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Proposal for teleportation of charge qubits via super-radiance

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Abstract. A scheme is proposed to teleport charge qubits via super-radiance. Reservoir-induced entanglement is generated between two semiconductor dots in a microcavity where a quantum state encoded in a third quantum dot is then tuned into collective decay with one of the entangled dots. Teleportation is achieved automatically in our scheme which we also extend to quantum wires.

Quantum entanglement has achieved a prime position in current research due to its central role in quantum information science, e.g., in quantum cryptography, quantum computing and teleportation [1]. Many efforts have been devoted to the study of entanglement induced by a direct interaction between the individual subsystems. Recently, attention has been focused on 'reservoir-induced' entanglement [2] with the purpose of shedding light on the generation of entangled qubits at remote separation.

Entangled states can also be generated via sub- and super-radiance, i.e. the collective spontaneous decay first introduced by Dicke [3]. For the simplest case of two identical two-level atoms interacting with the vacuum fluctuations of a common photon reservoir, entanglement naturally appears in the two intermediate states,

$$|T_0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle), \qquad |S_0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$
 (1)

of the two decay channels from the excited state $|T_1\rangle = |\uparrow\uparrow\rangle$ to the ground state $|T_{-1}\rangle = |\downarrow\downarrow\rangle$. An experimental demonstration of two-ion collective decay as a function of inter-ion separation was shown by DeVoe and Brewer in 1996 [4]. On the other hand, the possibility to modify decay

rates of individual atoms inside cavities (Purcell effect) [5] has been known for a long time, and enhanced and inhibited spontaneous rates for atomic systems were intensively investigated in the 1980s [6] by passing atoms through cavities.

Experiments of teleportation have already been realized in NMR [7], photonic [8] and atomic [9] systems. Turning to solid state systems, however, experimental demonstration of teleportation in charge qubits is still lacking and only a few theoretical schemes have been proposed [10]. In this work, we propose a teleportation scheme for atomic and solid state qubits (quantum dots (QDs)) and quantum wires), which is based on the Dicke effect and achieves, in contrast to usual schemes, a 'one-pass' teleportation by a joint measurement.

The simplest case. To address the role of super-radiance in teleportation, let us first consider a two-level atom passing through a cavity as shown in figure 1. In the strong coupling regime, the interaction between the atom and the cavity photon is

$$H' = \hbar g(\sigma_+ b^- + \sigma_- b^+),$$
 (2)

where g is the atom-cavity coupling strength, b^{\pm} and the Pauli matrices σ^{\pm} are the cavity photon and atom operators respectively. With the appropriate preparation of the initial state of atom 1 and the control of its passing time through the cavity, the singlet entangled state $\frac{1}{\sqrt{2}}(|0\rangle_c|\uparrow\rangle_1-|1\rangle_c|\downarrow\rangle_1)$ is created between atom 1 and the cavity photon. The notations $|\uparrow\rangle_1$ ($|\downarrow\rangle_1$) and $|0\rangle_c$ ($|1\rangle_c$) refer, respectively, to atom 1 in the excited (ground) state and no (one) photon in the cavity. If the quantum state α $|\uparrow\rangle_2 + \beta$ $|\downarrow\rangle_2$, generated by the coherent excitation of atom 2 with a laser pulse, is to be teleported, the next step to complete the teleportation is to trap both atoms close to each other. In this case, the total wave function of the system is given by

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle_{c}|\uparrow\rangle_{1} - |1\rangle_{c}|\downarrow\rangle_{1}) \otimes (\alpha|\uparrow\rangle_{2} + \beta|\downarrow\rangle_{2})$$

$$= |0\rangle_{c} \otimes \left(\frac{\alpha}{\sqrt{2}}|T_{1}\rangle_{12}\right) + |1\rangle_{c} \otimes \left(\frac{-\beta}{\sqrt{2}}|T_{-1}\rangle_{12}\right)$$

$$+ (\alpha|1\rangle_{c} + \beta|0\rangle_{c}) \otimes \frac{|S_{0}\rangle_{12}}{2} + (-\alpha|1\rangle_{c} + \beta|0\rangle_{c}) \otimes \frac{|T_{0}\rangle_{12}}{2}.$$
(3)

