for all } t.$$

Therefore, $h_{ij}(t) \geq 0$ and $\sum_{i=1}^{r_j} h_{ij}(t) = 1$ for all t.

III. Parallel Distributed Compensation

According to the decentralized control scheme, a set of model-based fuzzy controllers is synthesized via the technique of parallel distributed compensation (PDC) to stabilize the nonlinear multiple time-delay interconnected system N. The concept of PDC scheme is that each control rule is distributively designed for the corresponding rule of a T-S fuzzy model. The fuzzy controller shares the same fuzzy sets with the fuzzy model in the premise parts [22]. Since each rule of the fuzzy model is described by a linear state equation, a linear control theory can be used to design the consequent parts of a fuzzy controller. The resulting overall fuzzy controller, nonlinear in general, is achieved by fuzzy blending of each individual linear controller.

Hence, the jth model-based fuzzy controller can be described as follows:

$$\begin{aligned} &\text{Rule } i: \text{IF } x_{1j}(t) \text{ is } M_{i1j} \text{ and} \cdots \text{and } x_{gj}(t) \text{ is } M_{igj} \\ &\qquad \text{THEN } u_j(t) = -K_{ij}x_j(t) \end{aligned} \quad (5)$$

where K_{ij} is a local feedback gain matrix for $i = 1, 2, \cdots, r_j$. The final output of this fuzzy controller is

$$u_j(t) = -\frac{\sum_{i=1}^{r_j} w_{ij}(t)K_{ij}x_j(t)}{\sum_{i=1}^{r_j} w_{ij}(t)} = -\sum_{i=1}^{r_j} h_{ij}(t)K_{ij}x_j(t) \quad (6)$$

with

$$w_{ij}(t) \equiv \prod_{p=1}^{g} M_{ipj}\left(x_{pj}(t)\right), \quad h_{ij}(t) \equiv \frac{w_{ij}(t)}{\sum_{i=1}^{r_j} w_{ij}(t)} \quad (7)$$

in which $M_{ipj}(x_{pj}(t))$ is the grade of membership of $x_{pj}(t)$ in M_{ipj}.

IV. H^∞ CONTROL DESIGN VIA FUZZY CONTROL

Stabilizing the closed-loop nonlinear interconnected systems and attenuating the influence of the external disturbance $\phi_j(t)$ on the state variable $x_j(t)$ [16], [30], [32], [34], [39] is the objective of this paper. The influence of $\phi_j(t)$ will worsen the performance of fuzzy control systems. In order to guarantee the control performance by eliminating the influence of $\phi_j(t)$ is a significant problem in the control system. Hence, in this work, not only the stability of fuzzy control system is advised but also the H^∞ control performance is satisfied as follows:

$$\sum_{j=1}^{J}\int_0^{t_f} x_j^T(t)Q_jx_j(t)dt \le \sum_{j=1}^{J} x_j^T(0)P_jx_j(0) + \eta^2\sum_{j=1}^{J}\int_0^{t_f}\phi_j^T(t)\phi_j(t)dt \quad (8)$$

where t_f denotes the terminal time of the control, P_j are some positive definite matrices, η is a prescribed value which denotes the effect of $\phi_j(t)$ on $x_j(t)$, and Q_j is a positive definite weighting matrix. The physical meaning of (8) is that the effect of $\phi_j(t)$ on $x_j(t)$ must be attenuated below a desired level η from the viewpoint of energy [36].

V. ROBUSTNESS DESIGN OF FUZZY CONTROL

In this section, the stability of the nonlinear interconnected system N is examined under the influence of modeling error.

A. Modeling Error

Substituting (6) into (1) yields the jth $(j = 1, 2, \cdots, J)$ closed-loop nonlinear subsystem $\overline{N}_j$ as follows:

$$\begin{aligned}\dot{x}_j(t) = &\sum_{i=1}^{r_j}\sum_{l=1}^{r_j} h_{ij}(t)h_{lj}(t) \times \left[(A_{ij} - B_{ij}K_{lj})x_j(t) + \sum_{k=1}^{D_j} A_{ikj}x_j(t-\tau_{kj}) + \sum_{\substack{n=1\\n\ne j}}^{J} \hat{A}_{inj}x_n(t)\right] + \overline{f_j}\left(x_j(t)\right) \\ &+ \sum_{k=1}^{D_j} g_{kj}\left(x_j(t-\tau_{kj})\right) + \sum_{\substack{n=1\\n\ne j}}^{J} b_{nj}\left(x_n(t)\right) + \phi_j(t) \\ &- \sum_{i=1}^{r_j}\sum_{l=1}^{r_j} h_{ij}(t)h_{lj}(t) \times \left[(A_{ij} - B_{ij}K_{lj})x_j(t) + \sum_{k=1}^{D_j} A_{ikj}x_j(t-\tau_{kj}) + \sum_{\substack{n=1\\n\ne j}}^{J} \hat{A}_{inj}x_n(t)\right] \\ = &\sum_{i=1}^{r_j}\sum_{l=1}^{r_j} h_{ij}(t)h_{lj}(t) \times \left[(A_{ij} - B_{ij}K_{lj})x_j(t) + \sum_{k=1}^{D_j} A_{ikj}x_j(t-\tau_{kj}) + \sum_{\substack{n=1\\n\ne j}}^{J} \hat{A}_{inj}x_n(t)\right] \\ &+ \phi_j(t) + e_j(t) + \sum_{k=1}^{D_j} \overline{e}_j(t-\tau_{kj}) + \sum_{\substack{n=1\\n\ne j}}^{J} \hat{e}_{nj}(t) \end{aligned} \quad (9)$$

where

$$\overline{f_j}\left(x_j(t)\right) \equiv f_j\left(x_j(t), u_j(t)\right) \text{ with } u_j(t) = -\sum_{i=1}^{r_j} h_{ij}(t)K_{ij}x_j(t)$$

$$e_j(t) \equiv \left[\overline{f_j}\left(x_j(t)\right) - \sum_{i=1}^{r_j}\sum_{l=1}^{r_j} h_{ij}(t)h_{lj}(t) \times (A_{ij} - B_{ij}K_{lj})x_j(t)\right] \quad (10)$$

$$\begin{aligned}\overline{e}_j(t-\tau_{kj}) &\equiv g_{kj}\left(x_j(t-\tau_{kj})\right) - \sum_{i=1}^{r_j}\sum_{l=1}^{r_j} h_{ij}(t)h_{lj}(t)A_{ikj}x_j(t-\tau_{kj}) \\ &= g_{kj}\left(x_j(t-\tau_{kj})\right) - \sum_{i=1}^{r_j} h_{ij}(t)A_{ikj}x_j(t-\tau_{kj})\end{aligned} \quad (11)$$

$$\begin{aligned}\hat{e}_{nj}(t) &\equiv b_{nj}\left(x_n(t)\right) - \sum_{i=1}^{r_j}\sum_{l=1}^{r_j} h_{ij}(t)h_{lj}(t)\hat{A}_{inj}x_n(t) \\ &= b_{nj}\left(x_n(t)\right) - \sum_{i=1}^{r_j} h_{ij}(t)\hat{A}_{inj}x_n(t)\end{aligned} \quad (12)$$

and $\Delta\Phi_j(t) \equiv e_j(t) + \sum_{k=1}^{D_j}\overline{e}_j(t-\tau_{kj}) + \sum_{n=1, n\ne j}^{J}\hat{e}_{nj}(t)$ denotes the modeling error between the jth closed-loop nonlinear subsystem (9) and the closed-loop fuzzy model ((3)+(6)).

