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Neural-network-based optimal fuzzy controller design for nonlinear systems

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Abstract

A neural-learning fuzzy technique is proposed for T–S fuzzy-model identification of model-free physical systems. Further, an algorithm with a defined modelling index is proposed to integrate and to guarantee that the proposed neural-based optimal fuzzy controller can stabilize physical systems; the modelling index is defined to denote the modelling-error evolution, and to ensure that the training data for neural learning can describe the physical system behavior very well; the algorithm, which integrates the neural-based fuzzy modelling and optimal fuzzy controlling process, can implement off-line modelling and on-line optimal control for model-free physical systems. The neural-fuzzy inference network is a self-organizing inference system to learn fuzzy membership functions and fuzzy-subsystems' parameters as data feeding in. Based on the generated T–S fuzzy models for the continuous mass–spring–damper system and Chua's chaotic circuit, discrete-time model car system and articulated vehicle, their corresponding fuzzy controllers are formulated from both local-concept and global-concept fuzzy approach, respectively. The simulation results demonstrate the performance of the proposed neural-based fuzzy modelling technique and of the integrated algorithm of neural-based optimal fuzzy control structure.

Keywords: Riccati equation; Modelling index; *Linear* T–S fuzzy system; *Affine* T–S fuzzy system; Exponentially stable

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1. Introduction

Research in fuzzy modelling and fuzzy control has come of age [1,2,5,11,22,23]. There are two main ways to *theoretically* construct a T–S fuzzy model. One is from local linear approximation, which generates a linear consequent part with a constant term included in each rule; the other is via sector nonlinearity concept [7,14,15], which results in a constant-free linear consequence for each rule. Both are demonstrated to be universal approximations to any smooth nonlinear systems [16,20,28]. For simplification, these two kinds of fuzzy structures are, respectively, denoted as *linear* and *affine* T–S fuzzy systems by Tanaka and Wang [20]. It is noticed that the consequent part of each fuzzy rule in both models are represented by a linear state equation; the only difference between these two representations is that there exists a constant singleton in the fuzzy rule consequence for the *affine* T–S fuzzy model.

The T–S type with no constant term in the local linear consequent part of each rule (*linear* T–S fuzzy system) is the most popular fuzzy model for its further intrinsic analysis: T–S model-based fuzzy control has been successfully applied to many nonlinear systems [15]. The linear matrix inequality (LMI)-based fuzzy controller is to minimize the upper bound of the performance index [21]. Structure-oriented and switching fuzzy controller are further developed for more complicated systems [6,12,14]. The optimal fuzzy control technique is used to minimize the performance index from local-concept or global-concept approach [25–27]. Recently, Tanaka and Wang developed an integrated LMI approach to fuzzy modelling and controlling a nonlinear system with unknown parameters [10]. Three LMI conditions are derived to identify the parameters of T–S fuzzy models, and a robust controller is developed to compensate the identification error. The membership functions and fuzzy rule numbers are chosen as known parameters in the aforementioned approach. And in order to decrease the computational cost, much research focuses on rule and consequence order reduction [6,15,17] and on rule switching technique [12]. Advanced research for fuzzy modelling of more complicated systems is still open. Further, the aforementioned research is available only for model-based nonlinear systems.

The approach of model-free nonlinear systems to guarantee the proposed fuzzy model under limited modelling error and the corresponding fuzzy control with desirable implementation is still developing. Yu and co-workers use a type-1 fuzzy neural network (FNN) with sliding-mode and gradient-decent learning to control a Duffing system [9]. Wai uses FNN to mimic a perfect control law and compensate the error by another compensator [19]. Lin and co-workers use FNN to approximate nonlinear functions and develop adaptive laws to attenuate approximation errors and external disturbance [4]. Hu and Liu fuzzy model a time-delay system *analytically*, then use adaptive RBF NN to approximate fuzzy modelling error and adopt H_∞ control to compensate the error [8]. Wang and co-workers use type-1 FNN with adaptive update law to approximate an optimal controller [24]. Most of them describe systems with fuzzy rules and use FNN to control the systems. There was no direct approach to identify T–S fuzzy systems of model-free nonlinear systems.

In this work, we propose a neural fuzzy network (NFN) to achieve identification of a *linear* T–S fuzzy model for model-based or model-free systems, which can self-learn the Gaussian-type membership functions and fuzzy subsystems' parameters of each rule consequence. The generated *linear* T–S fuzzy model can be used to develop fuzzy controllers such as an LMI-based fuzzy controller, structure-oriented and switching fuzzy controllers. In order to further ensure that the generated fuzzy model can approximate the original physical system and more to control the model-free system well, we propose an integrated algorithm, which integrates the proposed neural fuzzy network and previously proposed nonlinear optimal fuzzy controller, to guarantee the generated fuzzy system can describe the physical system behavior and the closed-loop neural-based optimal fuzzy control system is stable. The proposed structure is applied to fuzzy modelling and optimal controlling of a mass–spring–damper system, a chaotic Chua's circuit system, a model car system and an articulated vehicle system.

2. Neural-based fuzzy model and optimal controller

2.1. Neural-based fuzzy inference structure

As we know, the T–S fuzzy model is basically a locally linearized fuzzy model, which describes global behavior by fuzzily blending linear subsystems. Most T–S fuzzy models are identified by, respectively, local linear approximation and sector nonlinearity concept [7,15], which *fuzzily blends* the bounded values of each nonlinear term to achieve global or semi-global effect. Accordingly, two kinds of T–S fuzzy system representations, *affine* T–S fuzzy model and *linear* T–S fuzzy model, are generated. The difference between these two representations is that a singleton is included in the fuzzy subsystems of the *affine* T–S fuzzy model. Both fuzzy models are demonstrated to be universal approximations of any smooth nonlinear system to any desired accuracy. However, these two modelling techniques (local linear approximation and sector nonlinearity concept) are available for model-based systems only. Besides, since the controller design for *linear* T–S fuzzy model has been developed very well, it is important to propose a modelling technique to construct a *linear* T–S fuzzy system not only for model-based but also for model-free nonlinear physical systems.

Juang and Lin proposed a neural-fuzzy inference network with self-learning ability (SONFIN) [3], though an *affine* T–S fuzzy system can be obtained via this network by regarding external inputs as augmented state variables. However, the singleton in each fuzzy rule is the key consequence for learning and the state-dependent terms are just optional generated for compensation. The learning process will always diverge by just deleting the singleton from the rule consequence of Juang's algorithm directly. In other words, basic SONFIN structure will learn type-1 fuzzy system basically. We modified this neural fuzzy network such that the input- and state-dependent terms initially exist and the corresponding parameters are adapted by the gradient method; in other words, the learning process will focus on generating input- and state-dependent terms.

We here name the modified NFN to be *linear*-NFN and Juang's to be *affine*-NFN to denote the constructed fuzzy models to be *linear* T–S type and *affine* T–S type, respectively. Notice that even these two structures are similar in representation but the learning spirit is totally different. That is, the singleton is the key term and state-dependent terms are optional generated for compensation in the rule consequences of affine type, but the input- and state-dependent terms are now the key terms in those of linear type.

Fig. 1 describes the proposed six-layer *linear* NFN structure for realizing a *linear* T–S fuzzy model. This structure is similar to Juang's except for the rule representation in the fifth layer. Each node in the structure possesses finite weighted fan-in connections to the last-layer nodes and fan-out connections to the next-layer nodes. An integration function is associated with the fan-in operation to integrate information, activation and evidence; in other words, the integration function is the net input of a node. For example, for the *i*th node in the *k*th layer, we have

$$net-input_i^k = f(u_{1_i}^k, u_{2_i}^k, \ldots, u_{p_i}^k, w_{1_i}^k, w_{2_i}^k, \ldots, w_{p_i}^k),$$

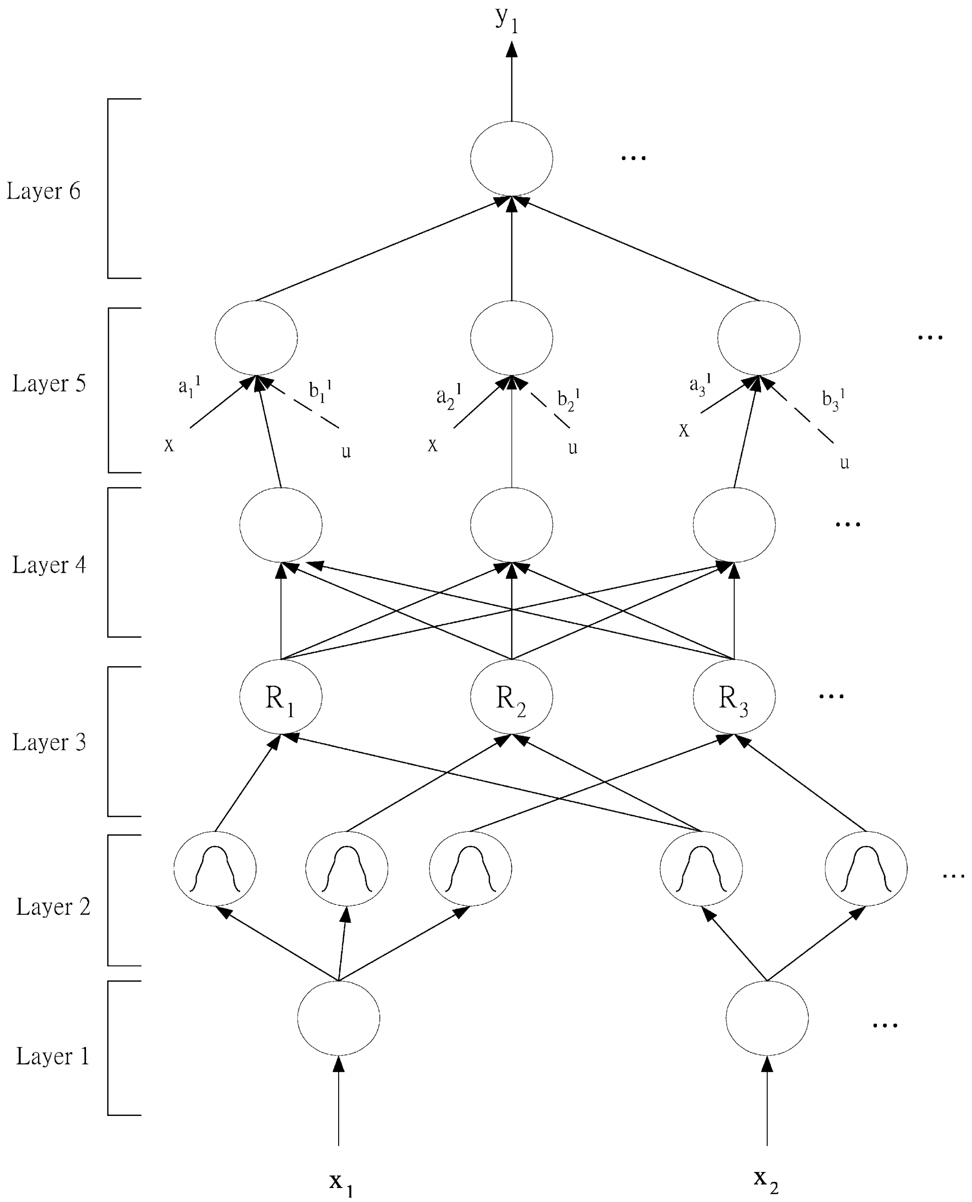

Fig. 1. *linear*-SONFIN structure.

where $u^k_{1_i}, u^k_{2_i}, \ldots, u^k_{p_i}$ are the inputs of the ith node and $w^k_{1_i}, w^k_{2_i}, \ldots, w^k_{p_i}$ are the associated weights. The output operation o^k_i is then proceeded by an activation function $a(\cdot)$,

$$o^k_i = a(net - input^k_i).$$

We now briefly describe the proposed six-layer NFN structure as follows.

