Physics Letters B 758 (2016) 37–41

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Stability of Einstein static state universe in the spatially flat branemodels

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ARTICLE INFO

Article history:
Received 9 February 2016
Received in revised form 21 April 2016
Accepted 25 April 2016
Available online 27 April 2016
Editor: M. Trodden

ABSTRACT

With the assumption that a perfect fluid with a constant equation of state is the only energy component on the brane, we study the stability of Einstein static state solution under homogeneous and inhomogeneous scalar perturbations in both spatially flat Randall–Sundrum (RS) and Shtanov–Sahni (SS) braneworlds. We find that if the perfect fluid has a phantom-like property and the "Weyl fluid" originating from the projection of the bulk Weyl tensor onto the brane behaves like a radiation with positive energy density, the Einstein static state solution is stable in the SS braneworld, but unstable in the RS one. Furthermore, we demonstrate that the static state solution is also stable in the bulk with a timelike extra dimension. Thus, in the model where the extra dimension is timelike, our universe can stay at the Einstein static state past-eternally, which means that the big bang singularity might be resolved successfully by an emergent scenario.

1. Introduction

Braneworld scenario, based on superstring theory (M theory), assumes that our $1+3$ dimensional observable universe ("brane") is embedded in a $1+3+d$ dimensional spacetime ("bulk"), and gravity can propagate freely in the bulk while ordinary particles and fields are confined on the brane. Thus the hierarchy problem could be resolved by the existence of extra dimensions. In most braneworld models, such as the famous Randall and Sundrum (RS) [1,2] and DGP [3] models, the extra dimension is spacelike, so the manifold of the bulk is Lorentzian. However, it is still plausible that timelike extra dimensions may exist. The simplest braneworld with a timelike extra dimension was constructed by Shtanov and Sahni [4]. In this model our Universe contracts at the beginning and then undergoes a nonsingular bounce [4]. It was also found that in both the spatially flat and positively-curved cases the Einstein static state solution is stable against homogeneous perturbations [5,6]. So, the authors in [5,6] argued that the big bang singularity problem may be resolved successfully since our universe can stay at the Einstein static state past-eternally and then evolve into an inflationary era naturally.

The idea, which uses the Einstein static state to solve the problem of big bang singularity, was first proposed by Ellis et al., and it was named the emergent scenario [7,8]. It is easy to see that the existence of a stable Einstein static state universe is a prerequisite for the emergent theory. Otherwise our universe is impossible to stay at the static state past-eternally. The emergent mechanism is unsuccessful for the avoidance of big bang singularity in the theory of general relativity since the Einstein static state solution is unstable. In the very early universe, due to that the cosmic energy density is very large, it is reasonable to consider some other effects, such as those from quantum gravity and modified gravity, which might help to stabilize the Einstein static state. It has been found that the Einstein static state universe is stable against homogeneous scalar perturbations in massive gravity [9,10], loop quantum cosmology [11], Horava–Lifshifz gravity [12], $f(T)$ gravity [13], braneworld scenario [14,15], Jordan–Brans–Dick theory [16], hybrid metric-Palatini gravity [17], modified Gauss–Bonnet gravity [18], $f(R)$ gravity [19], and some other theories [20]. However, inhomogeneous perturbations violate the stability of Einstein static state solution in modified Gauss–Bonnet gravity [21], and $f(R)$ gravity [22]. Therefore, the stability of Einstein static state solution under inhomogeneous perturbations must be investigated when

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http://dx.doi.org/10.1016/j.physletb.2016.04.049
0370-2693/

the emergent scenario is used to resolve the big bang singularity problem.

As is mentioned in the above, in the SS braneworld, only the stability of Einstein static state solution under homogeneous perturbations was investigated. Whether it is stable against inhomogeneous perturbations remains unclear. So, in this paper, we analyze the effect of inhomogeneous scalar perturbations on the stability of Einstein static state in the spatially flat braneworld. Besides the SS braneworld model, whose extra dimension is timelike, we also consider the RS braneworld one, which has a spacelike extra dimension. In addition, we discuss the stability of Einstein static state solution under these perturbations in the bulk with a timelike extra dimension.