Since atoms 1 and 2 are now close enough, the common photon reservoir will drive them to decay collectively with four possibilities for the detector's results: zero photon, two photons, one photon via the super-radiant channel, or one photon via the sub-radiant channel. If the measurement outcome is a single photon with a suppressed decay rate, the teleportation is achieved automatically. As for the result of one photon with enhanced decay rate, all we have to do is to perform a phase-gate operation on the cavity photon state to complete the teleportation.

Since decay time is a statistical average, one might ask how to distinguish between suband super-radiant photons via the decay time in one single shot? We would like to point out that because of the collective decay, the momentum of the emitted photon \vec{k} depends on the separation of the two atoms \vec{r} , i.e. $\vec{k} \cdot \vec{r} = 0$ or π corresponds to the emission of super- or sub-radiant photon respectively [4]. Therefore, sub- and super-radiance can be distinguished by placing detectors at appropriate angles as shown in figure 1(b). After monitoring all possible emission directions, one can further make sure whether the number of the emitted photon is one or not. The teleportation can then be tested by repeating this scheme over many cycles and probing the state of the cavity (one or no photon) after each cycle.

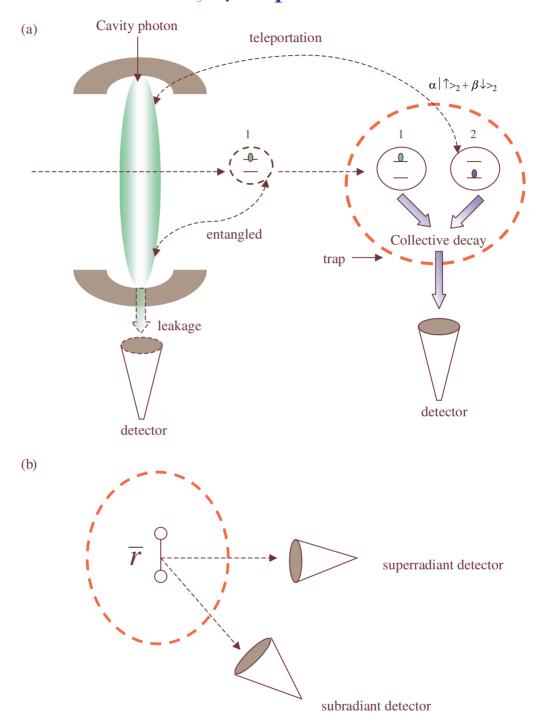


Figure 1. (a) Schematic description of teleportation by collective decay in a cavity QED system. Firstly, the singlet entangled state is generated between atom 1 and the cavity photon as the atom has passed through the cavity. Atoms 1 and 2 then decay collectively. If the measurement outcome is a single photon with sub-radiant decay rate, the teleportation to the cavity photon state is achieved automatically. (b) To distinguish between super- and sub-radiant decay, two detectors are placed at appropriate angles such that the emitted photon momentum \vec{k} satisfy the condition of $\vec{k} \cdot \vec{r} = 0$ and π respectively.