Layer 1: Each node in this layer is correspondent to one input variable and transmits the input variable to the next layer directly; that is,

$$f = u_i^1, \quad a^1 = f,$$

where the linking weight w_i^1 is unity in this layer. *Layer 2*: Each node in this layer denotes a linguistic label; that is, the input variables are fuzzified in this layer. We choose Gaussian distribution as the membership function and the operation is performed as

$$f(u_{ij}^2) = -\frac{(u_i^2 - m_{ij})^2}{\sigma_{ij}^2}, \quad a^2(f) = \mathrm{e}^f,$$

where m_{ij} and σ_{ij} are the mean and the standard deviation of the Gaussian membership function of the jth term of the ith input variable u_i^2.

Layer 3: The fuzzily blending operation is performed in this layer and hence each node represents one fuzzy logic rule; that is,

$$f(u_i^3) = \prod_{i=1}^{n} u_i^3 = \mathrm{e}^{-[D_i(x-m_i)]^t[D_i(x-m_i)]}, \quad a^3(f) = f,$$

where n is the number of Layer-2 nodes joining with the ith rule precondition, $D_i = diag(1/\sigma_{i1}, 1/\sigma_{i2}, \ldots, 1/\sigma_{in})$ and $m_i = (m_{i1}, m_{i2}, \ldots, m_{in})$. Notice that the weighted factor for each fan-in stream is unity in this layer and the node output is in fact the firing strength of the corresponding fuzzy rule.

Layer 4: This layer is for normalization and hence generalizes the normalized fire strength of the fuzzy rule in the following way:

$$f(u_i^4) = \sum_i u_i^4, \quad a^4(f) = \frac{u_i^4}{f},$$

where all weighted factors are unity in this layer.

Layer 5: This layer is the consequence layer. Each node in Layer 4 has its corresponding node. Notice that the node outputs in Layer 4 are the key consequences of the fuzzy rules in Juang's NFN; but now, they are only the basic node inputs to store the fire strength information. Not only the input variables in Layer 1 but also the external inputs of the physical system are included as the node inputs to generate the consequence condition for the corresponding fuzzy rule. In other words, the activity function in this layer is

$$f = \sum_j a_{ji} x_j + \sum_m b_{mi} u_m, \quad a^5(f) = f \cdot u_i^5.$$

Layer 6: Each node in this layer is correspondent to one system output variable. This layer is to integrate the actions from Layer 5, and hence to perform the defuzzifier operation for the fuzzy logic system. In other words,

$$f(u_i^6) = \sum_i u_i^6, \quad a^6(f) = f.$$

Hence, the input variables x_j are fuzzified as fuzzy variables whose corresponding term sets T_{ji} have Gaussian membership function with mean m_{ji} and standard deviation σ_{ji}; the corresponding output for the neural network is

$$SX(t) = A_i X(t) + B_i u(t),\ i = 1, \ldots, r.$$

In other words, the proposed NFN structure is in fact a neural-based *linear* T–S fuzzy modelling structure. Via neural learning technique, this structure will proceed the structure and parameter learning concurrently and generate the following *linear* T–S fuzzy system:

$$\begin{aligned} R^i: &\text{If } x_1 \text{ is } T_{1i}(m_{1i}, \sigma_{1i}), \ldots,\ x_n \text{ is } T_{ni}(m_{ni}, \sigma_{ni}),\ \text{then } Y(t) = CX(t) \\ &SX(t) = A_i X(t) + B_i u(t),\ i = 1,\ \ldots, r, \end{aligned} \tag{1}$$

where R^i denotes the ith rule of the fuzzy model; $x_1,\ \ldots, x_n$ are system states; $T_{ji}(m_{ji}, \sigma_{ji}),\ j = 1, \ldots, n$, is the fuzzy term of the input fuzzy variable x_j in the ith rule with m_{ji} and σ_{ji} being the mean and standard deviation of the Gaussian membership function; $SX(t)$ denotes $\dot{X}(t)$ for the continuous case and $X(t+1)$ for the discrete case; $X(t) = [x_1,\ \ldots, x_n]^t \in \Re^n$ is the state vector, $Y(t) = [y_1,\ \ldots, y_{n'}]^t \in \Re^{n'}$ is the system output vector, and $u(t) \in \Re^m$ is the system input (i.e., control output); and A_i, B_i and C are, respectively, $n \times n$, $n \times m$ and $n' \times n$ matrices.

Structure learning includes both precondition and consequence identification of a fuzzy IF–THEN rule. Precondition identification (input-space partition) is formulated as the combinational optimization problem to minimize the number of generated rules and the number of fuzzy term sets for each input fuzzy variable, where the input space is partitioned in a flexible way via the aligned clustering-based algorithm. Consequence identification is to decide the significant terms (states and inputs) to be added via projected-based correlation measure of each rule. The combined precondition and consequence structure identification scheme can set up an economical and dynamically growing network automatically. In other words, this NFN structure possesses the self-construction ability to generate its rule nodes, term set nodes and linking weights between nodes. As for the parameter learning, based on the supervised learning algorithm, the least mean square algorithm is adopted to adjust the parameters in the rule consequence, and the back-propagation algorithm for minimizing a given cost function is adopted to adjust the parameters in the rule precondition.

2.2. *Neural-fuzzy-based optimal fuzzy controller*

Though the proposed NFN structure can obtain the linear T–S fuzzy model for the model-free systems, the critical issue is how to ensure the training data sufficiently enough for describing the system behavior effectively. As we know, once the designed optimal fuzzy controller $u^*(t)$ is applied to a real physical system, then the deviation between real and estimated output comes from modelling error and controlling error. Via our previous papers [25–27], we know the proposed optimal fuzzy controller can exponentially stabilize the corresponding *linear* T–S fuzzy system once each fuzzy subsystem is completely controllable (c.c.) and completely observable (c.o.). In other words, the closed-loop real system compensated with the optimal fuzzy controller is exponentially stable in the case of zero modelling error; that is, the neural-learning-based T–S fuzzy system is consistent with the real nonlinear system. For measuring the modelling error, we define a modelling index as

$$I_{\mathrm{M}}(t) = \frac{Y^{\mathrm{cl}}_{\mathrm{Lsonfin}}(t) + \theta}{Y^{cl}(t) + \theta}, \tag{2}$$

where $Y_{\mathrm{Lsonfin}}(t)$ is the output of the proposed neural-learning-based T–S fuzzy closed-loop system and $Y(t)$ is the output of the real physical closed-loop system; θ is a small constant to ensure a nonzero denominator. Accordingly to the stability of the optimal fuzzy closed-loop system [25–27], we know the index must approach unity as time goes to infinity once the fuzzy model can approximate the real physical system very well. Therefore, we further integrate the neural-fuzzy modelling process and the optimal fuzzy controlling design scheme into an integrated neural-fuzzy modelling and controlling (INFMC) algorithm in Fig. 2. Via this INFMC algorithm, we can guarantee that the proposed neural-learning-based T–S fuzzy models can describe the real physical systems well and obtain the corresponding optimal fuzzy controller. In the rest of this subsection, the adopted local- and global-based optimal fuzzy controllers are described briefly as follows.

Based on the generated T–S fuzzy model from Section 2.1, we assume all desired controllers are in the form of

$$R^i \ : \ \text{If } y_1 \text{ is } S_{1i}, \ \ldots, \ y_{n'} \text{ is } S_{n'i}, \ \text{then } u(t) = r_i(t), \ i = 1, \ \ldots, \delta, \tag{3}$$

where $y_1, \ \ldots, y_{n'}$ are the elements of output vector $Y(t)$, $S_{1i}, \ \ldots, S_{n'i}$ are the input fuzzy terms in the ith control rule, and the plant input (i.e., control output) vector $u(t)$ or $r_i(t)$ is in $\Re^m$ space. Our quadratic optimal fuzzy control problem is then described as follows:

Problem 1. Given the rule-based fuzzy system in Eq. (1) with $X(t_0) = X_0 \in \Re^n$ and a rule-based fuzzy controller in Eq. (3), find the individual optimal control law, $r_i^*(\cdot)$, $i = 1, \ldots, \delta$, such that the composed optimal controller, $u^*(\cdot)$, can minimize the quadratic cost functional, $J(u(\cdot))$, over all possible inputs $u(\cdot)$.

$$J(u(\cdot)) = \int_{t_0}^{\infty} [X^t(t)L(t)X(t) + u^t(t)u(t)]\,\mathrm{d}t \quad \text{(continuous)}, \tag{4}$$

$$J(u(\cdot)) = \sum_{t=t_0}^{\infty} [X^t(t)L(t)X(t) + u^t(t)u(t)] \quad \text{(discrete-time)}, \tag{5}$$

where $X^t(t)L(t)X(t)$ is state-trajectory penalty with $L(t)$ belonging to a symmetric positive semi-definite $n \times n$ matrix and $u^t(t)u(t)$ is fuel consumption.

For the local approach, we first adopt the principles of dynamic programming to transform the quadratic optimization problem into a successively ongoing dynamic problem with regard to the state resulting from the previous decision. Then, based on the *additive property of energy*, we know that, *at any time-step* t, if we can find the *optimal local decision* (*optimal control law*) for minimizing

$$J_t(u_t) = \int_{t}^{\infty} (X_l^t L_l X_l + u_l^t u_l)\,\mathrm{d}l, \ \ t \in [t_0, \infty) \ \ \text{(continuous)}, \tag{6}$$

$$J_t(u_t) = \sum_{t}^{\infty} [X_l^t L_l X_l + u_l^t u_l], \ \ t \in [t_0, \infty) \ \ \text{(discrete)} \tag{7}$$