2. Einstein static state solution in the braneworlds

In braneworld scenario, our four-dimensional world is considered as a brane which is the boundary of a five-dimensional bulk. Its action has the following general form [4]

$$S = M^3 \int_{bulk} (\mathcal{R} - 2^{(5)}\Lambda)\sqrt{-\epsilon\, g}\, d^5x - 2\epsilon M^3 \int_{brane} K\sqrt{-h}\, d^4x + \int_{brane} (m^2 R - 2\sigma)\sqrt{-h}\, d^4x + \int_{brane} L\sqrt{-h}\, d^4x\,. \quad (1)$$

Here, g_{ab} is the five-dimensional metric in the bulk, and $\mathcal{R}$ is the scalar curvature of five dimensional spacetime. R is the scalar curvature of the induced metric on the brane which is defined by $h_{ab} = g_{ab} - \epsilon n_a n_b$, and n^a is the vector field of the inner unit normal to the brane. $K = h^{ab}K_{ab}$ is the trace of the symmetric tensor of extrinsic curvature $K_{ab} = h^c_a \nabla_c n_b$ of the brane. $\epsilon = 1$ or -1, which corresponds to a spacelike or timelike extra dimension respectively. M and m denote the five- and four-dimensional Planck masses, respectively. ${}^{(5)}\Lambda$ is the five-dimensional cosmological constant and σ is the brane tension. g and h are determinants of five-dimensional and four-dimensional metrics, respectively. L denotes the Lagrangian density of a perfect fluid restricted on the brane.

Varying the action given in Eq. (1) with respect to the metric h_{ab}, we obtain the Einstein field equation on the brane:

$$m^2 G_{ab} + \sigma h_{ab} = \epsilon M^3 (K_{ab} - K h_{ab}) + T_{ab}\,, \quad (2)$$

where G_{ab} and T_{ab} are the Einstein's tensor and stress-energy tensor of a perfect fluid on the brane, respectively. As Eq. (2) involves the extrinsic curvature tensor K_{ab}, it is not closed with respect to the intrinsic evolution on the brane. Using the Gauss–Codazzi identities and projecting the field equations onto the brane, the effective equation [23,24] which involves only four-dimensional quantities, can be obtained

$$G_{ab} + \Lambda_{\rm eff} h_{ab} = 8\pi G_{\rm eff} T_{ab} + \frac{1}{\alpha+1}\left(\frac{\epsilon}{M^6} Q_{ab} - W_{ab}\right). \quad (3)$$

Here $\alpha = \frac{2\epsilon\sigma m^2}{3M^6}$ is a dimensionless parameter, and $\Lambda_{\rm eff} = \frac{1}{\alpha+1}\left(\frac{{}^{(5)}\Lambda}{2} + \frac{\epsilon\sigma^2}{3M^6}\right)$ is the effective cosmological constant, which, for simplicity, we set to be zero in the following.[1] $8\pi G_{\rm eff} = \frac{2\epsilon\sigma}{3(\alpha+1)M^6}$ with $G_{\rm eff}$ being the effective gravitational constant, and Q_{ab} is a quadratic term defined by $Q_{ab} = \frac{1}{3}BB_{ab} - B_{ac}B^c{}_b + \frac{1}{2}\left(B_{cd}B^{cd} - \frac{1}{3}B^2\right)h_{ab}$, where $B_{ab} \equiv m^2 G_{ab} - T_{ab}$ is the 'bare' Einstein equation on the brane, and $B = h^{ab}B_{ab}$. For $m = 0$, which corresponds to RS or SS limit, one has $\alpha = 0$ and $B_{ab} = -T_{ab}$. Thus, Q_{ab} becomes the quadratic term of stress-energy tensor of the perfect fluid on the brane. The last term $W_{ab} \equiv n^c n^d W_{acbd}$ is the projection of the bulk Weyl tensor W_{abcd} onto the brane. With boundary conditions [25], the tensor W_{ab} characterizes the stress-energy tensor of a "Weyl fluid" [26] with the equation of state like that of "dark radiation" [27], i.e., $\gamma_w - 1 = \frac{1}{3}$.

