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Theory of the fluctuation diamagnetism of strongly layered superconductor in strong magnetic field

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Abstract. Recent experiments in $Bi_2Sr_1Ca_1Cu_2O_{8+\delta}$[7] demonstrated that the fluctuation diamagnetism in strong magnetic field at the vicinity of the critical temperature significantly differs from the behavior predicted by fluctuation diamagnetism theories based on Ginzburg-Landau model within the lowest Landau level approximation. In this study, we extend previous theoretical result to the $T < T_c$ region and incorporate to the theory contributions from higher Landau levels. Comparison with experimental results shows a good agreement with the theory for an optimally doped sample, while for under-doped sample the agreement is poor at low temperature.

1. Introduction

Fluctuating behavior near upper critical field, $H_{c2}(T)$, in type-II superconductors has been a subject of an intense interest in experiments and theory from the 90s. Theoretically, the problem of fluctuations can be investigated in framework of Ginzburg-Landau (GL) free energy functional, with the fluctuation of order parameter confined to the lowest-Landau level (LLL). The GL-LLL theory shows single-parameter scaling properties of various thermodynamic quantities, such as magnetization and heat capacity which are confirmed by numerous experiments [2, 3]. The LLL scaling functions are found associated with characteristic dimensionality of the system: Thermodynamic functions scale with scaling variable $a_T = [T - T_c(H)]/(HT)^\gamma$ where $\gamma = 1/2$ for quasi-2D system and $\gamma = 2/3$ for anisotropic 3D system.

The scaling behavior might not hold due to the layer structure of superconductor [4, 5, 9, 10] or due to strong fluctuations which enter higher Landau levels (HLLs)[1, 8]. Recent study of qusi-2D system by Ong [7] shows that the magnetization curve deviates from the LLL scaling behaviors (see Fig.1(a)) and disagrees with theoretical explanations based on GL[1, 8].

In this study, we calculate explicitly the free energy for Lawrence-Doniach model with all Landau levels correction by variational Gaussian approximation (VGA). The contribution from LLL is carefully considered using Borel-Pade (BP) resummation technique in ref.[5].

2. Layered superconductor in Magnetic field

Analysis of layered superconductor is accounted for using the Lawrence-Doniach model. The Boltzman factor is

$$F_{LD}[\Psi] = \frac{1}{T}\sum_{l}\int_{r} D\left\{-\alpha T_c(1-t)|\Psi_l|^2 + \frac{1}{2}\beta|\Psi_l|^4 + \frac{\hbar^2}{2m^*_{ab}}\left|(\nabla - i\frac{e^*}{\hbar c}\vec{A})\Psi_l\right|^2 + \frac{\hbar^2}{2m^*_c S^2}|\Psi_l - \Psi_{l-1}|^2\right\} \tag{1}$$

where Ψ_l is the order parameter of the l-th superconducting layer, $\vec{A}$ is vector potential, S is inter-layer spacing and D is the thickness of each layer. The free energy density is defined as $f = -(T/Vol)ln\int_\Psi exp\{-F_{LD}[\Psi]\}$ and magnetization is $M = -\partial f/\partial H$. We will use rescaled quantities from now on: $T = tT_c$, $H = bH_{c2}$, $\vec{R} = \vec{r}\xi$, $S = s\xi_c$, $D = d\xi_c$ and $\Psi^2 = \frac{2\alpha T_c}{\beta}\psi^2$. The length scales are coherence lengths which are different in xy (ξ) and z direction (ξ_c). To discuss topic related to charged particles in the presence of uniform magnetic field, expanding the fluctuations in Landau basis is a convenience choice, thus

$$\psi_l(r) = \sum_{n,k,k_z} c_{n,\vec{k},k_z}\phi_{n,\vec{k}}(r)\exp(ik_z l), \tag{2}$$

where n, k, k_z are the quantum numbers of the system. In this study, a rescaled gauge $A = (0, bx, 0)$ is chosen to describe the magnetic field b in z direction.