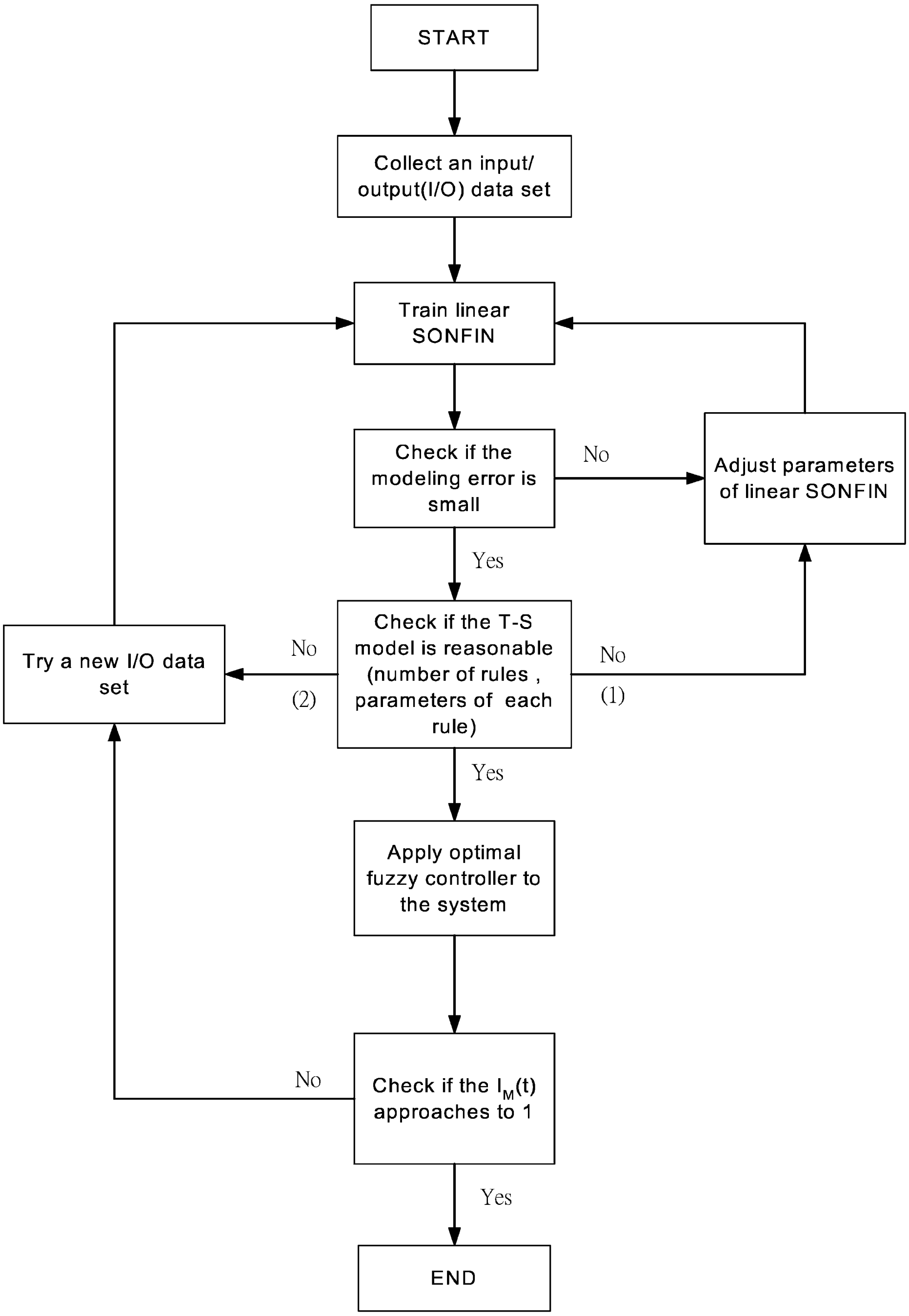

Fig. 2. Integrated neural-fuzzy modelling and controlling (INFMC) algorithm.

with regard to a *fuzzy subsystem*, the composed global decision can be a *global minimizer* of total cost with regard to a *fuzzy system*. In other words, based on the local viewpoint of the global optimal fuzzy control, we know that solving the quadratic optimal control problem is to find *only one corresponding optimal* solution of the fuzzy controller for *each* rule of the fuzzy model. Thereupon, both the fuzzy model and *admissible* fuzzy controller have, more precisely, the same input variables and same input space partition, and there exists only one optimal fuzzy control rule for each fuzzy subsystem described by a fuzzy rule in the fuzzy model. In short, the local-concept optimization technology is first adopted to rewrite the quadratic optimization problems into the following successively ongoing dynamic problems with regard to the state resulting from the previous decision [25].

Problem 1.1. Given the fuzzy subsystem,

$$\dot{X}_l = A_{i_l} X_l + B_{i_l} r_{i_l},\ \ l \in [t, \infty), i = 1, \ldots, r \ \ \text{(continuous)}, \tag{8}$$

$$X_{l+1} = A_{i_l} X_l + B_{i_l} r_{i_l},\ \ l \in [t, \infty), i = 1, \ldots, r \ \ \text{(discrete)} \tag{9}$$

with the initial state resulting from the previous decision, i.e., $X_{0_t} = X_t^*$,
(1) find the optimal local decision at time instant t, $r_{i_t}^*$, for minimizing the cost functional,

$$J_t(r_{i_t}) = \int_t^{\infty} (X_l^t L_l X_l + r_{i_l}^t r_{i_l})\, \mathrm{d}l,\ \ t \in [t_0, \infty) \ \ \text{(continuous)}, \tag{10}$$

$$J_t(r_{i_t}) = \sum_t^{\infty} (X_l^t L_l X_l + r_{i_l}^t r_{i_l}],\ \ t \in [t_0, \infty) \ \ \text{(discrete)}; \tag{11}$$

(2) obtain the optimal global decision at time instant t, u_t^*, for minimizing the cost functional $J_t(u_t)$ in Eqs. (6) and (7), by fuzzily blending each local decision, i.e., $u_t^* = \sum_{i=1}^r h_i(X_t^*) r_{i_t}^*$.

Since the local fuzzy system (i.e., fuzzy subsystem) is linear, its quadratic optimization problem is the same as the general linear quadratic issue. Therefore, it is realizable that solving the optimal control problem for a fuzzy *subsystem* can be achieved by simply generalizing the classical theorem from the deterministic case to the fuzzy case. Hence, we have the following corresponding local-concept optimal continuous fuzzy controller design schemes.

Proposition 1 (*Local-concept continuous Wu and Lin [25]*). *For a continuous fuzzy controller, respectively, in Eq.* (3) *and the continuous fuzzy system in Eq.* (1), *let* A_i, B_i, C, L *be given constant matrices. If* (A_i, B_i) *is c.c. and* (A_i, C) *is c.o. for* $i = 1, \ldots, r$, *then*

(1) *there exists a unique* $n \times n$ *symmetric positive semi-definite solution* π_∞^i *of the steady-state Riccati equation* (*SSRE*)

$$A_i^t K + K A_i - K B_i B_i^t K + L = 0; \tag{12}$$

(2) *the asymptotically local optimal fuzzy control law is*

$$r_i^*(t) = -B_i^t \pi_\infty^i X^*(t), \qquad i = 1, \ldots, r, \tag{13}$$

and their "blending" global minimizer $u^*(t) = \sum_{i=1}^n h_i(X^*) r_i^*(t)$ *minimizes* $J(u(\cdot))$ *in Eq.* (4);

(3) *and the optimal global feedback fuzzy subsystem*

$$\dot{X}^*(t) = \sum_{i=1}^{n} h_i(X^*)(A_i - B_i B_i^t \pi_\infty^i) X^*(t) \tag{14}$$

is exponentially stable.

Proof. From the inference in the above, we can get local optimal fuzzy control law $r_i^*(t)$ in Eq. (13) and the *local* feedback fuzzy subsystem, $\dot{X}^*(t) = (A_i - B_i B_i^t \pi_\infty^i) X^*(t)$, is exponentially stable. We demonstrate the stability of the composed *global* feedback fuzzy subsystem in Eq. (14) in the Appendix.

For the discrete-time system, we have the following corresponding local-concept optimal discrete-time fuzzy controller design schemes.

Proposition 2 (*Local-concept discrete-time, Wu and Lin [25]*). *For the discrete-time fuzzy controller in Eq.* (3) *and the discrete-time fuzzy system in Eq.* (1), *let A_i, B_i, C, L be given constant matrices. If (A_i, B_i) is stabilizable and (A_i, C) is detectable for $i = 1, \ldots, r$, then*

(1) *there exists a unique symmetric positive semi-definite solution $\pi^i(\infty)$ of the following algebraic SSRE*,

$$V(\infty) = L + A_i^t V(\infty)[I_n + B_i B_i^t V(\infty)]^{-1} A_i, \tag{15}$$

$$V(\infty) = L + A_i^t V(\infty) A_i - A_i^t V(\infty) B_i [I_n + B_i^t V(\infty) B_i]^{-1} B_i^t V(\infty) A_i; \tag{16}$$

(2) *the asymptotically local optimal fuzzy control law is*

$$r_i^*(t) = -[I_n + B_i^t \pi^i(\infty) B_i]^{-1} B_i^t \pi^i(\infty) A_i X^*(t), \qquad t = t_0, \ldots, N-1, \tag{17}$$

and the resultant global controller $u^*(t)$ minimizes $J(u(\cdot))$ in Eq. (5);

(3) *moreover*, *the optimal local feedback fuzzy subsystem*,

$$X^*(t+1) = [I_n + B_i B_i^t \pi^i(\infty)]^{-1} A_i X^*(t), \tag{18}$$

is asymptotically and exponentially stable.

As for the global-concept technique, since each penalty term in the performance index is with regard to the entire fuzzy system and controller, we fuzzily blend the distributed fuzzy subsystems and rule-based fuzzy controller into the entire fuzzy system and entire fuzzy controller formulations, and unify the individual matrices into synthetical matrices to form a *linear-like* global system representation of a fuzzy system,

$$\begin{aligned} SX(t) &= H(X(t))A(t)X(t) + H(X(t))B(t)W(Y(t))R(t), \\ Y(t) &= C(t)X(t), \end{aligned} \tag{19}$$

where $H(X(t)) = [h_1(X(t))I_n \ldots h_r(X(t))I_n]$, $W(Y(t)) = [w_1(Y(t))I_m \ldots w_\delta(Y(t))I_m]$,

$$A(t) = \begin{bmatrix} A_1(t) \\ \vdots \\ A_r(t) \end{bmatrix}, B(t) = \begin{bmatrix} B_1(t) \\ \vdots \\ B_r(t) \end{bmatrix}, R(t) = \begin{bmatrix} r_1(t) \\ \vdots \\ r_\delta(t) \end{bmatrix}$$

with I_n and I_m denoting the identity matrices of dimension n and m, respectively. And, $h_i(X(t))$ and $w_i(Y(t))$ are the normalized fire strengths for the ith fuzzy rule in the fuzzy system and in the fuzzy controller, respectively. Furthermore, a multistage-decomposition approach is adopted to transform the optimal control problem into an ongoing stage-by-stage dynamic issue [26,27].

Notice that the *formulation* and simplification of a *quadratic optimal fuzzy control* problem is achieved by fuzzily merging distributed rule-based T–S type fuzzy subsystems into an *entire* fuzzy system. This can initiate and activate the research in *global optimal* fuzzy controller design. The unification of *individual* matrices ($A_i(k)$ and $B_i(k)$, $i = 1, \ldots, r$) and normalized membership functions ($h_i(X(k))$, $i = 1, \ldots, r$, and $w_i(Y(k))$, $i = 1, \ldots, \delta$) into *synthetical* matrices ($A(k)$, $B(k)$, $H(X(k))$ and $W(Y(k))$) generates a *linear-like* global system representation of a fuzzy system with the value of each element of the nonlinear terms ($H(X(k))$ and $W(Y(k))$) being located in segment $[0, 1]$. This *linear-like* representation motivates us to develop the design scheme of a global optimal fuzzy controller in the way of general linear quadratic approach, i.e., calculus-of-variation method. Moreover, the multistage-decomposition approach is to transform the optimal control problem into an ongoing stage-by-stage dynamic issue; that is, the optimal solutions can be resolved from N *segmental* nonlinear TPBVP instead of the nonlinear TPBVP for the entire horizon. This decomposition operation can speed up numerical solution and keep the global optimality at the same time. Furthermore, $\bar{N}$ denotes the number of stages at which membership functions can be assumed to be invariant during the whole single stage and is assumed to make the *backward recursive* Riccati-like equation *available*. This avoids the high computational complexity of the collocation method at the expense of *approximate optimality* due to the time-invariant assumption. Furthermore, a procedure including a dynamical decomposition algorithm is proposed to justify the time-invariant assumption in practice [26].

According to the derivation above, we can obtain the global-concept-based optimal fuzzy controller for both continuous and discrete-time fuzzy systems as follows.