Suppose that there exist bounding matrices ΔH_{ilj} such that

$$\|\Delta\Phi_j(t)\| \le \left\|\sum_{i=1}^{r_j}\sum_{l=1}^{r_j} h_{ij}(t)h_{lj}(t)\Delta H_{ilj}x_j(t)\right\| \quad (13)$$

for the trajectory $x_j(t)$, and the bounding matrix ΔH_{ilj} can be described as follows:

$$\Delta H_{ilj} = \delta_{ilj}\overline{H}_j \tag{14}$$

where $\|\delta_{ilj}\| \le 1$, for $i,l = 1,2,\cdots,r_j$, $j = 1,2,\cdots,J$. From (13), (14), we have

$$\begin{aligned}
&\Delta\Phi_j^T(t)\Delta\Phi_j(t)\\
&\le \left\{\sum_{i=1}^{r_j}\sum_{l=1}^{r_j} h_{ij}(t)h_{lj}(t)\Delta H_{ilj}x_j(t)\right\}^T\\
&\quad\times\left\{\sum_{i=1}^{r_j}\sum_{l=1}^{r_j} h_{ij}(t)h_{lj}(t)\Delta H_{ilj}x_j(t)\right\}\\
&\le \left\|\sum_{i=1}^{r_j}\sum_{l=1}^{r_j} h_{ij}(t)h_{lj}(t)\delta_{ilj}^T\right\|\\
&\quad\times\left\|\sum_{i=1}^{r_j}\sum_{l=1}^{r_j} h_{ij}(t)h_{lj}(t)\delta_{ilj}\right\| \left[\overline{H}_j x_j(t)\right]^T\left[\overline{H}_j x_j(t)\right]\\
&\le \left[\overline{H}_j x_j(t)\right]^T\left[\overline{H}_j x_j(t)\right].
\end{aligned} \tag{15}$$

Namely, the modeling error $\Delta\Phi_j(t)$ is bounded by the specified structured bounding matrix $\overline{H}_j$.

Remark 1 ([36]): The procedures for determining δ_{ilj} and $\overline{H}_j$ are described in the following. Assuming that the possible bounds for all elements in ΔH_{ilj} are

$$\Delta H_{ilj} = \begin{bmatrix}\Delta h_{ilj}^{11} & \Delta h_{ilj}^{12}\\ \Delta h_{ilj}^{21} & \Delta h_{ilj}^{22}\end{bmatrix}$$

where $-\varepsilon_j^{rs} \le \Delta h_{ilj}^{rs} \le \varepsilon_j^{rs}$ for some ε_j^{rs}, $r,s = 1,2$ and $i,l = 1,2,\cdots,r_j$; $j = 1,2,\cdots,J$.

One possible description for the bounding matrix ΔH_{ilj} is

$$\Delta H_{ilj} = \begin{bmatrix}\delta_{ilj}^{11} & 0\\ 0 & \delta_{ilj}^{22}\end{bmatrix}\begin{bmatrix}\varepsilon_j^{11} & \varepsilon_j^{12}\\ \varepsilon_j^{21} & \varepsilon_j^{22}\end{bmatrix} = \delta_{ilj}\overline{H}_j$$

where $-1 \le \delta_{ilj}^{rr} \le 1$ for $r = 1,2$. It is noticed that δ_{ilj} can be chosen by other forms as long as $\|\delta_{ilj}\| \le 1$. Then, we check the validity of (13) in the simulation. If it is not satisfied, we can expand the bounds for all elements in ΔH_{ilj} and repeat the design procedures until (13) holds.

B. Stability in the Presence of Modeling Error

In the following, a stability criterion is proposed to guarantee the stability of the closed-loop nonlinear multiple time-delay interconnected system $\overline{N}$ which consists of J closed-loop subsystems described in (9). Prior to examination of stability of $\overline{N}$, a useful inequality is given below.

Lemma 1 ([33], [40]): For any matrices A and B with appropriate dimensions, we have $A^TB + B^TA \le \sigma A^TA + \sigma^{-1}B^TB$ where σ is a positive constant.

Theorem 1: The trajectories of closed-loop nonlinear multiple time-delay interconnected system $\overline{N}$ in the absence of disturbances $\phi_j(t)$ $(j = 1,2,\cdots,J)$ are asymptotically stable and the H^∞ control performance can be achieved, if there exist symmetric positive definite matrices P_j and positive constants α, β_j, γ and the feedback gains K_{ij}s shown in (6) are chosen to satisfy

$$\begin{aligned}
&(A_{ij} - B_{ij}K_{ij})^TP_j + P_j(A_{ij} - B_{ij}K_{ij}) + \sum_{k=1}^{D_j} R_{kj}\\
&+ \sum_{k=1}^{D_j} P_jA_{ikj}R_{kj}^{-1}A_{ikj}^TP_j + \sum_{n=1}^{J}\alpha^{-1}P_j\hat{A}_{inj}\hat{A}_{inj}^TP_j\\
&+ \alpha(J-1)I + \beta_j\overline{H}_j^T\overline{H}_j + \beta_j^{-1}P_j^2 + \gamma^{-1}P_j^2 + Q_j\\
&< 0,\ \text{for}\ i = 1,2,\cdots,r_j;\ j = 1,2,\cdots,J.
\end{aligned} \tag{16}$$

Proof: See Appendix.