[1] It is worth noting that ${}^{(5)}\Lambda < 0$ is required for $\epsilon = 1$ which allows a zero four-dimensional cosmological constant [23].

To find an Einstein static solution, we assume that our universe on the brane is described by a spatially flat Friedmann–Robertson–Walker (FRW) metric

$$ds^2 = -dt^2 + a(t)^2\left(dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right), \quad (4)$$

where a is the scale factor and t is the cosmic time. We further assume that the perfect fluid has a constant equation of state, which means that its energy density ρ and pressure p satisfy $p = (\gamma_m - 1)\rho$ with γ_m being a constant. The energy momentum tensor of the perfect fluid has the form

$$T_{ab} = p\, h_{ab} + (\rho + p)\, U_a U_b\,, \quad (5)$$

where U_a is the four-velocity vector. In addition, we find that W_{ab} can also be expressed as [28]

$$W_{ab} = p_w h_{ab} + (\rho_w + p_w)\, U_a U_b\,, \quad (6)$$

with

$$\rho_w = -3C/a^4\,, \qquad p_w = (\gamma_w - 1)\rho_w = 1/3\rho_w\,, \quad (7)$$

where C is an integration constant characterizing the "dark radiation" contributed by the projection of five-dimensional Weyl tensor onto the brane. The energy conservation law gives

$$\nabla^a T_{ab} = 0\,. \quad (8)$$

Together with Bianchi identity, one can obtain

$$\nabla^a\left(Q_{ab} - M^6 W_{ab}\right) = 0\,. \quad (9)$$

From Eqs. (3), (5) and (6), one can obtain the Friedmann equations of RS and SS braneworlds

$$H^2 = \frac{1}{3}\left(\rho + \epsilon\frac{\rho^2}{\rho_c}\right) - \frac{1}{3}\rho_w = \frac{1}{3}\left(\rho + \epsilon\frac{\rho^2}{\rho_c}\right) + \frac{C}{a^4} \quad (10)$$

$$H^2 + 2\frac{\ddot a}{a} = (1-\gamma_m)\rho + \epsilon(1-2\gamma_m)\frac{\rho^2}{\rho_c} - \frac{C}{a^4}\,, \quad (11)$$

where $H = \frac{\dot a}{a}$ is the Hubble parameter, $\rho_c \equiv 2\epsilon\sigma$ and $8\pi G_{\rm eff} \equiv 1$ are set for simplicity. In this paper, an overdot denotes a derivative with respect to t.

The Einstein static state solution satisfies the conditions $\dot a = 0$ and $\ddot a = 0$, which imply

$$a = a_0\,, \quad \rho = \rho_0\,, \quad H(a_0) = 0\,. \quad (12)$$

Combining Eqs. (10) and (11), we find that, in a static state universe, the energy density of the perfect fluid and the cosmic scale factor must satisfy

$$\rho_0 = \frac{4 - 3\gamma_m}{2(3\gamma_m - 2)}\epsilon\rho_c\,, \quad a_0^4 = \frac{(3\gamma_m - 2)^2}{\gamma_m(3\gamma_m - 4)}\cdot\frac{4\epsilon C}{\rho_c}\,. \quad (13)$$

3. Perturbations on the brane

Since only isotropic scalar perturbations are considered in our analysis, it is convenient to express the perturbed metric in the longitudinal gauge

$$ds^2 = -(1-2\Psi)dt^2 + a^2(1+2\Phi)\left(dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right) . \tag{14}$$