Proposition 3 (*Global-concept continuous, Wu and Lin [26]*). *Consider the time-invariant fuzzy system in Eq.* (1) *and fuzzy controller in Eqs.* (3); *if* $N > \bar{N}$, (A_i, B_i) *is c.c. and* (A_i, C) *is c.o., for all* $i = 1, \ldots, r$, *then*

$$(X^*_\infty(t), R^*_\infty(t)) = (X^{i*}_\infty(t), R^{i*}_\infty(t)), \forall t \in [t^i_0, t^i_1], t^1_0 = t_0, t^N_1 = \infty, i = 1, \ldots, N, \tag{20}$$

where $R^{i*}_\infty(t)$ *is the* i*th-stage asymptotically optimal control law,*

$$R^{i*}_\infty(t) = -W^t_i[W_i W^t_i]^{-1} B^t H^t_i \pi^i_\infty X^{i*}_\infty(t), \ t \in [t^i_0, \infty), \tag{21}$$

which minimizes $J^i_\infty(R(\cdot)) = \int_{t^i_0}^{\infty} [X^t(t) L X(t) + R^t(t) W^t_i W_i R(t)] \, \mathrm{d}t$, *and* $X^{i*}_\infty(t)$ *is the corresponding asymptotically optimal trajectory that satisfies*

$$\dot{X}^{i*}_\infty(t) = (H_i A - H_i B B^t H^t_i \pi^i_\infty) X^{i*}_\infty(t), \ t \in [t^i_0, \infty), \tag{22}$$

where π^i_∞ *is the unique symmetric positive semidefinite solution of the SSRE,*

$$A^t H^t_i K + K H_i A - K H_i B B^t H^t_i K + C^t C = 0. \tag{23}$$

Proposition 4 (*Global-concept discrete-time, Wu and Lin [27]*). *Consider the time-invariant fuzzy system and fuzzy controller described, respectively, by Eqs.* (1) *and* (3) *with* $L = C^t C$. *If there exists* $\bar{N}$ *such* *that if* $N > \bar{N}$, (A_i, B_i) *is c.c. and* (A_i, C) *is c.o.*, $i = 1, \ldots, r$, *then, for each stage*, $(X^*_\infty(k), R^*_\infty(k)) = X^{i*}_\infty(k), R^{i*}_\infty(k)), \forall k \in [k^i_0, k^i_1 - 1]$, $k^1_0 = k_0$, $k^N_1 = \infty$, *where the ith-stage asymptotically optimal control law*,

$$R^{i*}_\infty(k) = -W^t_i[W_i W^t_i]^{-1} B^t H^t_i \pi^i_\infty [I_n + H_i B B^t H^t_i \pi^i_\infty]^{-1} H_i A X^{i*}_\infty(k), \; k \in [k^i_0, \infty), \tag{24}$$

which minimizes $J^i_\infty(R(\cdot)) = \sum_{k=k^i_0}^{\infty} [X^t(k) L X(k) + R^t(k) W^t_i W_i R(k)]$; $X^{i*}_\infty(k)$ *is the corresponding asymptotically optimal trajectory*,

$$X^{i*}_\infty(k+1) = [I_n + H_i B B^t H^t_i \pi^i_\infty]^{-1} H_i A X^{i*}_\infty(k), \; k \in [k^i_0 \infty), \tag{25}$$

where π^i_∞ *is the unique symmetric positive semidefinite solution of the discrete-time algebraic Riccati-like equation*,

$$K = L + A^t H^t_i K [I_n + H_i B B^t H^t_i K]^{-1} H_i A, \tag{26}$$

$$K = L + A^t H^t_i K H_i A - A^t H^t_i K H_i B [I_n + B^t H^t_i K H_i B]^{-1} B^t H^t_i K H_i A. \tag{27}$$

3. Physical system modelling and controlling

In this section, we shall generate the T–S fuzzy models and design the optimal controllers for four complicated nonlinear physical systems. The INFMC algorithm is adopted to integrate the neural-fuzzy modelling and optimal fuzzy controlling process, and more to guarantee that the proposed neural-learning-based T–S fuzzy models can approximate the original physical systems very well. The neural-fuzzy-based optimal fuzzy controller are designed from both local and global concept, respectively. Simulation results show that the proposed optimal fuzzy controllers can effectively drive the physical systems to the target points in a short time.

3.1. Neural-based T–S fuzzy modelling

In this section, we shall use the proposed *linear* NFN structure to generate the corresponding *linear* T–S fuzzy models for the mass–spring–damper system [11], the chaotic Chua's circuit system [21], the model car system [13] and the articulated vehicle system [18], respectively.

A mass–spring–damper system can be formulated as [11]

$$\ddot{x} = -0.1\dot{x}^3 - 0.02x - 0.67x^3 + u, \tag{28}$$

where $x \in [-1.5 \; 1.5]$ and $\dot{x} \in [-1.5 \; 1.5]$.

It is not necessary to train the input/output pattern repeatedly in the learning process. There initially exists no rule in the neural-fuzzy structure. As on-line feeding in the training data, the following operations are done simultaneously: the input/output spaces are partitioned, the fuzzy rules are generated, the consequent structure and the parameters in the structure are identified optimally. The training results are shown in Fig. 3 and the neural-learning-based T–S fuzzy model for the mass–spring–damper system is

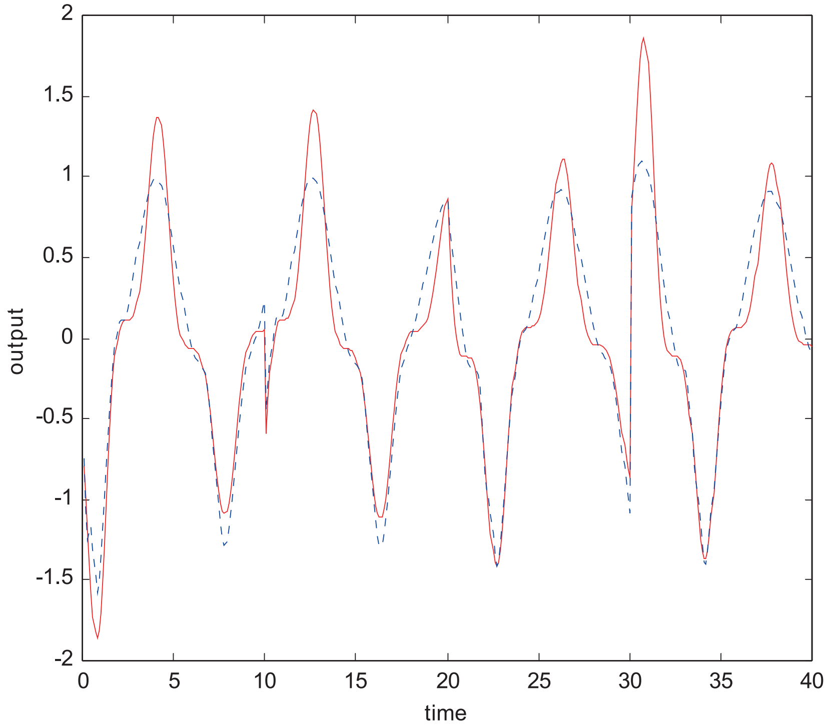

Fig. 3. Neural-based fuzzy modelling (solid line) for a continuous mass–spring–damper system (dashed line).

as follows:

$$R^i : \text{ If } x_1 \text{ is } T_{1i}(m_{1i}, \sigma_{1i}) \text{ and } x_2 \text{ is } T_{2i}(m_{2i}, \sigma_{2i}), \\ \text{then } \dot{X}(t) = A_i X(t) + B_i u(t), \;\; i = 1, \ldots, 5, \tag{29}$$

where fuzzy term sets $T_{11}(-0.4158, 0.6545)$, $T_{21}(0.3982, 0.5249)$, $T_{12}(-0.597, 0.7889)$, $T_{22}(-0.8596, 0.6376)$, $T_{13}(0.1681, 0.4798)$, $T_{23}(0.3514, 0.6428)$, $T_{14}(-0.5881, 0.7827)$, $T_{24}(-1.1486, 0.6783)$, $T_{15}(-0.5881, 0.7827)$, $T_{25}(1.1379, 1.0588)$;

$$A_1 = \begin{bmatrix} 0.3718 & 0.6995 \\ 1 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.0014 & 1.3836 \\ 1 & 0 \end{bmatrix} \quad A_3 = \begin{bmatrix} 0 & -0.1848 \\ 1 & 0 \end{bmatrix},$$
$$A_4 = \begin{bmatrix} 0 & -2.7786 \\ 1 & 0 \end{bmatrix}, \quad A_5 = \begin{bmatrix} -0.741 & -1.5384 \\ 1 & 0 \end{bmatrix},$$
$$B_i = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad i = 1, \ldots, 5;\; X(t) = \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}.$$

According to the controllability and observability analysis in [25], we know the generated fuzzy model in Eq. (29) is c.c. and c.o.

We next consider a more complex continuous nonlinear chaotic system, Chua's circuit, which is an electronic system with one inductor (L), two capacitors (C_1, C_2), one linear resistor (R) and one piecewise linear or nonlinear resistor (g) included. The dynamic behavior of Chua's circuit can be described as [21]

$$\begin{aligned}\dot{v}_{C_1} &= \frac{1}{C_1}\left(\frac{1}{R}(v_{C_2} - v_{C_1} - g(v_{C_1}))\right),\\ \dot{v}_{C_2} &= \frac{1}{C_2}\left(\frac{1}{R}(v_{C_1} - v_{C_2}) + i_L\right),\\ \dot{i}_L &= \frac{1}{L}(-v_{C_2} - R_0 i_L),\end{aligned} \tag{30}$$

where v_{C_1} and v_{C_2} are the voltage of capacitors and i_L is the instant current of the inductor; the nonlinear resistor is characterized as $g(v_{C_1}) = G_b v_{C_1} + \frac{1}{2}(G_a - G_b)(|v_{C_1} + E| - |v_{C_1} - E|)$ with parameters $G_a,\ G_b < 0$. We denote the state variable $X = [v_{C_1}, v_{C_2}, i_L]^t$ and choose $R = 10/7$, $R_0 = 0$, $L = 1/7$, $C_1 = 0.1$, $C_2 = 2$, $G_a = -4$, $G_b = -0.1$ and $E = 1$. After successful training in Fig. 4, the generated T–S fuzzy model is

$$R^i:\ \text{If } x_1 \text{ is } T_{1i}(m_{1i}, \sigma_{1i}),\ \text{then } \dot{X}(t) = A_i X(t),\ i = 1, \ldots, 4, \tag{31}$$

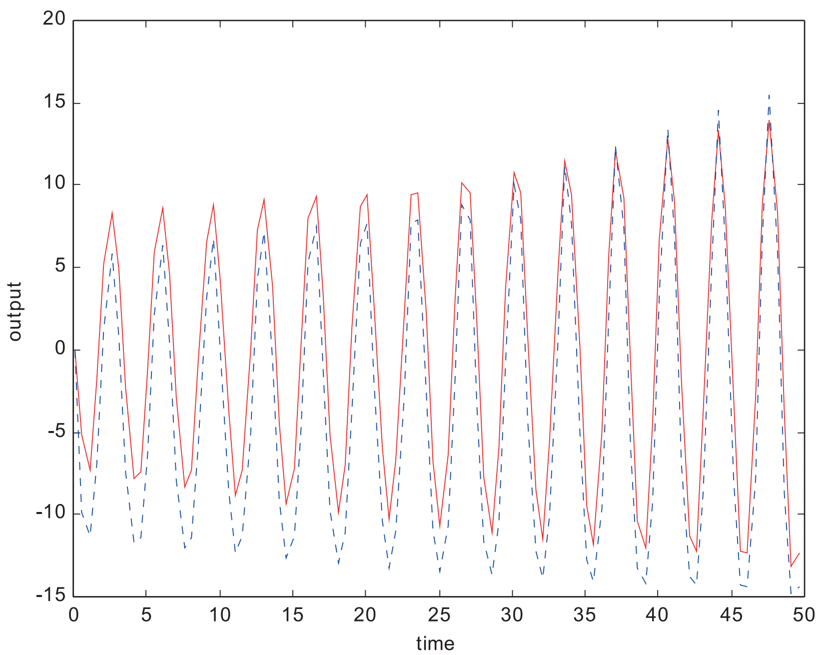

Fig. 4. Neural-based fuzzy modelling (solid line) for a continuous Chua's chaotic system (dashed line).