Remark 2: Equation (16) can be transformed to a linear matrix inequality (LMI) via the following procedure. By introducing new variables $W_j = P_j^{-1}$, $Y_{ij} = K_{ij}W_j$, $\overline{Q}_j = Q_j^{-1}$ and $\overline{R}_{kj} = R_{kj}^{-1}$, (16) is rewritten as follows:

$$\begin{aligned}
&W_jA_{ij}^T - (B_{ij}Y_{ij})^T + A_{ij}W_j - B_{ij}Y_{ij} + \sum_{k=1}^{D_j} W_j\overline{R}_{kj}^{-1}W_j\\
&+ \sum_{k=1}^{D_j} A_{ikj}\overline{R}_{kj}A_{ikj}^T + \sum_{n=1}^{J}\alpha^{-1}\hat{A}_{inj}\hat{A}_{inj}^T + \alpha W_j(J-1)\\
&\times W_j + \beta_jW_j\overline{H}_j^T\overline{H}_jW_j + \beta_j^{-1}I + \gamma^{-1}I + W_j\overline{Q}_j^{-1}W_j\\
&< 0
\end{aligned} \tag{17}$$

for $i = 1,2,\cdots,r_j$; $j = 1,2,\cdots,J$. Moreover, based on Schur complements [20], [30], [36], [41]–[43], it is easy to find that (17) is equivalent to the inequality (18) at the bottom of the next page, for $i = 1,2,\cdots,r_j$, $j = 1,2,\cdots,J$ where

$$\begin{aligned}
\psi_{ij} &= W_jA_{ij}^T - (B_{ij}Y_{ij})^T + A_{ij}W_j - B_{ij}Y_{ij}\\
&\quad+ \sum_{k=1}^{D_j} A_{ikj}\overline{R}_{kj}A_{ikj}^T + \sum_{n=1}^{J}\alpha^{-1}\hat{A}_{inj}\hat{A}_{inj}^T + \beta_j^{-1}I + \gamma^{-1}I
\end{aligned}$$

and the symbol * denotes the transposed elements in the symmetric positions, shown in (19) at the bottom of the next page.

Remark 3: Let $\overline{\alpha} \equiv \alpha^{-1}$, $\overline{\beta}_j \equiv \beta_j^{-1}$ and then (18) can be transformed into the LMI (19). After $\overline{\alpha}$ and $\overline{\beta}_j$ being solved from (19), α and β_j can be adjusted in (18) to find a suitable γ such that better H^∞ control performance can be achieved.

Therefore, based on the LMI technique as that in Hu *et al.* [43], Theorem 1 can be reformulated into an LMI problem and efficient interior-point algorithms are now available in Matlab toolbox to solve this problem.

VI. Example

A. TMD System

A passive TMD mounted on a shear structure is modeled as a two-degree-of freedom structure-TMD system as shown in Fig. 1 The parameters m_1, c_1 and k_1 represent mass, damping and stiffness in the subsystem 1; m_2, c_2 and k_2 represent mass, damping and stiffness in the subsystem 2; F and u represent external force and control input. The equation of motion with no control input can be written as [9], [44]

$$\begin{cases}\ddot{s}_1 + 2\xi_1\omega_1\dot{s}_1 - 2\mu\xi_2\omega_2(\dot{s}_2 - \dot{s}_1) + \omega_1^2s_1\\ \quad -\mu\omega_2^2(s_2 - s_1) = F\\ \ddot{s}_2 + 2\xi_2\omega_2(\dot{s}_2 - \dot{s}_1) + \omega_2^2(s_2 - s_1) = 0\end{cases} \tag{20}$$

where $\omega_1(\sqrt{k_1/m_1})$ is natural frequency of primary structure; $\omega_2(\sqrt{k_2/m_2})$ is the natural frequency of TMD; $\xi_1(c_1/2m_1\omega_1)$ is the damping ratio of primary structure; $\xi_2(c_2/2m_2\omega_2)$ is the damping ratio of TMD; $\mu(m_2/m_1)$ denotes mass ratio of

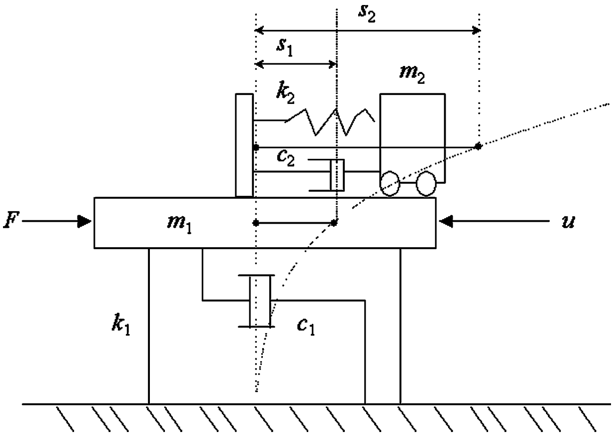

Fig. 1. Two-DOF structure-TMD system.

TMD to primary structure; $\overline{\omega}$ is the frequency of external force; $\omega(\overline{\omega}/\omega_1)$ frequency ratio.

Fig. 2 shows the effectiveness of a TMD system in reducing the response due to an external force with $m_1 = 1, \omega_1 = \omega_2 = 1.29$, $c_1 = 2.506 \times 10^{-3}$, $\xi_2 = 2.506 \times 10^{-5}$, $F = \cos(\overline{\omega} t)$, $\mu = 0.01, \overline{\omega} = 1.29$ and initial conditions $\dot{s}_1(0) = s_2(0) = \dot{s}_2(0) = 0$. Fig. 3 shows the dynamic magnification factor in 50 s where restoring force is a linear function. Hence, the passive TMD is appropriate when the frequency of external excitation is close to the structure. However, the restoring force of spring stiffness is nonlinear in actual systems. Moreover, TMD does not work shown in Figs. 4–6 with $k_1 = 1.664(1 - a^2 s_1^2)$, $k_2 = 0.016\,64(1 - a^2 s_2^2)$ and initial conditions $s_1(0) = \dot{s}_1(0) = s_2(0) = \dot{s}_2(0) = 0$. Therefore, a method of fuzzy control is proposed to guarantee the stability of nonlinear systems in next subsection.

In the above, a simple structural system without time delays is addressed. However, due to the information transmission between subsystems, time delays naturally exist in practical nonlinear interconnected systems. In the following, state-space representation is established to deal with stability problem of the structural system (20) and multiple time delays are considered in the meantime for real-world application.

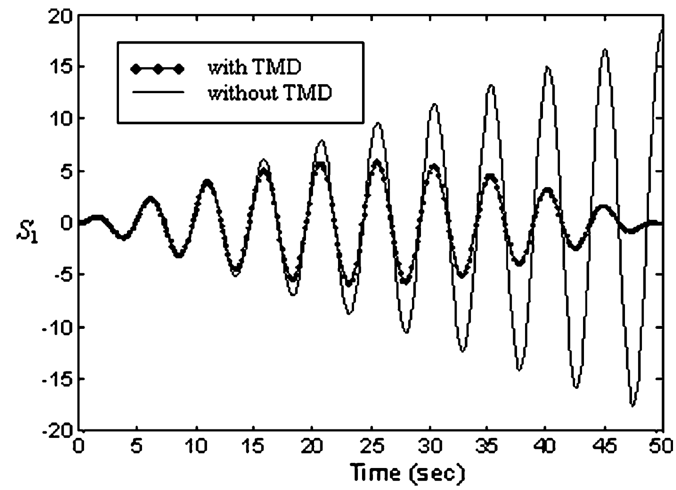

Fig. 2. Effectiveness of a TMD system.