According to VGA, a variational parameter ε of quadratic term is introduced to minimize the free energy. The functional can be separated into two parts: The dominant part K which is quadratic in order parameter and small part V which will be treated as a small parameter:

$$K = \frac{1}{\omega t}\sum_{l}^{N}\int_{r}\left\{\frac{1}{2}|(\nabla - ib)\psi_l|^2 + \frac{1}{s^2}|\psi_l - \psi_{l+1}|^2 + \varepsilon|\psi_l|^2\right\}, \tag{3}$$

$$V = \frac{1}{\omega t}\sum_{l}^{N}\int_{r}\left\{(-(1-t)-\varepsilon)|\psi_l|^2 + \frac{1}{2}|\psi|^4\right\} \tag{4}$$

where $\omega = \beta/4d\gamma\xi^3\alpha^2 T_c$ (with $\gamma = \sqrt{m^*_c/m^*}$) is proportional to $\sqrt{Gi}$. The rescaled free energy density is defined as $f_r = -\omega t ln\int_\psi exp\{-K[\psi] + V[\psi]\}$. In first order of V, the rescaled free energy density is written as

$$f_r(\varepsilon) \approx \frac{\omega t}{s}\left\{v(\varepsilon, b) + (-a_h - \varepsilon)\partial_\varepsilon v(\varepsilon, b) + \omega t(\partial_\varepsilon v(\varepsilon, b))^2\right\} \tag{5}$$

where

$$v(\varepsilon, b) = \frac{b}{2\pi}\sum_n\int_{-\pi/2}^{\pi/2} dk_z ln\left[nb + \varepsilon + (1 - cos[k_z s])s^{-2}\right] \tag{6}$$

$$= \frac{b}{2\pi}\sum_n 2\left\{ln\left[\sqrt{nb+\varepsilon} + \sqrt{nb+\varepsilon+2s^{-2}}\right] - ln2\right\} \tag{7}$$

$$\approx \frac{b}{2\pi}\left\{\sum_n ln[nb+\varepsilon] + \frac{1}{s^2}\sum_n\frac{1}{nb+\varepsilon}\right\}. \tag{8}$$

The limit for large s with $(1+x)^{1/2} \approx 1 + \frac{1}{2}x$ and $ln(1+x) \approx x$ are taken at the last step. Our main goal is to calculate the first term of eq.(8),

$$u(\varepsilon, b) = \frac{b}{2\pi}\sum_n ln[nb+\varepsilon] \tag{9}$$

$$\approx \frac{1}{2\pi}\left\{\Lambda(ln\Lambda - 1) + \left(\varepsilon - \frac{b}{2}\right)ln\Lambda\right\} + \frac{b}{2\pi}\left\{\left(\frac{1}{2} - \frac{\varepsilon}{b}\right)lnb + \varphi(\frac{\varepsilon}{b})\right\} \tag{10}$$

where $\varphi(x)$ is basically a loggamma function [8, 6] and Λ is a cutoff as $n_c = \Lambda/b - 1$. The first term of eq.(10) is divergent. However, similar to the renormalization of mass in field theory, the bare critical temperature is renormalized as $a_h^r = a_h - \frac{\omega t}{\pi} ln\Lambda$. Note also that the second term of eq.(7) is simply $\partial_\varepsilon u$. In low field limit, $u(\varepsilon, b) \approx -\frac{1}{24}\frac{b}{\varepsilon - \frac{b}{2}} + u(\varepsilon, 0)$ where $u(\varepsilon, 0)$ is a b independent diverging quantity.

Minimizing $f_r(\varepsilon)$ with respect to ε, one gets a gap equation,$\partial f_r(\varepsilon) = 0$,

$$\varepsilon \quad = \quad -a_h + 2\omega t \partial_\varepsilon v(\varepsilon, b). \tag{11}$$