where the fuzzy term sets $T_{11}(-0.105, 1.556)$, $T_{12}(7.45, 5.525)$, $T_{13}(0.031, 5.004)$, $T_{14}(-8.939, 7.622)$;

$$A_1 = \begin{bmatrix} 68.85 & 7 & 0 \\ 0.35 & -0.35 & 0.5 \\ 0 & -7 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -2.395 & 7 & 0 \\ 0.35 & -0.35 & 0.5 \\ 0 & -7 & 0 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 12.06 & 7 & 0 \\ 0.35 & -0.35 & 0.5 \\ 0 & -7 & 0 \end{bmatrix},$$

$$A_4 = \begin{bmatrix} -2.508 & 7 & 0 \\ 0.35 & -0.35 & 0.5 \\ 0 & -7 & 0 \end{bmatrix}.$$

We now step for fuzzy modelling the discrete-time nonlinear systems, the car model system [13],

$$\begin{aligned} x_1(k+1) &= x_1(k) + \frac{vt}{l\tan(u(k))}, \\ x_2(k+1) &= x_2(k) + vt\sin(x_1(k)), \\ x_3(k+1) &= x_3(k) + vt\cos(x_1(k)), \end{aligned} \quad (32)$$

where $x_1(k)$, $x_2(k)$ and $x_3(k)$ are, respectively, the angle of the car, the vertical and horizontal position of the rear end of the car; $u(k)$ is the steering angle, l is the length of the car, t is the sampling time and v is the constant speed. The parameters were chosen as $l = 2.8\,\mathrm{m}$, $v = 1.0\,\mathrm{m/s}$ and $t = 1.0\,\mathrm{s}$. After neuro-fuzzy modelling in Fig. 5, we have

$$R^i : \text{ If } x_i(k) \text{ is } T_{1i}(m_{1i}, \sigma_{1i}), \text{ then } X(k+1) = A_i X(k) + B_i u(k), \; i = 1, \ldots, 5, \quad (33)$$

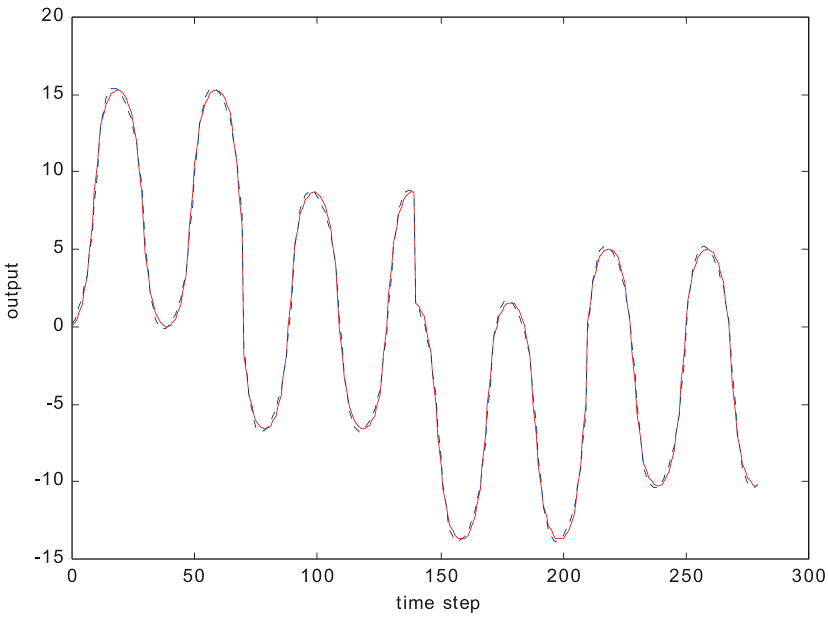

Fig. 5. Neural-based fuzzy modelling (solid line) for a discrete-time model car system (dashed line).

where fuzzy term sets $T_{11}(0.001, 0.224)$, $T_{12}(-0.57, 0.353)$, $T_{13}(0.567, 0.345)$, $T_{14}(-1.562, 0.618)$, $T_{15}(1.561.0.619)$; $X(k) = [x_1(k), x_2(k)]^t$;

$$A_1 = \begin{bmatrix} 1 & 0 \\ 1.01 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 0 \\ 0.969 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} 1 & 0 \\ 0.974 & 1 \end{bmatrix},$$

$$A_4 = \begin{bmatrix} 1 & 0 \\ 0.686 & 1 \end{bmatrix}, \quad A_5 = \begin{bmatrix} 1 & 0 \\ 0.672 & 1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.377 \\ -0.003 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.39 \\ 0.01 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 0.388 \\ 0.011 \end{bmatrix},$$

$$B_4 = \begin{bmatrix} 0.394 \\ -0.104 \end{bmatrix}, \quad B_5 = \begin{bmatrix} 0.403 \\ -0.15 \end{bmatrix},$$

which is c.c. and c.o.

We further concern ourselves with a multi-dimensional and more complicated discrete-time nonlinear articulated vehicle [18],

$$\begin{aligned} x_1(k+1) &= x_1(k) + \frac{v\triangle t}{l}\tan(u(k)), \\ x_2(k) &= x_1(k) - x_3(k), \\ x_3(k+1) &= x_3(k) + \frac{v\triangle t}{L}\sin(x_2(k)), \\ x_4(k+1) &= x_4(k) + v\triangle t\cos(x_2(k))\sin\left(\frac{x_3(k+1)+x_3(k)}{2}\right), \\ x_5(k+1) &= x_5(k) + v\triangle t\cos(x_2(k))\cos\left(\frac{x_3(k+1)+x_3(k)}{2}\right), \end{aligned} \tag{34}$$

where $u(k)$ is the steering angle; $x_1(k)$, $x_2(k)$, $x_3(k)$, $x_4(k)$ and $x_5(k)$ are the angle of truck, the angle difference between truck and trailer, and the angle of trailer, the vertical and horizontal position of the rear end of the trailer, respectively. We set $l = 0.2$ m, $L = 0.32$ m, $v = -0.1$ m/s, $\triangle t = 0.5$. After neuro-fuzzy modelling in Fig. 6, we obtain the following corresponding *linear* T–S fuzzy model:

$$\begin{aligned} R^i \ :\ & \text{If } x_2(k) \text{ is } T_{2i}(m_{2i}, \sigma_{2i}),\ x_3(k) \text{ is } T_{3i}(m_{3i}, \sigma_{3i}) \text{ and } x_4(k) \text{ is } T_{4i}(m_{4i}, \sigma_{4i}), \\ & \text{then } X(k+1) = A_i X(k) + B_i u(k),\ i = 1, \ldots, 4, \end{aligned} \tag{35}$$

where fuzzy term sets $T_{21}(0.178, 0.182)$, $T_{31}(1.316, 0.188)$, $T_{41}(0.63, 0.18)$, $T_{22}(0.809, 0.123)$, $T_{32}(0.532, 0.801)$, $T_{42}(-0.07, 0.578)$, $T_{23}(0.809, 0.123)$, $T_{33}(183.6, 254.1)$ $T_{43}(-1.03, 0547)$, $T_{24}(-0.043, 1.19)$, $T_{34}(-1.614, 0.975)$, $T_{44}(0.409, 0.89)$, $T_{25}(-0.043, 1.19)$, $T_{35}(0.935, 2.059)$, $T_{45}(-1.44, 2.32)$;

$$A_1 = \begin{bmatrix} -0.388 & 0.155 & 0.42 \\ -2.848 & 0.215 & 3.039 \\ -5.085 & -0.889 & 3.476 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.232 & 0.133 & -0.051 \\ -1.246 & -0.167 & 0.114 \\ -2.688 & -6.848 & 1.669 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0.062 & 0.641 & -0.242 \\ -1.076 & 2.063 & -2.07 \\ -11.16 & 9.032 & -0.596 \end{bmatrix}, \quad A_4 = \begin{bmatrix} 3.808 & -0.562 & 0.057 \\ -0.129 & 0.995 & 0.000 \\ -0.504 & 0.069 & 0.99 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1.602 \\ 3.662 \\ 5.345 \end{bmatrix},$$

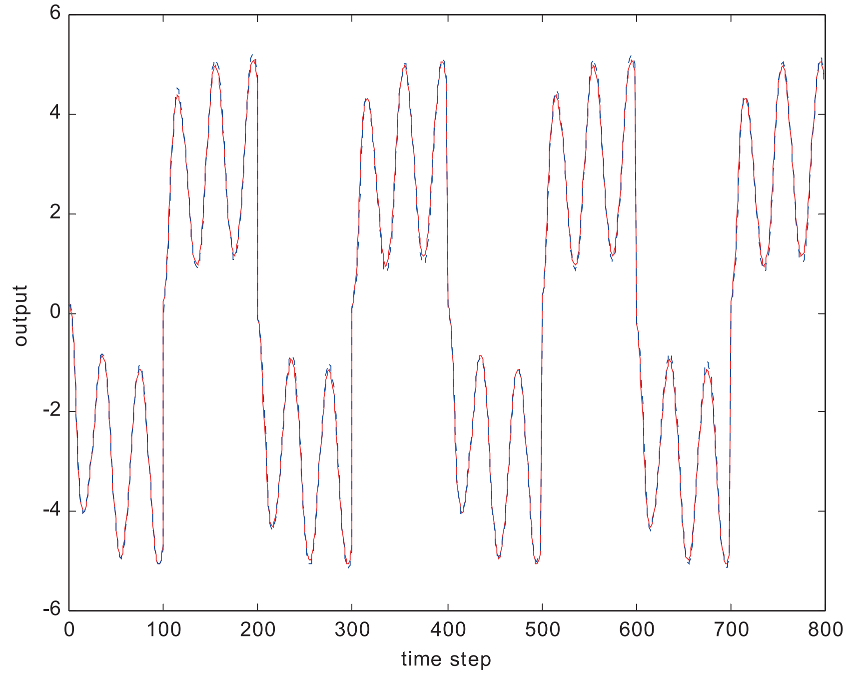

Fig. 6. Neural-based fuzzy modelling (solid line) for a discrete-time articulated vehicle system (dashed line).

$$A_5 = \begin{bmatrix} 2.674 & -0.348 & 0.029 \\ -0.17 & 1.004 & -0.0003 \\ -0.366 & 0.042 & 0.992 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1.329 \\ 0.391 \\ -23.31 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 1.162 \\ 12.56 \\ 128.5 \end{bmatrix}, \quad B_4 = \begin{bmatrix} -2.395 \\ -0.017 \\ 0.404 \end{bmatrix},$$

$$B_5 = \begin{bmatrix} -1.433 \\ 0.012 \\ 0.281 \end{bmatrix}; \quad X(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{bmatrix}^t,$$

which is also c.c. and c.o.

3.2. *Optimal fuzzy controlling*

Based on the proposed T–S fuzzy model in Eq. (29) for the continuous mass–spring–damper system, the fuzzy model in Eq. (31) for continuous Chua's circuit, the fuzzy model in Eq. (33) for the discrete-time model-car system and the fuzzy model in Eq. (35) for the discrete-time articulated vehicle system, we can now obtain the corresponding optimal fuzzy controllers, which can achieve global minimum effect under quadratic performance consideration defined on the *entire* fuzzy system and fuzzy controller.