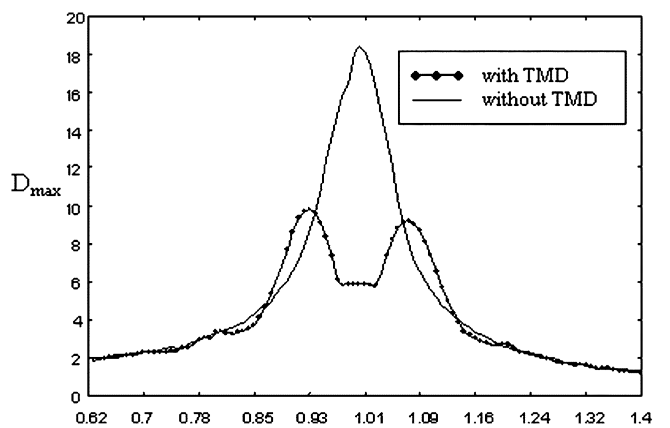

Fig. 3. Effectiveness of a TMD system with linear stiffness $k(\mathrm{x})$.

$$\begin{bmatrix} \psi_{ij} & * & * & * & * & * \\ W_j & -(\alpha(J-1)I + \beta_j \overline{H}_j^T \overline{H}_j)^{-1} & * & * & * & * \\ W_j & 0 & -\overline{Q}_j & * & * & * \\ W_j & 0 & 0 & -\overline{R}_{1j} & * & * \\ W_j & 0 & 0 & 0 & \ddots & * \\ W_j & 0 & 0 & 0 & 0 & -\overline{R}_{D_j j} \end{bmatrix} < 0 \quad (18)$$

$$\begin{bmatrix} \psi_{ij} & \overline{H}_j W_j & * & * & * & * & * \\ (\overline{H}_j W_j)^T & -\overline{\beta}_j & * & * & * & * & * \\ W_j & 0 & -\overline{\alpha}(J-1)^{-1} I & * & * & * & * \\ W_j & 0 & 0 & -\overline{Q}_j & * & * & * \\ W_j & 0 & 0 & 0 & -\overline{R}_{1j} & * & * \\ W_j & 0 & 0 & 0 & 0 & \ddots & * \\ W_j & 0 & 0 & 0 & 0 & 0 & -\overline{R}_{D_j j} \end{bmatrix} < 0 \quad (19)$$

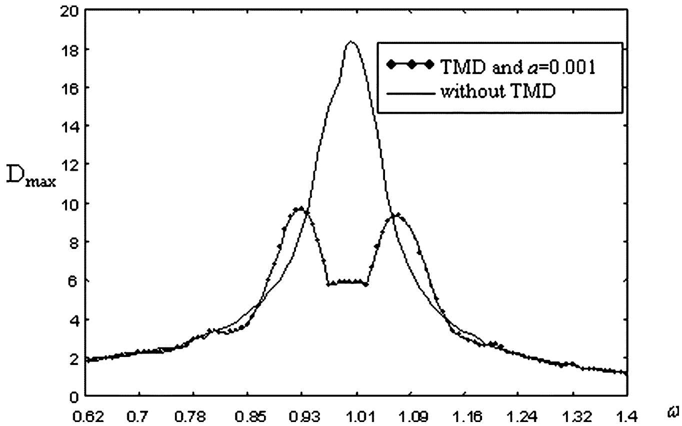

Fig. 4. Dynamic magnification factor of a TMD system with nonlinear stiffness $k(\mathrm{x})$.

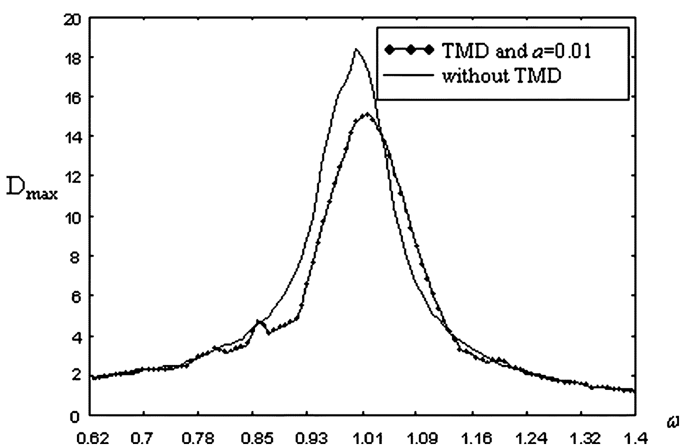

Fig. 5. Dynamic magnification factor of a TMD system with nonlinear stiffness $k(\mathrm{x})$.

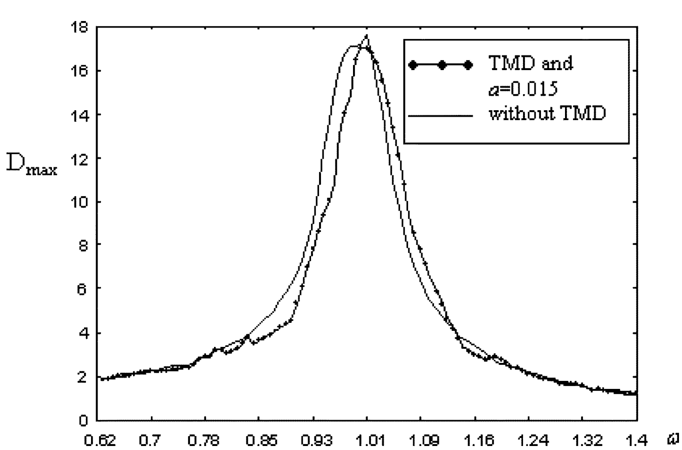

Fig. 6. Dynamic magnification factor of a TMD system with nonlinear stiffness $k(\mathrm{x})$.

B. PDC Fuzzy Controllers

The objective here is to synthesize a set of T-S fuzzy controller to stabilize the nonlinear interconnected system N which is composed of two subsystems with multiple time delays described in (21) and (22). Furthermore, the parameters in (20) are shown as follows:

$$\begin{aligned} a &= 0.01 \\ m_1 &= 1 \\ \omega_1 &= \omega_2 = 1.29 \\ c_1 &= 2.506 \times 10^{-3} \\ \xi_2 &= 2.506 \times 10^{-5} \\ F &= \cos(\overline{\omega} t) \\ \mu &= 0.01 \\ \overline{\omega} &= 1.29. \end{aligned}$$

Subsystem 1:

$$\begin{cases} \dot{x}_{11}(t) = 10x_{21}(t) + 0.01x_{11}(t-0.5) + 0.01x_{11}^2(t-0.5) \\ \qquad + 0.025x_{21}(t-0.5) + 0.01x_{21}^2(t-0.5) \\ \qquad + 0.01x_{11}(t-0.85) + 0.01x_{11}^2(t-0.85) \\ \qquad + 0.01x_{21}(t-0.85) + 0.01x_{21}^2(t-0.85) \\ \dot{x}_{21}(t) = -0.1681x_{11}(t) + 1.6641 \times 10^{-7}x_{11}^3(t) \\ \qquad - 2.531 \times 10^{-3}x_{21}(t) + 1.6641 \times 10^{-3}x_{12}(t) \\ \qquad - 1.6641 \times 10^{-9}x_{12}^3(t) + 1.6641 \\ \qquad \times 10^{-9}x_{11}(t)x_{12}^2(t) + 2.506 \times 10^{-5}x_{22}(t) \\ \qquad + 0.01x_{11}(t-0.5) + 0.01x_{21}(t-0.5) \\ \qquad + 0.0135x_{11}(t-0.85) + 0.01x_{21}(t-0.85) \\ \qquad + \cos(1.29t) + 5u_1(t) \end{cases} \tag{21}$$