Here, Ψ is the "Bardeen" potential, and Φ represents the perturbation to the spatial curvature. The perturbed energy-momentum tensors of the perfect fluid and "dark radiation", respectively, have the forms [25]

$$\delta T^a{}_b = \delta\rho U^a U_b + U^a D_b q + U_b D^a q + \delta p \mathcal{P}^a{}_b , \tag{15}$$

$$\delta W^a{}_b = \delta\rho_w U^a U_b + U^a D_b q_w + U_b D^a q_w + \delta p_w \mathcal{P}^a{}_b , \tag{16}$$

where q and q_w are related respectively to the velocity perturbations of the perfect fluid and the "Weyl fluid". $\mathcal{P}^a{}_b$ and D_a are given by

$$\mathcal{P}^a{}_b = \delta^a{}_b + U^a U_b , \quad D_a = \mathcal{P}^b{}_a \partial_b . \tag{17}$$

Assuming adiabatic perturbations, one has that $\delta p = (\gamma_m - 1)\delta\rho$ and $\delta p_w = \frac{1}{3}\delta\rho_w$.

Now, for convenience we perform a harmonic decomposition for the perturbations:

$$\begin{aligned} \Psi &= \Psi_n(t)\mathcal{H}_n(\theta^i), & \Phi &= \Phi_n(t)\mathcal{H}_n(\theta^i), \\ \delta\rho &= \delta\rho_n(t)\mathcal{H}_n(\theta^i), & \delta\rho_w &= \delta\rho_{wn}(t)\mathcal{H}_n(\theta^i), \\ q &= q_n(t)\mathcal{H}_n(\theta^i), & q_w &= q_{wn}(t)\mathcal{H}_n(\theta^i). \end{aligned} \tag{18}$$

Here, n is larger than zero and is a continuous real number. The harmonic function $\mathcal{H}_n = \mathcal{H}_n(\theta^i)$ satisfies

$$\Delta\mathcal{H}_n = -k^2\mathcal{H}_n, \quad k^2 = n^2 \geq 0, \tag{19}$$

where Δ denotes the Laplacian operator on the three-dimensional spatial sections. $n = 0$ corresponds to the case of homogeneous scalar perturbations.

Using the perturbed metric and linearizing the field equation given in Eq. (3) and the energy conservation laws (Eqs. (8), (9)), we obtain the following linear perturbation equations

$$\Phi_n - \Psi_n = 0 , \tag{20}$$

$$\epsilon\rho_c a_0^2 \delta\rho_{wn} - (2\rho_0 + \epsilon\rho_c)a_0^2\delta\rho_n + 2\epsilon\rho_c k^2\Phi_n = 0 , \tag{21}$$

$$\epsilon\rho_c(2\dot{\Phi}_n + q_{wn}) - (2\rho_0 + \epsilon\rho_c)q_n = 0 , \tag{22}$$

$$-2\epsilon\rho_c\ddot{\Phi}_n + \left(2(1-2\gamma_m)\rho_0 + \epsilon(1-\gamma_m)\rho_c\right)\delta\rho_n + \frac{1}{3}\epsilon\rho_c\delta\rho_{wn} = 0 , \tag{23}$$

$$a_0^2\left(3\gamma_m\rho_0\dot{\Phi}_n + \delta\dot{\rho}_n\right) + k^2 q_n = 0 , \tag{24}$$

$$-\gamma_m\rho_0\Psi_n + (\gamma_m - 1)\delta\rho_n + \dot{q}_n = 0 , \tag{25}$$

$$-\frac{12C}{a_0^4}\rho_c\dot{\Phi}_n + \rho_c\delta\dot{\rho}_{wn} - 6\gamma_m\rho_0^2\dot{\Phi}_n - 2\rho_0\delta\dot{\rho}_n + \frac{2k^2\rho_0}{a_0^2}q_n = 0 , \tag{26}$$

$$\rho_c\dot{q}_{wn} - 2\rho_0\dot{q}_n + \frac{1}{3}\rho_c\delta\rho_{wn} - 2(2\gamma_m - 1)\rho_0\delta\rho_n + 2\gamma_m\rho_0^2\Psi_n + 4\rho_c\frac{C}{a_0^4}\Psi_n = 0 . \tag{27}$$