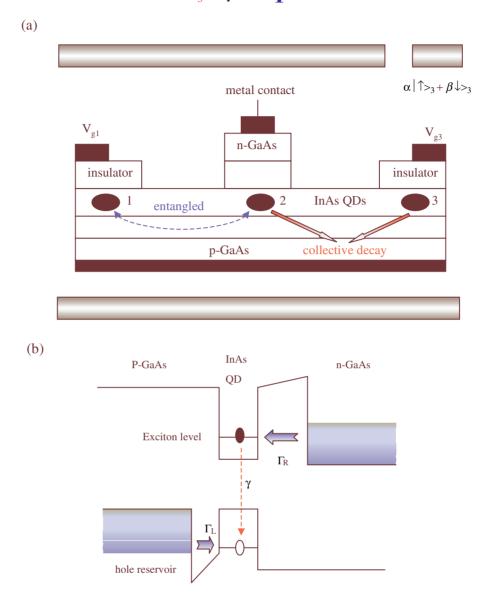


Figure 2. (a) Schematic description of teleportation by the Dicke effect in QDs. Firstly, a sub-radiance–induced singlet entangled state is generated between QDs 1 and 2. The energy bandgap of the exciton in QD 3 (1) is then tuned to be (non)-resonant with that in QD 2. Finally, a joint measurement is performed naturally by collective decay of QDs 2 and 3. (b) Energy band diagram of an InAs QD embedded inside a p-i-n junction.

Quantum dots. We now proceed to consider three semiconductor QDs embedded inside a p-i-n junction as shown in figure 2. The Fermi level of the p(n)-side hole (electron) is assumed to be slightly lower (higher) than the hole (electron) sub-band in the dot. After a hole is injected into the hole sub-band in the QD, the n-side electron can tunnel into the exciton level because of the Coulomb interaction between the electron and hole. The effective Hilbert space of the closed system is spanned by $|\uparrow\rangle = |e,h\rangle$ and $|\downarrow\rangle = |0,0\rangle$, which correspond to the exciton and ground state in the QD [11]. The interactions between the ith QD and the reservoirs can then be

written as

$$H_V = \sum_{\mathbf{q},i} (V_{\mathbf{q}} c_{\mathbf{q}}^{\dagger} \widehat{s_{R,i}} + W_{\mathbf{q}} d_{\mathbf{q}}^{\dagger} \widehat{s_{L,i}} + \text{H.c.}), \tag{4}$$

where $\widehat{s_R} = |h\rangle \langle \uparrow|$, $\widehat{s_L} = |h\rangle \langle \downarrow|$, $V_{\bf q}$ and $W_{\bf q}$ couple the channels ${\bf q}$ of the electron and the hole reservoirs, and the extra state $|h\rangle$ means there is one hole in the QD. Here, $c_{\bf q}$ and $d_{\bf q}$ denote the electron operators in the left and right reservoirs respectively. The state $|e\rangle$ can be neglected by using a thicker barrier on the electron side so that there is little chance for an electron to tunnel in advance.

If the energy level differences of the first and second dot excitons are tuned to be resonant with each other, super-radiant decay occurs between them with the interaction

$$H_{P} = \sum_{\mathbf{k}} \frac{1}{\sqrt{2}} \{ D_{\mathbf{k}} b_{\mathbf{k}} [(1 + e^{i\mathbf{k} \cdot \mathbf{r}}) | S_{0} \rangle \langle T_{-1} | + (1 - e^{i\mathbf{k} \cdot \mathbf{r}}) | T_{0} \rangle \langle T_{-1} |] + \text{H.c.} \}, \quad (5)$$

where $|S_0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2$ and $|T_0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\downarrow\rangle_2 + |\downarrow\rangle_1|\uparrow\rangle_2$ represent the singlet and triplet entangled states in these two QDs (1 and 2). Here, $b_{\bf k}$ is the photon operator, $D_{\bf k}$ is the coupling strength and ${\bf r}$ is the position vector between dots 1 and 2.

The transient behaviour of the non-equilibrium problem caused by the interactions H_V and H_P can be solved by master-equation techniques. If the current is conducted through dot 2 only, however, maximum entangled states can be created in the long time limit and for the system being inside a rectangular microcavity: it turns out that for small radiative decay rates, the steady state density operator becomes independent of the coupling to the leads and approaches a pure singlet or triplet entangled state depending on the inter-dot distance [12]. This means that even for remote separation of the dots and under transport conditions, the entanglement can still be achieved.