Subsystem 2:

$$\begin{cases} \dot{x}_{12}(t) = 10x_{22}(t) + 0.01x_{12}(t-0.55) + 0.01x_{12}^2(t-0.55) \\ \qquad + 0.01x_{22}(t-0.55) + 0.01x_{22}^2(t-0.55) \\ \qquad + 0.01x_{12}(t-0.65) + 0.01x_{12}^2(t-0.65) \\ \qquad + 0.027x_{22}(t-0.65) + 0.01x_{22}^2(t-0.65) \\ \dot{x}_{22}(t) = -0.16641x_{12}(t) + 1.6641 \times 10^{-7}x_{12}^3(t) \\ \qquad - 2.506 \times 10^{-3}x_{22}(t) + 0.16641x_{11}(t) \\ \qquad - 1.664 \times 10^{-7}x_{11}(t)x_{12}^2(t) + 2.506 \times 10^{-3}x_{21}(t) \\ \qquad + 0.01x_{12}(t-0.55) + 0.01x_{22}(t-0.55) \\ \qquad + 0.01x_{12}(t-0.65) + 0.01x_{22}(t-0.65) + 4.5u_2(t) \end{cases} \tag{22}$$

where $x_{11} = 10s_1$, $x_{21} = \dot{s}_1$, $x_{12} = 10s_2$ and $x_{22} = \dot{s}_2$.

How do we synthesize two T-S fuzzy controllers to stabilize the nonlinear TMD system N?

Solution: We can solve this problem according to the following steps.

Step 1) Establish a T-S fuzzy model for each nonlinear interconnected subsystem by the concept of local linearization as that in [21], [36]. To minimize the design effort and complexity, we try to use as few rules as possible. Hence, the subsystems (21), (22) are approximated with the following fuzzy models:

1) T-S Fuzzy Model of Subsystem 1:

Rule 1 : IF $x_{11}(t)$ is M_{111} THEN

$$\dot{x}_1(t) = A_{11}x_1(t) + \sum_{k=1}^{2} A_{1k1}x_1(t-\tau_{k1}) + \hat{A}_{121}x_n(t) + B_{11}u_1(t)$$

Rule 2 : IF $x_{11}(t)$ is M_{211} THEN

$$\dot{x}_1(t) = A_{21}x_1(t) + \sum_{k=1}^{2} A_{2k1}x_1(t-\tau_{k1}) + \hat{A}_{221}x_n(t) + B_{21}u_1(t)$$

where

$$\begin{aligned} x_1^T(t) &= [x_{11}(t)x_{21}(t)] \\ \tau_{11} &= 0.5\ (s) \\ \tau_{21} &= 0.85\ (s) \\ A_{11} &= \begin{bmatrix} 0 & 10 \\ -0.1681 & -0.0025 \end{bmatrix} \\ A_{21} &= \begin{bmatrix} 0 & 10 \\ -0.1680 & -0.0025 \end{bmatrix} \\ A_{111} &= \begin{bmatrix} 0.01 & 0.02 \\ 0.01 & 0.01 \end{bmatrix} \\ A_{121} &= \begin{bmatrix} 0.01 & 0.01 \\ 0.014 & 0.01 \end{bmatrix} \\ B_{11} &= \begin{bmatrix} 0 \\ 5 \end{bmatrix} \\ A_{211} &= \begin{bmatrix} 0.01 & 0.03 \\ 0.01 & 0.01 \end{bmatrix} \\ A_{221} &= \begin{bmatrix} 0.01 & 0.01 \\ 0.013 & 0.01 \end{bmatrix} \\ B_{21} &= \begin{bmatrix} 0 \\ 5 \end{bmatrix} \\ \hat{A}_{121} &= \begin{bmatrix} 0 & 0 \\ 0.0017 & 0.000\,03 \end{bmatrix} \\ \hat{A}_{221} &= \begin{bmatrix} 0 & 0 \\ 0.0016 & 0.000\,03 \end{bmatrix} \end{aligned} \tag{23}$$

and the membership functions for Rule 1 and Rule 2 are

$$M_{111}(x_{11}(t)) = \frac{1}{\left[1 + \left|\frac{1-x_{11}(t)}{2}\right|\right]^2}$$

$$M_{211}(x_{11}(t)) = 1 - M_{111}(x_{11}(t)).$$

2) T-S Fuzzy Model of Subsystem 2:

Rule 1 : IF $x_{12}(t)$ is M_{112} THEN

$$\dot{x}_2(t) = A_{12}x_2(t) + \sum_{k=1}^{2} A_{1k2}x_2(t-\tau_{k2}) + \hat{A}_{112}x_n(t) + B_{12}u_2(t)$$

Rule 2 : IF $x_{12}(t)$ is M_{212} THEN

$$\dot{x}_2(t) = A_{22}x_2(t) + \sum_{k=1}^{2} A_{2k2}x_2(t-\tau_{k2}) + \hat{A}_{212}x_n(t) + B_{22}u_2(t)$$

where

$$\begin{aligned} x_2^T(t) &= [x_{12}(t)\ x_{22}(t)] \\ \tau_{12} &= 0.55\ (s) \\ \tau_{22} &= 0.65\ (s) \\ A_{12} &= \begin{bmatrix} 0 & 10 \\ -0.1664 & -0.0025 \end{bmatrix} \\ A_{22} &= \begin{bmatrix} 0 & 10 \\ -0.1663 & -0.0025 \end{bmatrix} \\ A_{112} &= \begin{bmatrix} 0.01 & 0.01 \\ 0.01 & 0.01 \end{bmatrix} \\ A_{122} &= \begin{bmatrix} 0.01 & 0.025 \\ 0.01 & 0.01 \end{bmatrix} \\ B_{12} &= \begin{bmatrix} 0 \\ 4.5 \end{bmatrix} \\ A_{212} &= \begin{bmatrix} 0.01 & 0.01 \\ 0.01 & 0.01 \end{bmatrix} \\ A_{222} &= \begin{bmatrix} 0.01 & 0.035 \\ 0.01 & 0.01 \end{bmatrix} \\ B_{22} &= \begin{bmatrix} 0 \\ 4.5 \end{bmatrix} \\ \hat{A}_{112} &= \begin{bmatrix} 0 & 0 \\ 0.1664 & 0.0025 \end{bmatrix} \\ \hat{A}_{212} &= \begin{bmatrix} 0 & 0 \\ 0.1663 & 0.0025 \end{bmatrix} \end{aligned} \tag{24}$$

and membership functions for Rule 1 and Rule 2 are

$$\begin{cases} M_{112}(x_{12}(t)) = \frac{2}{3\pi}x_{12}(t)+1 & \text{when } -\frac{3\pi}{2} \le x_{12}(t) \le 0 \\ M_{112}(x_{12}(t)) = -\frac{2}{3\pi}x_{12}(t)+1 & \text{when } 0 < x_{12}(t) \le \frac{3\pi}{2} \\ M_{112}(x_{12}(t)) = 0 & \text{otherwise} \end{cases}$$

$$M_{212}(x_{12}(t)) = 1 - M_{112}(x_{12}(t)).$$

Step 2) In order to stabilize the nonlinear interconnected system N, two model-based fuzzy controllers designed via the concept of PDC scheme are synthesized as follows.

