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Recovery of the Aharonov-Bohm oscillations in asymmetrical quantum rings

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We theoretically investigate suppression and recovery of the Aharonov-Bohm oscillations of the diamagnetic response of electrons (holes) confined in self-assembled $In_cGa_{1-c}As/GaAs$ semiconductor reflection asymmetrical quantum rings. Based on the mapping method and gauge-origin-independent definition for the magnetic vector potential we simulate the energies and wave functions of the electron (hole) under external magnetic and electric fields. We examine the transformation of the ground state wave function of the electron (hole) in reflection asymmetrical rings from localized in one of the potential valleys (dotlike shape of the wave function) to distributed over all volume of the ring (ringlike shape) under an appropriate lateral electric field. This transformation greatly recovers the electron (hole) diamagnetic coefficient and Aharonov-Bohm oscillations of the diamagnetic response of the ring. However, the recovering electric field for the first Aharonov-Bohm diamagnetic oscillation of the electron is a suppressing one for the hole (and vice versa). This can block the recovery of the optical Aharonow-Bohm effect in $In_cGa_{1-c}As/GaAs$ asymmetrically wobbled rings. However, the recovery of the Aharonov-Bohm oscillations for the independent electron (hole) by the external electric field remains interesting and feasible objective for the asymmetric rings. [http://dx.doi.org/10.1063/1.4958875]

I. INTRODUCTION

Semiconductor quantum rings present a special class of nano-structures that recently have attracted considerable attention due to a wide variety of the fundamental quantum-mechanical effects observed in these structures (see for instance Ref. 1 and numerous references therein). Meanwhile, potential applications of the rings are already suggested to be in high performance photodetectors,[2–5] solar cells,[6–8] lasers,[9] single photon sources,[10] etc. Nevertheless, the increasing interest to the semiconductor quantum rings arises mainly from the non-simply-connected topology of the rings and their unique magnetic properties.[1,11] The rings have become a useful tool for the observation of quantum phase coherence effects in the scattering-free limit (such as the Aharonov-Bohm-like effects[12]) and thus they have potential applications in quantum information processing.[13–17]

Quantum rings made from different semiconductor materials and with controllable geometrical parameters have been fabricated and investigated experimentally and theoretically (see, e.g., Refs. 1, 11, 18–38 and references therein). Additionally, impacts of imperfections (impurities, strain, geometry asymmetry, etc.[39–48]) as well as external electric fields[49–55] on the rings' magnetic and magneto-optical properties have been explored. More specifically, in Ref. 42, using a simplified model of a two-dimensional eccentric ring, the authors demonstrated that the destructive impact of the ring asymmetry (eccentricity) on the electron Aharonov-Bohm oscillations can be compensated by the external in-plane electric field. The possibility to tune the Aharonov-Bohm effect by using an

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electric field applied perpendicular to semiconductor cylindrically symmetrical quantum rings for both single particles and for excitons has been discovered in Ref. 54.

Recently it was found that the magnetic and optical properties of In_cGa_{1-c}/GaAs semiconductor self-assembled quantum rings (SAQRs) strongly depend on their actual geometry and material parameters.[31,33,47] A small asymmetry in the ring geometry and material content can generate a prompt change in the shape of the ground state wave function of the electron (hole) confined in the ring: from the reflection symmetrical (ringlike wave function which is extended around the ring volume) to a reflection non-symmetrical (strongly localized dotlike wave function). The change leads to the substantial decrease of the neutral exciton diamagnetic coefficient and can be a reason of the suppression of the optical Aharonov-Bohm effect in $In_cGa_{1-c}As$/GaAs wobbled rings. However, the wave function of the independent electron in general is less sensitive to the lack of the reflection symmetry then the independent hole's wave function. This helps to explain the observation of the electron Aharonov-Bohm magnetization oscillations even in quantum rings with possibly broken reflection symmetry.[19,30,47]