Fig. 7 shows the simulation results for the mass–spring–damper system in Eq. (28) at the initial conditions, $X(0) = (-1, \ -1)^t$, $(-1, \ 1)^t$, $(1, \ -1)^t$ and $(1, \ 1)^t$, and the designed *local-concept*

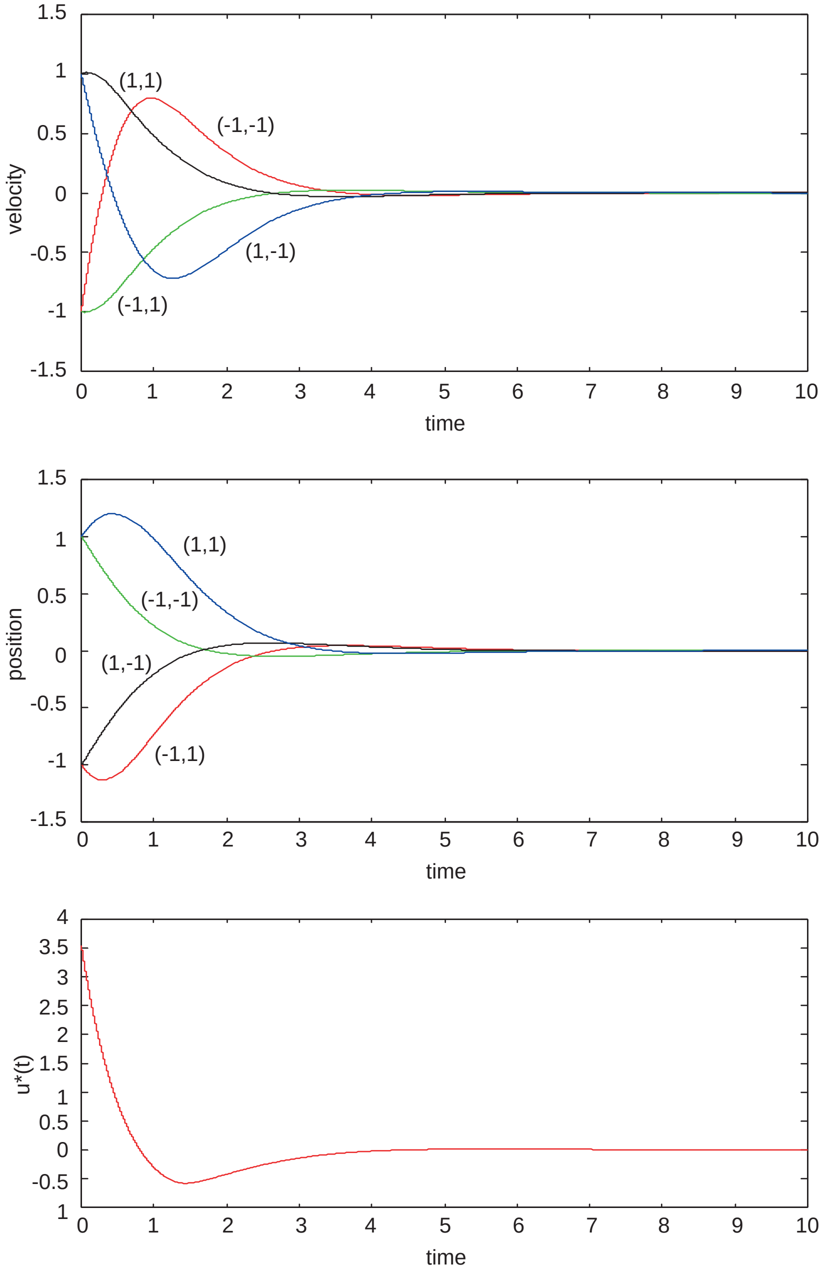

Fig. 7. (a) The state responses of the continuous mass–spring–damper system with the designed *local-concept* optimal controller at the four initial conditions: $X(0) = (-1, \; -1)^t$, $(-1, \; 1)^t$, $(1, \; -1)^t$ and $(1, \; 1)^t$; (b) the designed *local-concept* optimal controller with $X(0) = (-1, \; -1)^t$.

optimal controller with $X(0) = (-\pi/2, \; 10, \; 0)^t$. As for the automaton chaotic system, in order to control the chaotic behavior, the external forces are imposed on the Chua's circuit in Eq. (30); and hence the corresponding automaton forced-free fuzzy model in Eq. (31) is then rewritten as the following forced

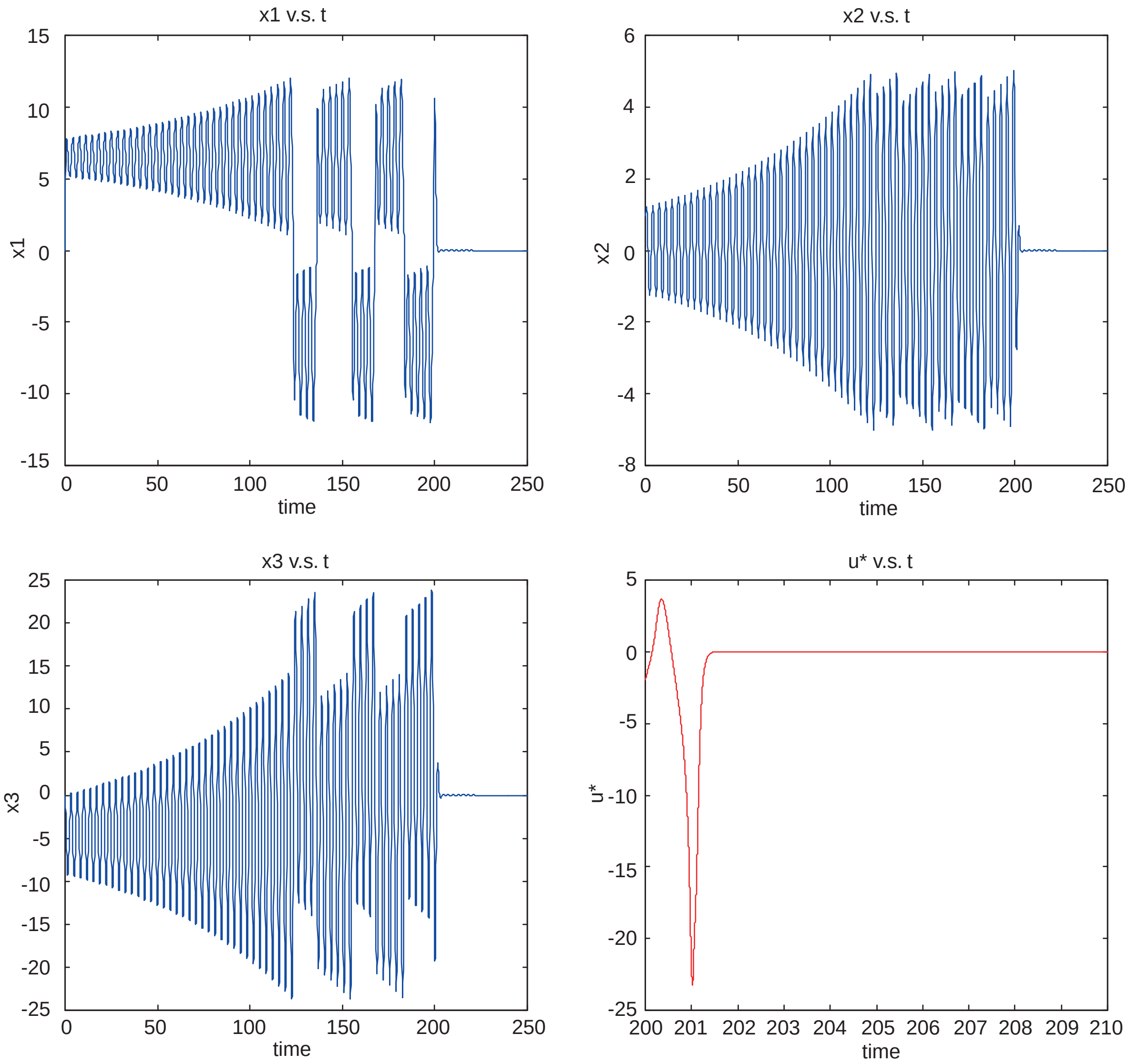

Fig. 8. The state responses of the continuous Chua's chaotic system with initial conditions $X(0) = (0,\ 1,\ 0)^t$, actuated by the designed *global-concept* optimal controller at $t = 200$.

fuzzy model,

$$R^i:\ \text{If } x_1 \text{ is } T_{1i},\ \text{then } \dot{X}(t) = A_i X(t) + B_i U(t),\ i = 1, \ldots, 3, \tag{36}$$

where

$$U(t) = \begin{bmatrix} u_1(k) \\ u_2(k) \\ u_3(k) \end{bmatrix}$$

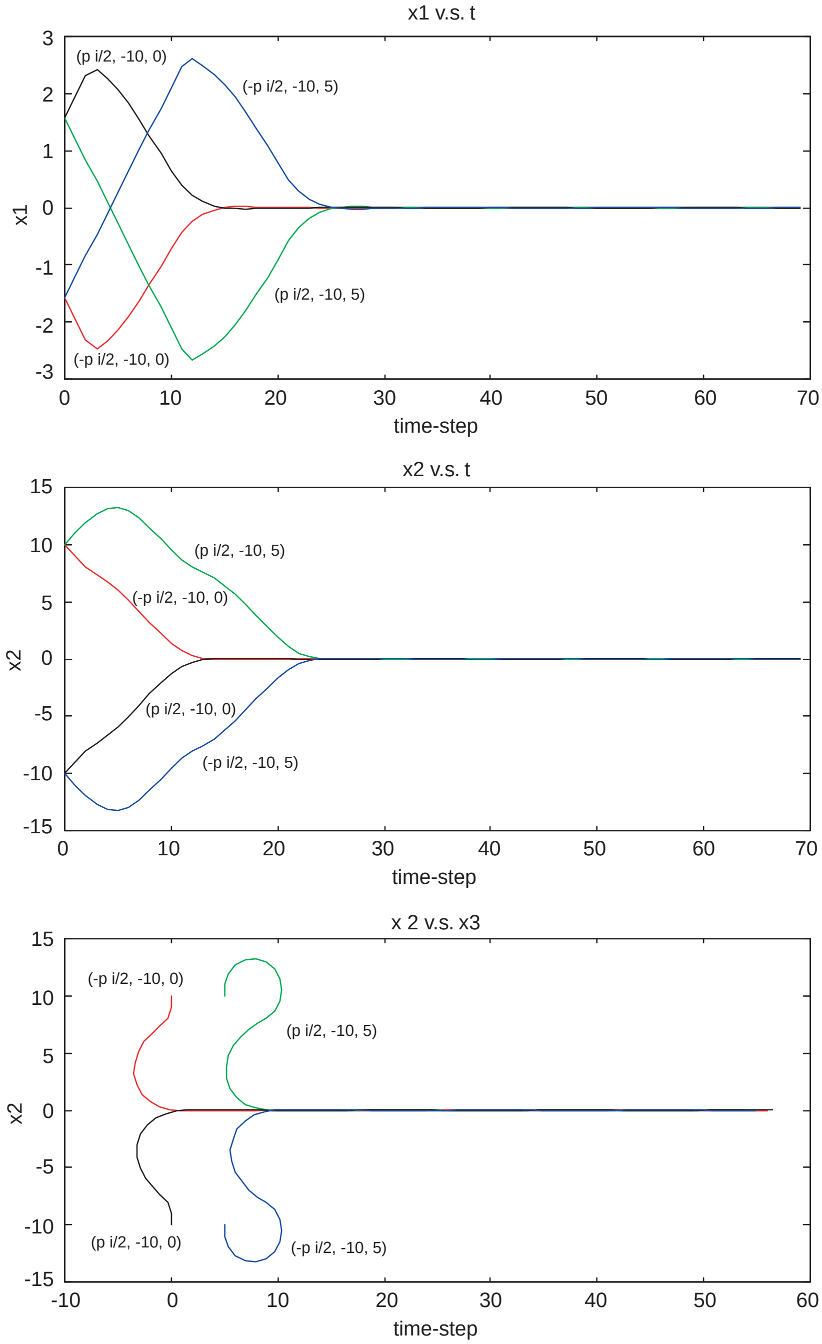

Fig. 9. The state responses and trajectories of the discrete-time model car system with the designed *local-concept* optimal controller at the four initial conditions: $X(0) = (-\pi/2,\ 10,\ 0)^t$, $(\pi/2,\ 10,\ 5)^t$, $(\pi/2,\ -10,\ 0)^t$ and $(-\pi/2,\ -10,\ 5)^t$.