Once the entangled state is formed, we then lower the energy bandgap of the QD 1 exciton using the Stark effect, such that its level spacing becomes smaller than the lowest mode of the cavity photon. Thus, the interaction between the photon and the exciton in QD 1 is turned off. To prevent further decoherence of the exciton, the couplings to the left and right electron reservoirs should also be switched off. If the quantum state $\alpha \mid \uparrow \rangle_3 + \beta \mid \downarrow \rangle_3$, generated by the coherent excitation of a laser pulse [13], in QD 3 is to be teleported, the next step to complete the teleportation is to tune the exciton in QD 3 to be resonant with that in dot 2. As we mentioned above, if the measurement outcome is a single photon with a suppressed (enhanced) decay rate, the teleportation is achieved automatically.

Unlike the spatially resolved scheme of super-radiant/sub-radiant photon in purely quantum optic systems, a time resolved measurement scheme is needed for the QDs in a microcavity. In this case, it is possible to have the vanishing sub-radiant decay rate if QDs 2 and 3 are placed at certain positions [12], rendering them indistinguishable from the $|T_{-1}\rangle$ state. Therefore, in the design of the device, the separation between QDs 2 and 3 should not be positioned exactly equal to the multiple (effective) wavelength of the emitted photon.³ On the other hand, however, the time resolved measurement scheme is actually a statistical average. To distinguish between these two channels, one has to define a border line of time, such that above (below) it is a sub-radiant (super-radiant) decay. From this point of view, the ratio of super-radiant to sub-radiant emission

³ In [12], we considered two QDs in a 'rectangular' microcavity with length λ_0 . The energy of the cavity photon is composed of the confined and unconfined parts. To preserve the energy conservation, the 'effective' wavelength in the unconfined direction is enhanced by a factor of $\sqrt{2}$.

rates should not be too small. This means the separation between the QDs is better to be designed as close as to the multiple (effective) wavelength of the emitted photon.

Based on these, we can now perform a simple estimation of the success rate: the superand sub-radiant decay times can be written as $(1 \pm \Delta)\tau_0$, where τ_0 is the spontaneous lifetime of a single QD exciton. The range of Δ is between 0 and 1, depending on the separation of the QDs. Assuming that the border line is τ_0 and the statistical distribution of the detections on the sub-radiant [super-radiant] emission time is a Gaussian form centreing at $(1 + \Delta)\tau_0$ [$(1 - \Delta)\tau_0$] with half-width $\sigma_-[\sigma_+]$, the probabilities for the successful detections of $|T_0\rangle$ and $|S_0\rangle$ states are

$$P_{S_0} = \frac{1}{N_{S_0}} \int_{\tau_0}^{\infty} e^{-[t - (1 + \Delta)\tau_0]^2/\sigma_-^2} dt, \qquad P_{T_0} = \frac{1}{N_{T_0}} \int_0^{\tau_0} e^{-[t - (1 - \Delta)\tau_0]^2/\sigma_+^2} dt, \qquad (6)$$

where N_{S_0} and N_{T_0} are the corresponding normalization constants. Together with equation (3), one can define the success rate as $P_{1D} \equiv (P_{S_0} + P_{T_0})/4$. For reasonable parameters of $\Delta = 0.7$ and assuming that the half-widths are equal to one-half of the centreing times ($\sigma_{\pm} = (1 \pm \Delta)\tau_0/2$), one finds that the success rate P_{1D} is about 0.47. This is a little bit lower than the ideal success probability of 0.5. In addition to the statistical part, one can further consider the detection efficiency η in a real experiment as pointed out in [14]. In the detection stage, one will be able to detect a fraction η of all the successful protocols. This means one will erroneously regard a fraction $2\eta(1-\eta)$ of the cases with two decays as successful cases. Then the success rate changes to $P_{\text{suc}}(\eta) = \eta P_{1D} + 2\eta(1-\eta)(1-P_{1D}-P_{ND})$, where P_{ND} is the probability of no decay during the detection.