In $2D$ limit, the gap equation is $\varepsilon = -a_h^r + 2\omega t \partial_\varepsilon u(\varepsilon, b)$with $a_h = (1 - t - b)/2$. The rescaled free energy density of the system ,f_r, can be obtained by substitution of the gap solution in eq.(5). The effective free energy density we used includes HLLs part (f_G^{HLL}) by VGA and LLL (f_{BP}^{LLL}) by BP: $f \approx f_G^{HLL} + f_{BP}^{LLL}$. The rescaled magnetization is given by $m = \partial_b f_r$,

$$m = \frac{\omega t}{s}\left\{\partial_b v + \frac{1}{2}\partial_\varepsilon v\right\}|_{\varepsilon=\varepsilon_{sol}}, \tag{12}$$

while the relation between rescaled quantities and phyiscal quantities are:

$$f = \frac{H_{c2}^2}{2\pi\kappa^2} f_r, M = \frac{H_{c2}}{2\pi\kappa^2} m. \tag{13}$$

3. Results and Analysis of quasi-2D system

The failure of LLL scaling in $BSCCO$[7] is shown in Fig.1. The magnetization curves do not collapse into a single line in the vicinity of $H_{c2}(T)$, i.e. $a_T = 0$, especially for under-doped sample. The rescaling parameters are collected in the first two columns of Tab.1. In Fig.2, one can see the contribution of HLLs on magnetization,with coupling between different levels and each HLL, at high temperature is important, it tends to approach a constant. The HLLs contribution suppress the diamagnetic property, as shown in Ref. [1], the magnetization is asymptotically approaching zero at high temperature. However, since our expansion parameter is ωt, thus our result is not reliable at very high temperature. While the contribution from HLLs decays exponentially at low temperature. The HLLs contribution for lower field, say $b = .1$ (blue), is stronger at high temperature while at high field, say $b = .5$ (red), is also important at lower temperatures. The contribution of HLLs can be enhanced by large Gi (see the dashed curve). High Landau level contribution is not negligible at the vicinity of T_c for strong fluctuating system. Comparison of optimally doped and under doped sample with all Landau corrections are shown in Fig.3 with the fitting parameters in Tab.(1).

Table 1. Fitting parameters of magnetization curves

	$T_c\,(K)$	$H_{c2}^{\prime scaling}(T/K)$	$H_{c2}^{\prime fitting}(T/K)$	Gi	κ
Optimally Doped	93.3	1.25	2.36	0.0035	72.3747
Under-doped	57	1.80	2.98	0.31	97.183

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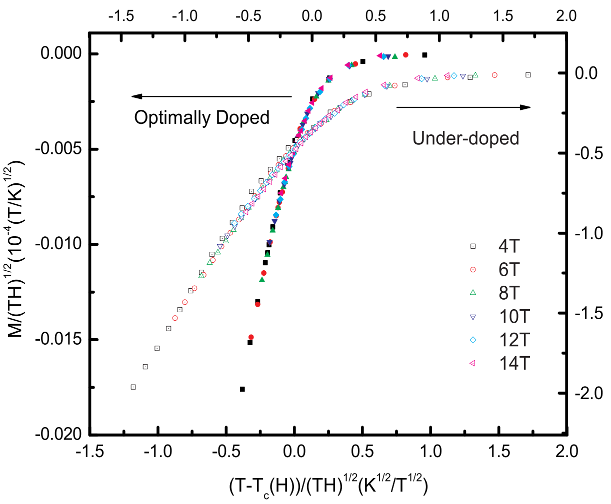

Figure 1. LLL magnetization scaling of $Bi_2Sr_1Ca_1Cu_2O_{8+\delta}$[7]. ■: optimally doped sample and □: under-doped sample. Different color denotes different magnetic fields.