1) Fuzzy controller of subsystem 1:

$$\begin{aligned} &\text{Rule 1: IF } x_{11}(t) \text{ is } M_{111}, \text{ THEN } u_1(t) = -K_{11}x_1(t) \\ &\text{Rule 2: IF } x_{11}(t) \text{ is } M_{211}, \text{ THEN } u_1(t) = -K_{21}x_1(t). \end{aligned} \tag{25}$$

2) Fuzzy controller of subsystem 2:

$$\begin{aligned} &\text{Rule 1: IF } x_{12}(t) \text{ is } M_{112}, \text{ THEN } u_2(t) = -K_{12}x_2(t) \\ &\text{Rule 2: IF } x_{12}(t) \text{ is } M_{212}, \text{ THEN } u_2(t) = -K_{22}x_2(t). \end{aligned} \tag{26}$$

Step 3) In accordance with Remark 1, specified structured bounding matrices are chosen as

$$\overline{H}_1 = \begin{bmatrix} 0 & 0 \\ 0 & 2.5 \times 10^{-7} \end{bmatrix}$$
$$\overline{H}_2 = \begin{bmatrix} 0.001 & 0.001 \\ 0.001 & 0.6 \end{bmatrix}$$
$$\delta_{ilj} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$\text{for} \quad i,l = 1,2; \quad j = 1,2. \tag{27}$$

Step 4) In order to guarantee the H^∞ control performance, the matrices Q_js in (16) must be chosen to be positive definite. At first, based on (23)–(27) and (18), we can get the common solutions W_j, Y_{ij}, and $\overline{R}_{kj}(k,j = 1,2)$ via Matlab LMI optimization toolbox with $\alpha = 0.1$, $\beta_1 = \beta_2 = 0.01$ and $\gamma = 0.1$.

$$W_1 = \begin{bmatrix} 13.6 & -14.683 \\ -14.683 & 39.271 \end{bmatrix}$$
$$W_2 = \begin{bmatrix} 18.623 & -12.271 \\ -12.271 & 33.925 \end{bmatrix}$$
$$\overline{R}_{11} = \begin{bmatrix} 4.6134 & 2.0926 \\ 2.0926 & 4.4518 \end{bmatrix}$$
$$\overline{R}_{21} = \begin{bmatrix} 4.2749 & 1.8496 \\ 1.8496 & 4.43 \end{bmatrix}$$
$$\overline{R}_{12} = \begin{bmatrix} 4.2604 & 2.0011 \\ 2.0011 & 4.5696 \end{bmatrix}$$
$$\overline{R}_{22} = \begin{bmatrix} 4.3972 & 1.7738 \\ 1.7738 & 4.0996 \end{bmatrix}$$
$$Y_{11} = [-24.032 \quad 208.08]$$
$$Y_{21} = [-15.913 \quad 204.52]$$
$$Y_{12} = [28.031 \quad 182.49]$$
$$Y_{22} = [28.031 \quad 182.49]$$
$$\overline{Q}_1 = \begin{bmatrix} 3.1632 & -3.0528 \\ -3.0528 & 3.6777 \end{bmatrix}$$
$$\overline{Q}_2 = \begin{bmatrix} 3.1632 & -3.0528 \\ -3.0528 & 3.6777 \end{bmatrix}$$
$$\overline{Q}_3 = \begin{bmatrix} 3.3052 & -2.4183 \\ -2.4183 & 2.7632 \end{bmatrix}.$$

Then, the following positive definite matrices $P_j(= W_j^{-1})$, $R_{kj}(= \overline{R}_{kj}^{-1})$, Q_js $(= \overline{Q}_j^{-1})$ and feedback gains K_{ij}s $(= Y_{ij}W_j^{-1})$ can be obtained such that (16) is satisfied

$$P_1 = \begin{bmatrix} 0.1233 & 0.0461 \\ 0.0461 & 0.0427 \end{bmatrix}$$
$$P_2 = \begin{bmatrix} 0.0705 & 0.0255 \\ 0.0255 & 0.0387 \end{bmatrix} \tag{28}$$
$$R_{11} = \begin{bmatrix} 0.2755 & -0.1295 \\ -0.1295 & 0.2855 \end{bmatrix}$$

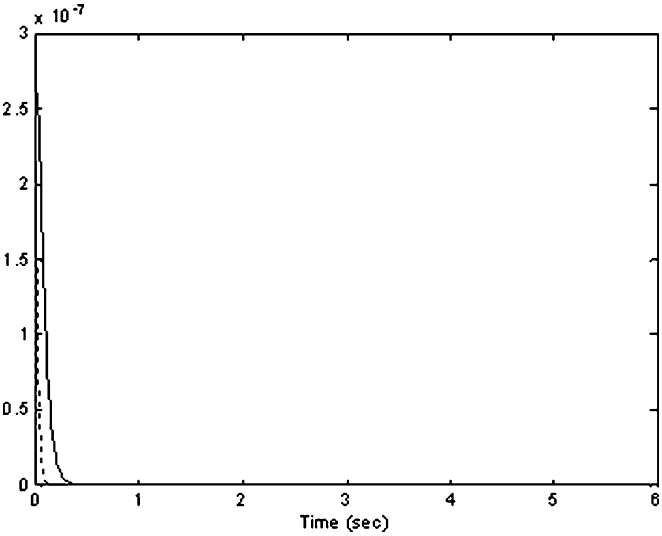

Fig. 7. Plots of $\|\overline{f_1}(x_1(t)) - \sum i = 1^2 \sum_{l=1}^2 h_{i1}(t)h_{l1}(t)(A_{i1} - B_{i1}K_{l1})x_1(t)\|$ (dashed line) and $\|\sum_{i=1}^2 \sum_{l=1}^2 h_{i1}(t)h_{l1}(t)\Delta H_{il1}x_1(t)\|$ (solid line).