Combining the above equations, we arrive at two independent equations

$$\ddot{\Phi}_n = -\frac{k^2}{3a_0^2}\Phi_n + \left(\epsilon(4-6\gamma_m)\frac{\rho_0^2}{3\rho_c} + \frac{1}{6}(4-3\gamma_m)\rho_0\right)\delta_n , \tag{28}$$

$$\ddot{\delta}_n = \left(\frac{(\gamma_m - 1)}{a_0^2}k^2 + \epsilon\gamma_m(6\gamma_m - 4)\frac{\rho_0^2}{\rho_c} + \frac{1}{2}\gamma_m(3\gamma_m - 4)\rho_0\right)\delta_n . \tag{29}$$

Here, $\delta = \delta\rho/\rho$ is the relative density perturbation, and $\delta_n = \delta\rho_n/\rho$. Expressing the above two equations in a matrix form, $\frac{d^2\mathbf{u}}{dt^2} = \ddot{\mathbf{u}} = \mathbf{A}\mathbf{u}$, where $\mathbf{u} = \begin{pmatrix}\Phi_n \\ \delta_n\end{pmatrix}$ and $\mathbf{A}$ is the coefficient matrix of Eqs. (28) and (29), one can see that the solution of this second-order system has the form:

$$\mathbf{u}(t) = \mathbf{u}_1\left(c_1 e^{+i\omega_1 t} + c_2 e^{-i\omega_1 t}\right) + \mathbf{u}_2\left(c_3 e^{+i\omega_2 t} + c_4 e^{-i\omega_2 t}\right), \tag{30}$$

where c_1, c_2, c_3 and c_4 are some constants, $\mathbf{u}_1$ and $\mathbf{u}_2$ are eigenvectors of $\mathbf{A}$ with λ_1 and λ_2 being the corresponding eigenvalues, respectively. It is easy to obtain the frequencies, which can be written as

$$\begin{aligned} \omega_1^2 &= -\lambda_1 = \frac{k^2}{3a_0^2} , \\ \omega_2^2 &= -\lambda_2 = \frac{(1-\gamma_m)}{a_0^2}k^2 + \epsilon\gamma_m(4-6\gamma_m)\frac{\rho_0^2}{\rho_c} \\ &\quad + \frac{1}{2}\gamma_m(4-3\gamma_m)\rho_0 . \end{aligned} \tag{31}$$

If the eigenvalues λ_1 and λ_2 are always negative for any k, which means that the frequencies ω_1 and ω_2 are always real, the exponential functions $e^{\pm i\omega_1 t}$ and $e^{\pm i\omega_2 t}$ oscillate and do not grow up. Then, the static state solution is neutrally stable against scalar perturbations. Apparently, the stability requires that $\omega_1^2 \geq 0$ and $\omega_2^2 \geq 0$, which are determined only by the values of constants γ_m and C.

4. Stability

Now we study the stability of the critical point given in Eq. (13) against scalar perturbations in braneworlds with a timelike or spacelike extra dimension.

4.1. SS model

In this case, the extra dimension is timelike and $\epsilon = -1$. The Einstein static state solution shown in Eq. (13) becomes

$$\rho_0 = \frac{3\gamma_m - 4}{2(3\gamma_m - 2)}\rho_c , \quad a_0^4 = -\frac{(3\gamma_m - 2)^2}{\gamma_m(3\gamma_m - 4)} \cdot \frac{4C}{\rho_c} . \tag{32}$$

Since the energy density and scalar factor should be positive, the existence of an Einstein static state solution requires

$$\begin{aligned} &C < 0, \ \ \gamma_m < 0 \text{ or } \gamma_m > \frac{4}{3} ; \\ &C > 0, \ \ 0 < \gamma_m < \frac{2}{3} . \end{aligned} \tag{33}$$