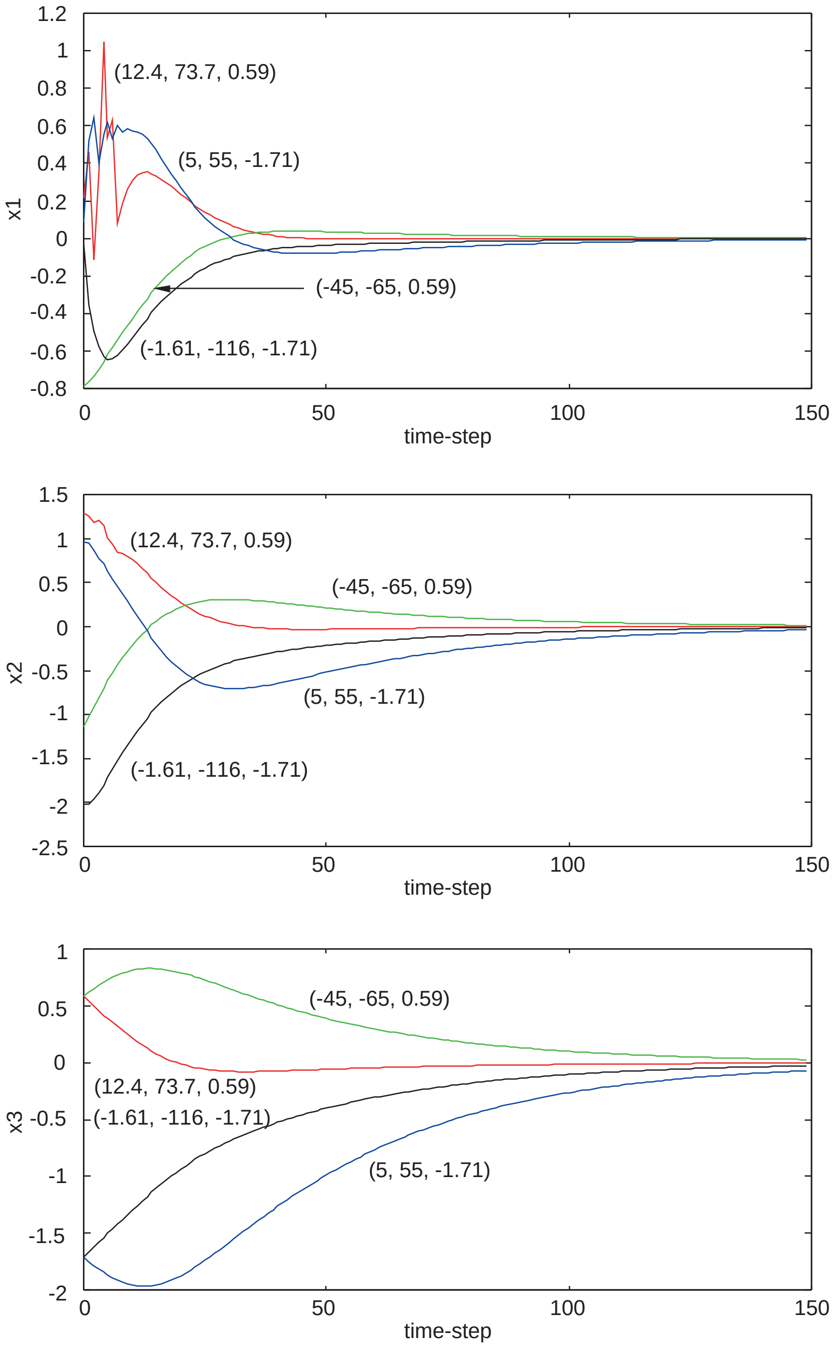

Fig. 10. The state responses of the discrete-time articulated vehicle system with the designed *global-concept* optimal controller at the four initial conditions: $X(0) = (-86.1^\circ, \underline{12.4^\circ, 73.7^\circ, 0.59}, -0.41)^t$, $(-110^\circ, \underline{-45^\circ, -65^\circ, 0.59}, -0.61)^t$, $(-118^\circ, \underline{-1.61^\circ, -116^\circ, -1.71}, -0.41)^t$ and $(60^\circ, \underline{5^\circ, 55^\circ, -1.71}, -0.61)^t$.

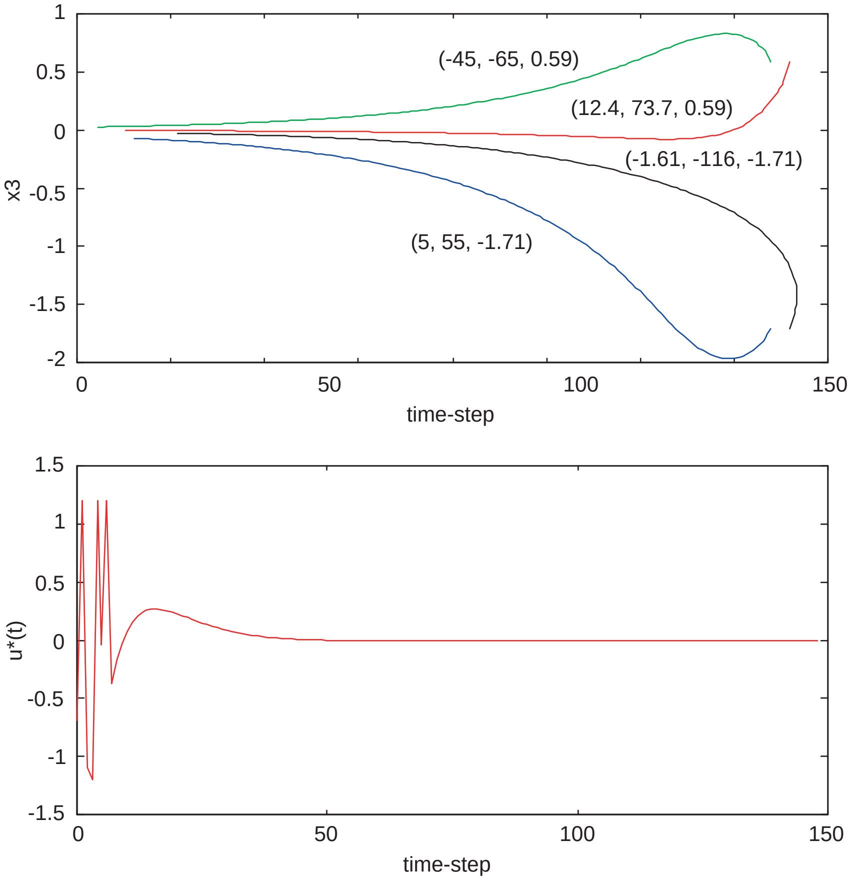

Fig. 11. (a) The trajectory of the discrete-time articulated vehicle system with the designed *global-concept* optimal controller at the four initial conditions: $X(0) = (-86.1°,\ 12.4°,\ 73.7°,\ 0.59,\ -0.41)^t$, $(-110°,\ -45°,\ -65°,\ 0.59,\ -0.61)^t$, $(-118°,\ -1.61°,\ -116°,\ -1.71,\ -0.41)^t$ and $(60°,\ 5°,\ 55°,\ -1.71,\ -0.61)^t$; (b) the designed global-concept optimal controller with $X(0) = (-86.1°,\ 12.4°,\ 73.7°,\ 0.59,\ -0.41)^t$.

is the imposed external input and $B_i(t)$, $i = 1, \ldots, 3$, is chosen as the identity matrix with dimension 3×3. Fig. 8 shows the state responses of the continuous Chua's chaotic system with initial conditions $X(0) = (0,\ 1,\ 0)^t$, controlled by the designed *global-concept* optimal controller applied at $t = 200$.

As for discrete-time system, the steering angle of the model car is restricted to $u(k) < \pi/2$. Hence, we assume the controller output for the model car system is $u(k) < 1.2$. Based on the proposed fuzzy model and the corresponding *local-concept* fuzzy controller, we have the simulation results for four initial conditions, $X(0) = (-\pi/2,\ 10,\ 0)^t$, $(\pi/2,\ 10,\ 5)^t$, $(\pi/2,\ -10,\ 0)^t$ and $(-\pi/2,\ -10,\ 5)^t$, in Fig. 9. Fig. 10 shows the state response of the articulated vehicle closed-loop system controlled by the proposed *global-concept* optimal fuzzy controller at initial conditions, $X(0) = (-86.1°,\ 12.4°,\ 73.7°,\ 0.59,\ -0.41)^t$, $(-110°,\ -45°,\ -65°,\ 0.59,\ -0.61)^t$, $(-118°,\ -1.61°,\ -116°,\ -1.71,\ -0.41)^t$ and $(60°,\ 5°,\ 55°,\ -1.71,\ -0.61)^t$. Fig. 11 is the trajectory at initial conditions, $X(0) = (-86.1°,\ 12.4°,\ 73.7°,\ 0.59,\ -0.41)^t$, $(-110°,\ -45°,\ -65°,\ 0.59,\ -0.61)^t$, $(-118°,\ -1.61°,\ -116°,\ -1.71,\ -0.41)^t$

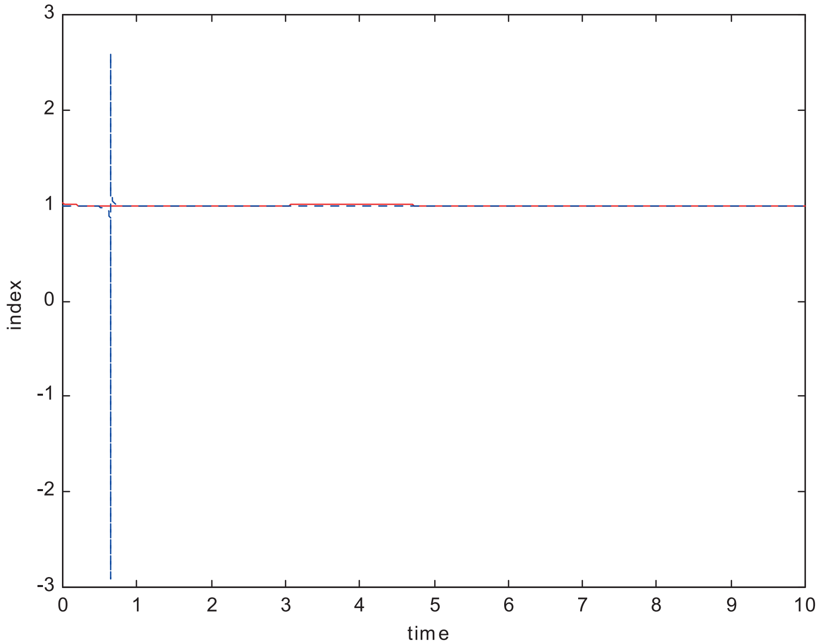

Fig. 12. The modelling index $I_M(t)$ for local-concept optimal fuzzy controller actuated mass–spring–damper system with initial condition $X(0) = [-1, \ -1]^t$, where the dashed line denotes output $Y(t) = x(t)$ and the solid line denotes $Y(t) = \dot{x}(t)$.

and $(60°,\ 5°,\ 55°,\ -1.71,\ -0.61)^t$, and the proposed *global-concept* optimal controller with $X(0) = (-86.1°,\ 12.4°,\ 73.7°,\ 0.59,\ -0.41)^t$.