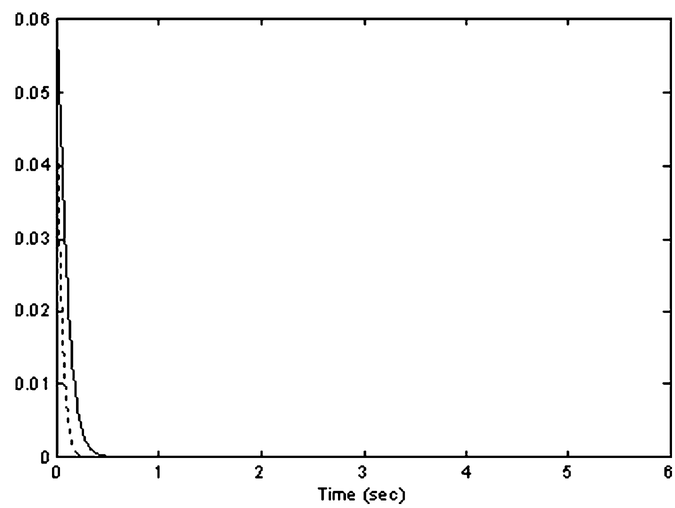

Fig. 8. Plots of $\|\overline{f_2}(x_2(t)) - \sum_{i=1}^2 \sum_{l=1}^2 h_{i2}(t)h_{l2}(t)(A_{i2} - B_{i2}K_{l2})x_2(t)\|$ (dashed line) and $\|\sum_{i=1}^2 \sum_{l=1}^2 h_{i2}(t)h_{l2}(t)\Delta H_{il2}x_2(t)\|$ (solid line).

$$R_{21} = \begin{bmatrix} 0.2855 & -0.1192 \\ -0.1192 & 0.2755 \end{bmatrix}$$
$$R_{12} = \begin{bmatrix} 0.2955 & -0.1294 \\ -0.1294 & 0.2755 \end{bmatrix}$$
$$R_{22} = \begin{bmatrix} 0.2755 & -0.1192 \\ -0.1192 & 0.2955 \end{bmatrix} \tag{29}$$
$$K_{11} = [11.9664 \quad 9.9995]$$
$$K_{21} = [7.4664 \quad 7.9995]$$
$$K_{12} = [6.6297 \quad 7.7772]$$
$$K_{22} = [6.6297 \quad 7.7772] \tag{30}$$
$$Q_1 = \begin{bmatrix} 1.5897 & 1.3196 \\ 1.3196 & 1.3673 \end{bmatrix}$$
$$Q_2 = \begin{bmatrix} 1.5897 & 1.3196 \\ 1.3196 & 1.3673 \end{bmatrix}$$
$$Q_3 = \begin{bmatrix} 0.8412 & 0.7362 \\ 0.7362 & 1.0062 \end{bmatrix}. \tag{31}$$

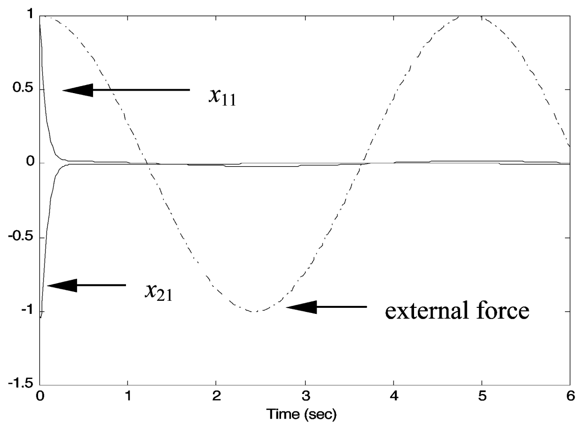

Fig. 9. State response of subsystem 1.

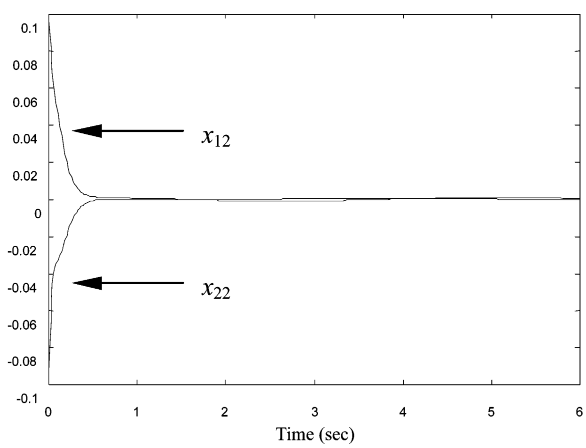

Fig. 10. State response of subsystem 2.

Furthermore, the assumption of

$$\|\Delta\Phi_j(t)\| \leq \left\| \sum_{i=1}^{r_j} \sum_{l=1}^{r_j} h_{ij}(t)h_{lj}(t)\Delta H_{ilj}x_j(t) \right\| \quad \text{for } n,\ j=1,2$$

are satisfied from the illustration in Figs. 7 and 8 with initial conditions, $x_{11}(0) = 1$, $x_{21}(0) = -1$, $x_{12}(0) = 0.1$, and $x_{22}(0) = -0.1$.

Therefore, based on Theorem 1, the T-S fuzzy controllers described in (25) and (26) can stabilize the nonlinear interconnected TMD system N. Simulation results of each closed-loop subsystem $\overline{N}_j$ $(j = 1, 2)$ are illustrated in Figs. 9 and 10. From Figs. 9 and 10, we have that the nonlinear interconnected system is stable because the trajectories of two subsystems starting from nonzero initial states both approach close to the origin under harmonic excitation.

VII. Conclusion

In this paper, a stability criterion is derived for nonlinear multiple time-delay interconnected systems via Lyapunov's direct method. An example of nonlinear TMDs is given to demonstrate the validity of the proposed controller design and it shows TMD is not suitable to reduce responses in nonlinear systems. A systematic design of fuzzy control is therefore proposed to ensure the stability of nonlinear multiple time-delay interconnected systems. According to the stability criterion and the decentralized control scheme, a set of model-based fuzzy controllers via the technique of PDC is proposed to overcome the influence of modeling error and stabilize the nonlinear multiple time-delay interconnected TMD systems. So, the proposed H^∞ control performance of fuzzy control can be applied to the robust control design of nonlinear interconnected systems with multiple time delays.