4.1.1. Homogeneous perturbations

Homogenous perturbations corresponds to the case of $k = 0$. Then, Eq. (31) reduces to

$$\omega_1^2 = 0 , \quad \omega_2^2 = \frac{\gamma_m(4-3\gamma_m)^2\rho_c}{4(3\gamma_m - 2)} , \tag{34}$$

where Eq. (32) has been used. The stable conditions require

$$\gamma_m \leq 0 \text{ or } \gamma_m > \frac{2}{3} \tag{35}$$

Combining Eqs. (33) and (35), we obtain the conditions for stable Einstein static state solution under homogeneous scalar perturbations

$$C < 0, \quad \gamma_m < 0 \text{ or } \gamma_m > \frac{4}{3}. \tag{36}$$

4.1.2. Inhomogeneous perturbations

The inhomogeneous perturbations correspond to $k > 0$. Substituting Eq. (32) into Eq. (31), we obtain the frequencies

$$\omega_1^2 = \frac{k^2}{3a_0^2},$$
$$\omega_2^2 = \frac{\gamma_m(4-3\gamma_m)^2}{4(3\gamma_m-2)}\rho_c - \frac{k^2(\gamma_m-1)}{2|2-3\gamma_m|}\sqrt{\frac{\gamma_m(4-3\gamma_m)\rho_c}{C}}. \tag{37}$$

Since ω_1^2 is always positive, we only need to discuss the requirement for $\omega_2^2 \geq 0$, which gives

$$\gamma_m < 0, \qquad C < 0;$$
$$0 < \gamma_m < \frac{2}{3}, \quad 0 < C \leq \frac{(\gamma_m-1)^2}{\gamma_m(4-3\gamma_m)^3}\frac{4k^4}{\rho_c};$$
$$\gamma_m > \frac{4}{3}, \quad C \leq \frac{(\gamma_m-1)^2}{\gamma_m(4-3\gamma_m)^3}\frac{4k^4}{\rho_c}. \tag{38}$$

For an any given value of C, since $k \in (0, \infty)$, the requirements shown in the second and third lines of the above expression are violated. Thus, the conditions for stable Einstein static state solution against inhomogeneous scalar perturbations are

$$\gamma_m < 0, \qquad C < 0. \tag{39}$$

Combining the conditions for homogeneous and inhomogeneous scalar perturbations, we find that in the SS braneworld, the Einstein static state universe is stable if γ_m and C satisfy Eq. (39). Therefore, in the SS braneworld a successful implementation of emergent scenario requires that the perfect fluid on the brane should be the phantom-like and the Weyl fluid has a positive energy density.

4.2. RS model

The RS model has a spacelike extra dimension, which means that $\epsilon = 1$. The Einstein static state solution (Eq. (13)) has the form

$$\rho_0 = \frac{4-3\gamma_m}{2(3\gamma_m-2)}\rho_c, \quad a_0^4 = \frac{(3\gamma_m-2)^2}{\gamma_m(3\gamma_m-4)}\cdot\frac{4C}{\rho_c}. \tag{40}$$

The requirements of a positive energy density and a positive scalar factor give

$$C < 0, \quad \frac{2}{3} < \gamma_m < \frac{4}{3}. \tag{41}$$

For homogenous scalar perturbations ($k = 0$), Eq. (31) reduces to

$$\omega_1^2 = 0, \quad \omega_2^2 = -\frac{\gamma_m(4-3\gamma_m)^2\rho_c}{4(3\gamma_m-2)}. \tag{42}$$

$\omega_2^2 \geq 0$ leads to that

$$0 \leq \gamma_m < \frac{2}{3}, \text{ or } \gamma_m = \frac{4}{3}. \tag{43}$$

From Eqs. (41) and (43), one can see that there is no overlap for the allowed regions of γ_m. Thus, the Einstein static state solution is unstable in the spatially-flat RS model.