3.3. Performance and stability

The modelling index I_M in Eq. (2) not only provides an index to integrate the neural-fuzzy modelling and controlling for both the model-based and model-free physical system due to the stability properties mentioned in our previous paper, but also serves as a modelling-error index for the neural-fuzzy modelling process in both transient and infinite-time states. Fig. 12 shows the modelling index evolution of the proposed neural-learning-based T–S fuzzy model in Eq. (29) for the mass–spring–damper system in Eq. (28) with initial condition set to be $X(0) = [-1, -1]^t$. The index is nearly coincident with one except in some trivial points; in other words, the modelling error approaches zero in the large. Hence, with the INFMC algorithm, we can self-organize a T–S fuzzy model of a physical system under limited modelling error.

Furthermore, since the proposed neural-based T–S fuzzy model can approximate the real physical very well, the properties of the closed-loop system (the real physical system compensated with the proposed optimal fuzzy controller) are the same as those of the closed-loop fuzzy system (the T–S fuzzy model compensated with the proposed optimal fuzzy controller). For the mass–spring–damper system, since each fuzzy system in Eqs. (29) is c.c. and c.o., we know the local-concept closed-loop fuzzy systems and the global-concept closed-loop fuzzy system, and then their corresponding closed-loop real physical systems are exponentially stable [25–27]. The same properties can be found in Chua's circuit, model-car and articulated vehicle closed-loop systems.

4. Conclusions

A neural-learning-based fuzzy inference network, which emphasizes physical system input- and state-dependence consequences in each fuzzy rule, is proposed to achieve the *linear* T–S fuzzy modelling. Both the local-concept and global-concept optimal fuzzy controller design scheme are adopted to stabilize the nonlinear system. Furthermore, based on the guaranteed stability properties, an INFMC algorithm with defined modelling index included is proposed to integrate the neural-fuzzy modelling and optimal fuzzy controlling. Via the proposed INFMC algorithm, the neural-based fuzzy model for a nonlinear system is ensured; hence, the intrinsic properties of the closed-loop physical system can be captured by those of the corresponding closed-loop fuzzy system. Two continuous and two discrete-time physical systems are concerned in implementation of the modified neural-fuzzy structure and the proposed INFMC algorithm. Simulation results demonstrate that the proposed NFN can self-organize the *linear* T–S fuzzy models for those real systems with limited modelling errors and that the proposed neural-based optimal fuzzy controller can drive the physical systems to desired targets in a short time.

Appendix A.

Proof of Proposition 1. Via the converse theorem, we know the stability of the resultant feedback fuzzy system concurs with that of the linearized fuzzy system (with respect to $X_{\rm o}$)

$$\dot{X}^*(t) = \sum_{i=1}^{r} h_i(X_{\rm o})[A_i - B_i B_i^T \pi_\infty^i]X^*(t). \tag{37}$$

For clarity, we introduce the notation A_{ci} to denote the local feedback system matrix. Then, as we know each feedback fuzzy subsystem is exponentially stable, which means the *spectrum* of A_{ci}, $i = 1, \ldots, r$, denoted by $\boldsymbol{\sigma}[A_{ci}]$, is located in the open left-half plane of the complex space, $\mathscr{C}_-^{\rm o}$, i.e., $\boldsymbol{\sigma}[A_{ci}] \subset \mathscr{C}_-^{\circ}$, $i = 1, \ldots, r$. Accordingly, we have $\boldsymbol{\sigma}[h_i(X_{\rm o})A_{ci}] \subset \mathscr{C}_-^{\rm o}$, $i = 1, \ldots, r$, via the spectral mapping theorem and $h_i(X_{\rm o}) \in [0, 1]$ for all $X_{\rm o} \in \Re^n$. Hence, the zero solution of $\dot{X}(t) = h_i(X_{\rm o})A_{ci}X(t)$ on $t \geqslant t_0$ is exponentially stable; in other words, there exists constants $a_i > 0$ and $m_i > 0$ such that for all $t_0 \in \Re_+$

$$\| \mathrm{e}^{h_i(X_o)A_{ci}(t-t_0)} \| \leqslant m_i \mathrm{e}^{-a_i(t-t_0)}, \ \forall t \geqslant t_0, \ i = 1, \ldots, r.$$

Then, the state transition matrix, $\phi(t, t_0)$, of the linearized fuzzy system in Eq. (37) is

$$\|\phi(t, t_0)\| = \|\mathrm{e}^{\sum_{i=1}^r h_i(X_o)A_{ci}(t-t_0)}\| \leqslant \prod_{i=1}^{r} \|\mathrm{e}^{h_i(X_o)A_{ci}(t-t_0)}\| \leqslant \prod_{i=1}^{r} m_i \mathrm{e}^{-a_i(t-t_0)} \leqslant m\mathrm{e}^{-a(t-t_0)},$$

where $m \triangleq \prod_{i=1}^r m_i > 0$ and $a \triangleq \sum_{i=1}^r a_i > 0$. Therefore, the linearized fuzzy system and also the feedback fuzzy system are exponentially stable.

References

[1] S.G. Cao, N.W. Rees, G. Feng, Analysis and design for a class of complex control systems, Part I: Fuzzy modelling and identification, Automatica 33 (6) (1997) 1017–1028.
[2] S.G. Cao, N.W. Rees, G. Feng, Analysis and design for a class of complex control systems, Part II: Fuzzy controller design, Automatica 33 (6) (1997) 1029–1039.
[3] C.F. Juang, C.T. Lin, An on-line self-constructing neural fuzzy inference network and its applications, IEEE Trans. Fuzzy Systems 6 (1) (1998).
[4] T.C. Lin, C.H. Wang, H.L. Liu, Observer-based indirect adaptive fuzzy-neural tracking control for nonlinear SISO systems using VSS and H_∞ approach, Fuzzy Sets and Systems 143 (2004) 211–232.
[5] X.J. Ma, Z.Q. Sun, Y.Y. He, Analysis and design of fuzzy controller and fuzzy observer, IEEE Trans. Fuzzy Systems 6 (1) (1998) 41–51.
[6] H. Ohtake, K. Tanaka, A construction method of switching Lyapunov function for nonlinear systems, FUZZ-IEEE'02, Hawaii, 2002, pp. 221–226.
[7] H. Ohtake, K. Tanaka, H.O. Wang, Fuzzy modelling via sector nonlinearity concept, Proc. IFSA/NAFIPS, Canada, 2001, pp. 127–132.
[8] S. Su, Y. Liu, Robust H_∞ control of multiple time-delay uncertain nonlinear system using fuzzy model and adaptive neural network, Fuzzy Sets and Systems 146 (2004) 403–420.
[9] S. Su, X. Yu, Z. Man, A fuzzy neural network approximator with fast terminal sliding mode and its applications, Fuzzy Sets and Systems 148 (2004) 469–486.
[10] K. Tanaka, T. Hori, K. Yamafuji, H.O. Wang, An integrated fuzzy control system design for nonlinear systems, in: Proceedings of the Decision and Control, 1999, pp. 4349–4354.
[11] K. Tanaka, T. Ikeda, H.O. Wang, Robust stabilization of a class of uncertain nonlinear systems via fuzzy control: quadratic stabilizability, H^∞ control theory and linear matrix inequalities, IEEE Trans. Fuzzy Systems 4 (1) (1996) 1–13.
[12] K. Tanaka, M. Iwazaki, H.O. Wang, Switching control of an R/C hovercraft: stabilization and smooth switching, IEEE Trans. Fuzzy Systems 31 (2001) 853–863.
[13] K. Tanakan, M. Sano, Trajectory stabilization of a model car via fuzzy control, Fuzzy Sets and Systems 70 (1995) 155–170.
[14] K. Tanaka, T. Taniguchi, S. Hori, H.O. Wang, Structure-oriented design for a class of nonlinear systems, in: Proceedings of the FUZZ-IEEE'01, Australia, 2001, pp. 696–699.
[15] K. Tanaka, T. Taniguchi, H.O. Wang, Generalized Takagi–Sugeno fuzzy systems: rule reduction and robust control, in: Proceedings of the FUZZ-IEEE'00, San Antonio, 2000, pp. 688–693.
[16] K. Tanaka, H.O. Wang, Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach, Wiley, New York, 2001.
[17] T. Taniguchi, K. Tanaka, H. Ohtake, H.O. Wang, Model construction, rule reduction and robust compensation for generalized form of Takagi–Sugeno fuzzy system, IEEE Trans. Fuzzy Systems 9 (4) (2001) 525–538.
[18] M. Tokunaga, H. Ichihashi, Backer-upper control of a trailer trunck by neuro-fuzzy optimal control, in: Proceedings of the 8th Fuzzy System Symposium (in Japanese), 1992, pp. 49–52.
[19] R.J. Wai, Robust fuzzy neural network control for nonlinear motor-toggle servomechanism, Fuzzy Sets and Systems 139 (2003) 185–208.
[20] H.O. Wang, J. Li, D. Niemann, K. Tanaka, T–S fuzzy model with linear rule consequence and PDC controller: a universal framework for nonlinear control system, in: Proceedings of the FUZZ-IEEE'00, 2000, pp. 549–554.
[21] H.O. Wang, K. Tanaka, An LMI-based stable fuzzy control of nonlinear systems and its application to control of chaos, in: Proceedings of the FUZZ-IEEE'96, 1996.
[22] H. Wang, K. Tanaka, M. Griffin, Parallel distributed compensation of nonlinear systems by Takagi and Sugeno's fuzzy model, in: Proceedings of the FUZZ-IEEE'95, Yokahama, 1995, pp. 531–538.
[23] H.O. Wang, K. Tanaka, M.F. Griffin, An approach to fuzzy control of nonlinear system: stability and design issues, IEEE Trans. Fuzzy Systems 4 (1) (1996) 14–23.
[24] W.Y. Wang, Y.G. Leu, T.T. Lee, Output-feedback control of nonlinear systems using direct adaptive fuzzy-neural controller, Fuzzy Sets and Systems 140 (2003) 341–358.
[25] S.J. Wu, C.T. Lin, Optimal fuzzy controller design: local concept approach, IEEE Trans. on Fuzzy Systems 8 (2) (2000) 171–185.

[26] S.J. Wu, C.T. Lin, Optimal fuzzy controller design in continuous fuzzy system: global concept approach, IEEE Trans. Fuzzy Systems 8 (6) (2000) 713–729.
[27] S.J. Wu, C.T. Lin, Discrete-time optimal fuzzy controller design: global concept approach, IEEE Trans. Fuzzy Systems 10 (1) (2002) 21–38.
[28] X.J. Zeng, M.G. Singh, Approximation theory of fuzzy systems-MIMO case, IEEE Trans. Fuzzy Systems 3 (2) (1995) 219–235.