Appendix
Proof of Theorem 1

Let the Lyapunov function for the closed-loop nonlinear multiple time-delay interconnected system $\overline{N}$ be defined as

$$\begin{aligned} V(t) &= \sum_{j=1}^{J} v_j(t) \\ &= \sum_{j=1}^{J} \left[x_j^T(t)P_jx_j(t) + \sum_{k=1}^{D_j} \right. \\ &\qquad \left. \times \int_0^{\tau_{kj}} x_j^T(t-\tau)R_{kj}x_j(t-\tau)d\tau \right] \end{aligned} \tag{32}$$

where the weighting matrices $P_j = P_j^T > 0$ and $R_{kj} = R_{kj}^T > 0$. We then evaluate the time derivative of $V(t)$ on the trajectories of (9) to get

$$\begin{aligned} \dot{V} &= \sum_{j=1}^{J} \dot{v}_j(t) \\ &= \sum_{j=1}^{J} \sum_{i=1}^{r_j} \sum_{l=1}^{r_j} h_{ij}(t)h_{lj}(t)x_j^T(t) \\ &\quad \cdot \left[(A_{ij} - B_{ij}K_{lj})^T P_j + P_j(A_{ij} - B_{ij}K_{lj}) \right] x_j \\ &\quad + \sum_{j=1}^{J} \sum_{i=1}^{r_j} \sum_{l=1}^{r_j} \sum_{\substack{n=1 \\ n\neq j}}^{J} h_{ij}(t)h_{lj}(t) \\ &\quad \times \left[x_n^T(t)\hat{A}_{inj}^T P_j x_j(t) + x_j^T(t)P_j\hat{A}_{inj}x_n(t) \right] \\ &\quad + \sum_{j=1}^{J} \sum_{i=1}^{r_j} \sum_{l=1}^{r_j} \sum_{k=1}^{D_j} h_{ij}(t)h_{lj}(t) \\ &\quad \cdot \left[x_j^T(t-\tau_{kj})A_{ikj}^T P_j x_j(t) + x_j^T(t)P_jA_{ikj}x_j(t-\tau_{kj}) \right] \\ &\quad + \sum_{j=1}^{J} \left[\Delta\Phi_j^T(t)P_jx_j(t) + x_j^T(t)P_j\Delta\Phi_j(t) \right] \\ &\quad + \sum_{j=1}^{J} \left[\phi_j(t)P_jx_j(t) + x_j^T(t)P_j\phi_j(t) \right] \\ &\quad + \sum_{j=1}^{J} \sum_{k=1}^{D_j} \left(x_j^T(t)R_{kj}x_j(t) - x_j^T(t-\tau_{kj}) \right. \\ &\qquad \left. \times R_{kj}x_j(t-\tau_{kj}) \right). \end{aligned} \tag{33}$$

Based on Lemma 1, (33) and (15), we have

$$\begin{aligned}\dot{V} \leq & \sum_{j=1}^{J}\sum_{i=1}^{r_j}\sum_{l=1}^{r_j} h_{ij}(t)h_{lj}(t)x_j^T(t) \\ & \times \Bigg[(A_{ij}-B_{ij}K_{lj})^T P_j + P_j(A_{ij}-B_{ij}K_{lj}) \\ & \quad + \sum_{k=1}^{D_j} R_{kj} + \sum_{k=1}^{D_j} P_j A_{ikj} R_{kj}^{-1} A_{ikj}^T P_j \Bigg] x_j(t) \\ & + \sum_{j=1}^{J}\sum_{i=1}^{r_j}\sum_{n=1}^{J} h_{ij}(t) \\ & \times \Bigg[\alpha\left(\frac{J-1}{J}\right) x_j^T(t)x_j(t) + \alpha^{-1} \\ & \quad \times \Big(x_j^T(t)P_j\hat{A}_{inj}\hat{A}_{inj}^T P_j x_j(t) \\ & \qquad - \frac{1}{J} x_j^T(t)P_j\hat{A}_{ijj}\hat{A}_{ijj}^T P_j x_j(t)\Big)\Bigg]^1 \\ & + \sum_{j=1}^{J}\Big\{\beta_j \left[\overline{H}_j x_j(t)\right]^T \left[\overline{H}_j x_j(t)\right] \\ & \qquad + \beta_j^{-1}\left[x_j^T(t)P_j^2 x_j(t)\right]\Big\} \\ & + \sum_{j=1}^{J}\left[\gamma^{-1}\left(x_j^T(t)P_j^2 x_j(t)\right) + \gamma\left(\phi_j^T(t)\phi_j(t)\right)\right]. \end{aligned} \tag{34}$$

In view of $\sum_{i=1}^{r_j} h_{ij}(t) = 1$ and $\sum_{i=1}^{r_j} h_{lj}(t) = 1$, we have

$$\begin{aligned}\dot{V} \leq & \sum_{j=1}^{J}\sum_{i=1}^{r_j}\sum_{l=1}^{r_j} h_{ij}(t)h_{lj}(t)x_j^T(t) \\ & \times \Big\{(A_{ij}-B_{ij}K_{ij})^T P_j + P_j(A_{ij}-B_{ij}K_{ij}) \\ & \quad + \sum_{k=1}^{D_j} R_{kj} + \sum_{k=1}^{D_j} P_j A_{ikj} R_{kj}^{-1} A_{ikj}^T P_j \\ & \quad + \sum_{n=1}^{J} \alpha^{-1} P_j \hat{A}_{inj}\hat{A}_{inj}^T P_j + \alpha(J-1)I \\ & \quad + \beta_j \overline{H}_j^T \overline{H}_j + \beta_j^{-1} P_j^2 + \gamma^{-1} P_j^2 \Big\} x_j(t) \\ & + \sum_{j=1}^{J} \gamma\left[\phi_j^T(t)\phi_j(t)\right]. \end{aligned} \tag{35}$$

Based on (16) and (35)

$$\begin{aligned}\dot{V} \leq & \sum_{j=1}^{J}\sum_{i=1}^{r_j}\sum_{l=1}^{r_j} h_{ij}(t)h_{lj}(t)x_j^T(t)\{-Q_j\}x_j(t) \\ & + \sum_{j=1}^{J}\gamma\left[\phi_j^T(t)\phi_j(t)\right] \\ \leq & \sum_{j=1}^{J} -x_j^T(t)Q_j x_j(t) + \sum_{j=1}^{J}\gamma \left\|\phi_{upj}(t)\right\|^2. \end{aligned} \tag{36}$$

[1] Based on the concept of interconnection, the matrix $\hat{A}_{ijj}$ is equal to zero.

This demonstrates that the trajectories of the closed-loop system (9) in the absence of disturbance are asymptotically stable. Integrating (36) from $t = 0$ to $t = t_f$ yields

$$V(t_f) - V(0) \leq -\sum_{j=1}^{J}\int_0^{t_f} x_j^T(t)Q_j x_j(t)dt + \sum_{j=1}^{J}\gamma\int_0^{t_f}\phi_j^T(t)\phi_j(t)dt. \tag{37}$$

From (32), we get

$$\sum_{j=1}^{J}\int_0^{t_f} x_j^T(t)Q_j x_j(t)dt \leq \sum_{j=1}^{J} x_j^T(0)P_j x_j(0) + \sum_{j=1}^{J}\gamma\int_0^{t_f}\phi_j^T(t)\phi_j(t)dt \tag{38}$$

and then the H^∞ control performance can be achieved with a prescribed $\gamma = \eta^2$.

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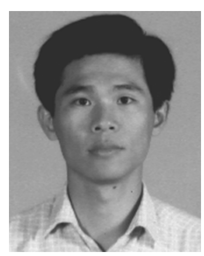

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