5. Perturbations in the bulk

It has been found that the static Horava–Witten braneworlds can be stable subject to finite energy deformations [29], while they are unstable from the higher-dimensional point of view [30,31]. Thus, it is interesting to discuss the stability of Einstein static state solutions in the bulk. Varying the action given in Eq. (1) with respect to the five-dimensional metric $g_{\alpha\beta}$, one can obtain the five-dimensional Einstein field equations [4]

$${}^{(5)}G_{\alpha\beta} = -{}^{(5)}\Lambda\, g_{\alpha\beta}, \tag{44}$$

where ${}^{(5)}\Lambda = -\frac{2\epsilon\sigma^2}{3M^6}$ since $\Lambda_{\text{eff}} = 0$. By dividing the five dimensional coordinates into (x^a, y) with y denoting the extra dimensional coordinate which is orthogonal to the brane situated at $y = 0$, we can express the RS and SS solutions as [32,33]

$${}^{(5)}ds^2 = e^{-2y/l}h_{ab}(x)dx^a dx^b + \epsilon dy^2, \tag{45}$$

where $l \equiv \frac{3\epsilon M^3}{\sigma}$ and the bulk coordinate is in the range $y \geq 0$. In Poincare coordinates, the above metric can be re-expressed as

$${}^{(5)}ds^2 = \frac{l^2}{z^2}\left[h_{ab}(x)dx^a dx^b + \epsilon dz^2\right], \tag{46}$$

where $z = le^{y/l}$ represents the extra dimensional coordinate. For the case of Einstein static states, the analysis in the proceeding Section shows that the metric on the brane is independent on x^a, which indicates that h_{ab} are constants.

To study the stability of Einstein static state solutions in the bulk, we need to discuss the stability of the metric given in Eq. (46) under perturbations. Since only on the SS brane are there stable Einstein static state solutions, in the following we focus on the case of $\epsilon = -1$. In the longitudinal gauge, the perturbed five-dimension metric can be described by [34,35]

$$\begin{aligned}{}^{(5)}ds^2 = \frac{l^2}{z^2}\Big[&(1-2{}^{(5)}\Psi)h_{00}dt^2 + (1+2{}^{(5)}\Phi)h_{ij}dx^i dx^j \\ &-(1-2{}^{(5)}\Gamma)dz^2\Big].\end{aligned} \tag{47}$$

Substituting Eq. (47) into Eq. (44) and using $\delta g^{\alpha}{}_{\beta} = 0$ and $\dot{h}_{ab} = 0$, we obtain the perturbed Einstein field equations in the bulk

$$\begin{aligned}\delta^{(5)}G^0{}_0 = &-\frac{12}{l^2}{}^{(5)}\Gamma + \frac{3z}{l^2}{}^{(5)}\Gamma' + \frac{9z}{l^2}{}^{(5)}\Phi' + \frac{z^2}{l^2}(2{}^{(5)}\Phi - {}^{(5)}\Gamma)^{|i}{}_{|i} \\ &-\frac{3z^2}{l^2}{}^{(5)}\Phi'' = 0,\end{aligned} \tag{48}$$

$$\begin{aligned}\delta^{(5)}G^5{}_5 = &-\frac{12}{l^2}{}^{(5)}\Gamma - \frac{3z}{l^2}{}^{(5)}\Psi' + \frac{9z}{l^2}{}^{(5)}\Phi' + \frac{z^2}{l^2}(2{}^{(5)}\Phi - {}^{(5)}\Psi)^{|i}{}_{|i} \\ &-\frac{3z^2}{l^2}{}^{(5)}\ddot{\Phi} = 0,\end{aligned} \tag{49}$$

$$\delta^{(5)}G^0{}_5 = \frac{3z^2}{l^2}{}^{(5)}\dot{\Phi}' - \frac{3z}{l^2}{}^{(5)}\dot{\Gamma} = 0, \tag{50}$$