Our model can be applied to electron transport through gate-controlled double QDs [15] as well, if the quantum states are replaced as: $|\uparrow\rangle \rightarrow |L\rangle = |N_L + 1, N_R\rangle$ and $|\downarrow\rangle \rightarrow |R\rangle = |N_L, N_R + 1\rangle$, which correspond to different many-body ground states with N_L electrons in the left and N_R electrons in the right dot. For such a system, the interaction with phonons $(\sum_{\bf q} \hbar \omega_{\bf q} a_{\bf q}^{\dagger} a_{\bf q})$ plays the dominant role and is given by $H_{\rm ph} = T_c[|L\rangle \langle R|X + |R\rangle \langle L|X^{\dagger}]$, where T_c is the coupling strength between two dots, and $X = \prod_{\bf q} D_{\bf q} (\frac{\alpha_{\bf q} - \beta_{\bf q}}{\omega_{\bf q}})$ is the phonon-dependent operator with $D_{\bf q}(z) = {\rm e}^{za_{\bf q}^{\dagger} - z^* a_{\bf q}}$. With the advances of nano-fabrication technologies, it is now possible to embed QDs in a free standing nano-size semiconductor slab (phonon cavity) [16]. Just like photon cavities, suppressed and enhanced spontaneous emission of phonons are both possible by adjusting energy level difference of the double dot [17]. The entangled state between two qubits can also be generated via sharing the common phonon reservoir [18].

Extension to quantum wires. The present scheme can also be applied to other solid state systems that exhibit super-radiance. High quality quantum wires in a microcavity can now be fabricated with advanced technologies [19], with the exciton and photon forming a new eigenstate called 'polariton' because of the crystal symmetry in the chain direction. The decay rate of quantum wire polaritons in a planar microcavity is [20]

$$\gamma_{1d} = \frac{2\pi e^2 \hbar}{m^2 c^2 d} \sum_{n=1}^{N_c} \frac{1}{L_c} \frac{|\epsilon \cdot \chi|^2}{\sqrt{\left(\frac{2\pi}{\lambda}\right)^2 - \left(\frac{\pi}{L_c}n\right)^2}},\tag{7}$$

where N_c is the total number of decay channels, ϵ is the polarization vector of the photon, χ the dipole moment of the exciton, d the lattice spacing and L_c is the cavity length. Because of energy conservation, the value of N_c must be smaller than $2L_c/\lambda$, where λ is the wavelength of the emitted photon. If the cavity length is designed to be roughly equal to $\lambda/2$, the Stark effect

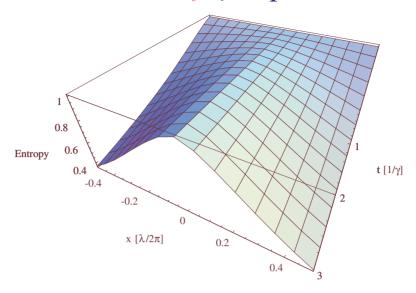


Figure 3. Effect of renormalized frequency shift on the degree of entanglement (the von Neumann entropy).

can be applied to vary the energy gap of the exciton and the polariton decay can be controlled according to equation (7).

As for teleportation, the additional advantage over the QD systems mentioned above is the fact that quantum wire polaritons are collective excitations and the time-resolved measurement of many outcoming photons therefore can be done much more easily. The difference is that the preparation and readout of the quantum states can only be accomplished by optical ways. The drawback here is the enhancement of the renormalized frequency shift Ω (counterpart of the decay rate) by a factor of λ/d . This has a strong effect on the entanglement and cannot be neglected: assuming a singlet entangled state is formed between the wires at t = 0, the quantum state of the system at t > 0 can be written as $\Phi(t) = b_1(t) |\exp(t)| 0$; $\exp(t) |0$; $\exp(t) |$