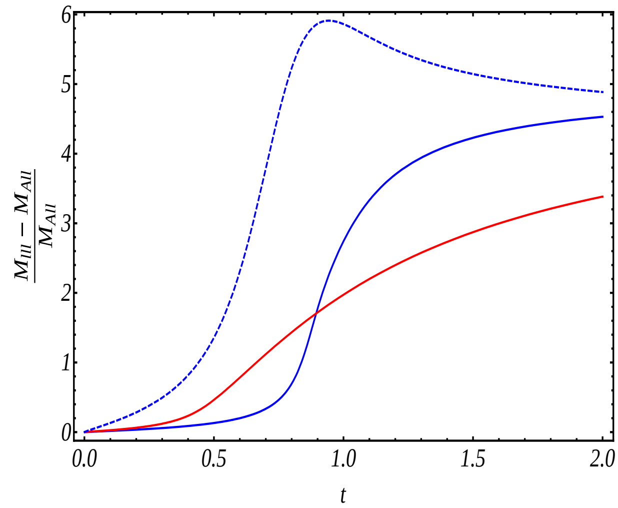

Figure 2. HLLs contributions of magnetization as a function of rescaled temperature in VGA. ——: $Gi = 10^{-5}$ and - - - -: $Gi = 10^{-3}$. $b = 0.1$ for Blue line and $b = 0.5$ for red line.

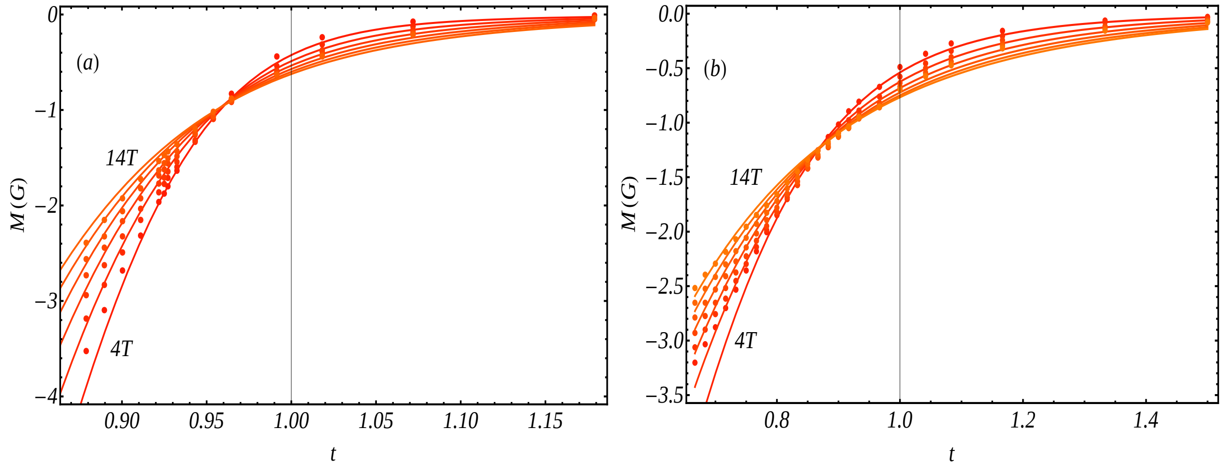

Figure 3. Magnetization curves as a function of temperature, t, in various magnetic fields: (a)Optimally doped BSCCO(b)Under-doped BSCCO. ● are experimental data and ——are theoretical curves

References

[1] Prange R E 1970 *Phys. Rev. B* **1** 2349
[2] Li Q, Suenaga M, Hikata T and Sato K 1992 *Phys. Rev. B.* **46** 5857
[3] Welp U, Fleshler S, Kwok WK, Klemm RA, Vinokur VM, Downey J, Veal B and Crabtree GW 1991 *Phys. Rev. Lett.* **67** 3180
[4] Tešanović Z , Xing L , Bulaevskii L , Li Q and Suenaga M 1992 *Phys. Rev. Lett.* **69** 3563
[5] Li D and Rosenstein B 2002 *Phys. Rev. B* **65** 024513
[6] Li D and Rosenstein B 2004 *Phys. Rev.B* **70** 144521
[7] Ong N P 2005,*Phys. Rev. Lett.* **95** 247002
[8] Larkin A and Varlamov A 2005 *Theory of Fluctuations in Superconductors* (Oxford University Press) chapter 2 pp 7-30
[9] Naughton M J 2000 *Phys. Rev. B* **61** 1605
[10] Salem-Sugui J S, Alvarenga A D, Goretta K C, Vieira V N , Veal B and Paulikas A P 2005 *J. of Low Temp Phys* **141** 83