$$\delta^{(5)}G^0{}_i = \left(-\frac{z^2}{l^2}{}^{(5)}\dot{\Gamma} + \frac{2z^2}{l^2}{}^{(5)}\dot{\Phi}\right)_{|i} = 0, \tag{51}$$

$$\delta^{(5)}G^5{}_i = \left(-\frac{3z}{l^2}{}^{(5)}\Gamma - \frac{z^2}{l^2}{}^{(5)}\Psi' + \frac{2z^2}{l^2}{}^{(5)}\Phi'\right)_{|i} = 0, \tag{52}$$

$$\delta^{(5)}G^i{}_j = \Big(-\frac{12}{l^2}{}^{(5)}\Gamma + \frac{3z}{l^2}{}^{(5)}\Gamma' + \frac{z^2}{l^2}{}^{(5)}\ddot{\Gamma} - \frac{3z}{l^2}{}^{(5)}\Psi' + \frac{z^2}{l^2}{}^{(5)}\Psi'' + \frac{6z}{l^2}{}^{(5)}\Phi' - \frac{2z^2}{l^2}{}^{(5)}\Phi'' - \frac{2z^2}{l^2}{}^{(5)}\ddot{\Phi} - \frac{z^2}{l^2}({}^{(5)}\Psi - {}^{(5)}\Phi + {}^{(5)}\Gamma)^{|k}{}_{|k} \Big)\delta^i{}_j + \frac{z^2}{l^2}({}^{(5)}\Psi - {}^{(5)}\Phi + {}^{(5)}\Gamma)^{|i}{}_{|j} = 0\,, \quad (53)$$

where the vertical bar denotes a covariant derivative with respect to three dimensional component of the metric, and a dot (prime) denotes the derivative with respect to time (the coordinate z). From Eq. (53) with $i \neq j$, we obtain that

$${}^{(5)}\Psi - {}^{(5)}\Phi + {}^{(5)}\Gamma = 0\,,$$

which implies that the anisotropic quantity of the perturbation in the bulk vanishes. This property can also be obtained in the case of the spacelike extra dimension [34]. Combining Eqs. (48–53), we find that

$$h_{ab}\widetilde{\nabla}^a\widetilde{\nabla}^{b\,(5)}\Phi = m^2\,{}^{(5)}\Phi = 0, \quad (54)$$

where $\widetilde{\nabla}$ denotes a covariant derivative with respect to four dimensional component of the metric and m is a constant. Since $m^2 \geq 0$ means that the solution is stable [35], Eq. (54) shows that in the bulk the Einstein static state solution is stable under scalar perturbations.

6. Conclusions

In this paper, we study the emergent scenario in spatially flat RS and SS braneworlds with the assumption that a perfect fluid with a constant equation of state is the only energy component on the brane. The existence of a stable Einstein static state solution requires that this perfect fluid has a phantom-like property since its equation of state must be less than −1, and the "Weyl fluid", which originates from the projection of the five dimension Weyl tensor onto the brane and behaves like a radiation, has a positive energy density. However, there is no stable Einstein static state solution for the RS braneworld. Furthermore, we find that in the bulk with a timelike extra dimension the static state solution is also stable under scalar perturbations. Thus, in the SS braneworld where the extra dimension is timelike, our universe can stay at the Einstein static state past-eternally, which means that it is possible to resolve the big bang singularity problem by an emergent scenario.

Acknowledgements

This work was supported by the National Natural Science Foundation of China under Grants No. 11222545, No. 11435006, No. 11375092, No. 11447104, and No. 11505004, the Anhui Provincial Natural Science Foundation of China under Grant No. 1508085QA17, the Research Culture Funds of Anhui Normal University under Grant No. 2013rcpy31, and the Ministry of Science and Technology (Taiwan) under Grant No. MOST 104-2112-M-009-020–MY3. K.Z. thanks Prof. Chao-Qiang Geng and the National Center for Theoretical Sciences, Taiwan, where part of the work was done, for the warm hospitality.

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