$$\begin{cases} \frac{\mathrm{d}}{\mathrm{d}t}b_1(t) = f_{11}b_1(t) + f_{12}b_2(t), \\ \frac{\mathrm{d}}{\mathrm{d}t}b_2(t) = f_{21}b_1(t) + f_{22}b_2(t), \end{cases}$$
(8)

where $f_{ij} = i\Omega_{ij} + \gamma_{ij}$, $\gamma_{ij} = \gamma_{1d} e^{i(\sqrt{k_0^2 - k_c^2})(x_i - x_j)}$, i, j = 1, 2 and

$$\Omega_{ij} = \frac{e^2 \hbar}{m^2 c^2 d} \sum_{n_c=1}^{N_c} \frac{1}{L_c} \int \frac{|\epsilon \cdot \chi|^2 e^{ik_x (x_i - x_j)} dk_x}{(k_0 - \sqrt{k_x^2 + k_c^2}) \sqrt{k_x^2 + k_c^2}}.$$
(9)

Here, $k_0 = 2\pi/\lambda$, $k_c = \frac{\pi}{L_c} N_c$ and x_i is the position of the *i*th quantum wire. Ω_{ij} is the position-dependent frequency shift, which contains both infrared and ultraviolet divergence and has to be renormalized properly [22]. Figure 3 shows how the degree of entanglement (the von Neumann

entropy, E) decreases as a function of inter-wire separation x and time t when the two wires are embedded inside a planar microcavity with cavity length $L_c = 3\lambda/4$. As seen, the larger the wire separation, the faster the decreasing of the entropy. This tells us that, to maintain high fidelity of teleportation, the quantum wires have to be fabricated as close as possible and the collective measurement has to be performed immediately once the singlet state is created. Otherwise, the enhanced frequency shift will inevitably diminish the fidelity of teleportation.

A few remarks about the comparisons between our scheme and other proposals should be mentioned here. In usual teleportation scheme, one has to perform Hadamard and CNOT transformations on one of the entangled particles and the teleported quantum state. After that, the information from the joint measurements of the two particles has to be sent to the other entangled particle in order to allow proper unitary operations. In our proposal, however, the Hadamard and CNOT transformations are omitted and the joint measurements are performed naturally by collective decay. This kind of 'one-pass' teleportation is similar to Bose's proposal [14], where the teleportation between two trapped atoms in two independent cavities is achieved by the leaked cavity photons impinging on a 50–50 beam splitter. Very recently, Beenakker and Kindermann [23] have also theoretically pointed out that 'one-pass' teleportation of spin states in quantum Hall system is possible. The key is the recombination of the electron and hole at the tunnel barrier. A disadvantage is that the success rate is small.

Just like Bose's protocol, our probabilistic proposal can be modified to teleportation with insurance, so that in the cases when the protocol is unsuccessful the original teleported state is not destroyed, but mapped on to another reserve atom (or dot) r. To accomplish this, the key step is the local redundant encoding of the teleported state [24] before the collective decay:

$$|\Psi\rangle_{\text{code}} = \beta(|\uparrow\rangle_2|\downarrow\rangle_r + |\downarrow\rangle_2|\uparrow\rangle_r) + \alpha(|\downarrow\rangle_2|\downarrow\rangle_r + |\uparrow\rangle_2|\uparrow\rangle_r).$$

If the teleportation is unsuccessful, the coded state is left with either state $\alpha \mid \uparrow \rangle_r + \beta \mid \downarrow \rangle_r$ or a state that can be converted to $\alpha \mid \uparrow \rangle_r + \beta \mid \downarrow \rangle_r$ by a known unitary transformation. In this case, one can repeat this procedure until teleportation is successful.

In summary, we have proposed an alternative way to accomplish quantum teleportation in solid state charge qubits. Super-radiance/sub-radiance achieves both entanglement generation and joint measurement during the teleported process. To the best of our knowledge, it is the first time that super-radiance is pointed out to be useful in quantum teleportation. Our examples of QDs and wires demonstrated possible experimental realizations and deserve to be tested with present technologies.

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