In a weak external magnetic field B the shift of the energy levels for the electron (hole) confined in the SAQRs energy is[56] $E_{e(h)}(B) - E_{e(h)}(0) = s\mu_B g_{e(h)} B + \alpha_{e(h)} B^2$, where μ_B stands for the Bohr magneton, $g_{e(h)}$ presents the electron (hole) Landé factor, $s = \pm 1/2$ is the particle spin, and $\alpha_{e(h)}$ is the electron (hole) diamagnetic coefficient. The diamagnetic coefficient is obviously associated with the second derivative of the carrier's energy with respect to the magnitude of the external magnetic field.[28,53,54] The second derivative of the carrier's energy is also related to the "differential diamagnetic coefficient" (which can be defined as $A(B) = \frac{1}{2} d^2E_{e(h)}/dB^2$) and the single particle differential magnetic susceptibility of the SAQR at absolute zero temperature[30,36,57] $\chi_{e(h)}(B) = -d^2E_{e(h)}(B)/dB^2$. Therefore, $\alpha_{e(h)} = A_{e(h)}(0) = -\frac{1}{2}\chi_{e(h)}(0)$ in the weak-field limit.

The observable transformations in the localization (topology) of the wave function of carriers confined in the SAQRs are regarded as the "iso-spin" operations. The iso-spin is a mark to the actual position of a particle in the structure and it can be used in quantum information processing in complete analogy to the particle's spin.[58] When the magnetic field is applied along a SAQR structure growth direction (z axis, see Fig. 1), the diamagnetic coefficient is defined as[56]: $\alpha_{e(h)} = e^2\rho^2_{e(h)}/8m_{e(h)}$, where $\rho_{e(h)}$ is the electron (hole) characteristic confinement length in the x-y plane, e is the elementary charge, and $m_{e(h)}$ stands for the electron (hole) effective mass. Hence, the particle diamagnetic coefficient gives an estimate for the electron (hole) lateral confinement length, effective position of the carrier in the SAQR, wave function shape (topology), and the actual status of the iso-spins in the SAQR.

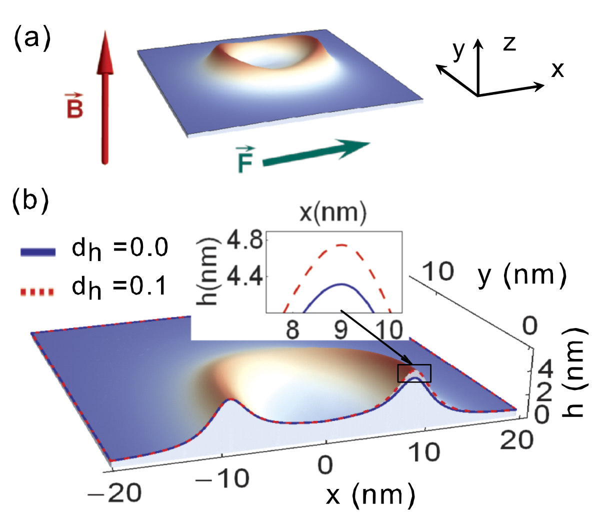

FIG. 1. (a) $In_cGa_{1-c}As$/GaAs wobbled SAQR in the presents of the external magnetic (**B**) and electric (**F**) fields. (b) Cross section of the SAQR by the $(x, 0, z)$ plane for different values of the shape asymmetry parameter d_h. Zoom of the cross section near $x = x_p$ is shown. See description in the text.

This work analyzes the effects induced by the broken reflection symmetry and external lateral electric field on the Aharonov-Bohm oscillations for the individual electron (hole) and confined in the $In_cGa_{1-c}As/GaAs$ wobbled SAQRs. Deploying the mapping method[36,47] and gauge-origin-independent definition for the magnetic vector potential[47] we simulate the electron and hole properties in the reflection symmetrical and asymmetrical (with respect to the reflection in the y-z plane, see Fig. 1) SAQRs under external magnetic and electric fields. We demonstrate how the ring-like shape of the ground state wave functions, diamagnetic coefficients, and the Aharonov-Bohm oscillations for the electrons and holes can be recovered in reflection asymmetrical SAQRs by an appropriate adjustment of the applied electric field.

II. THEORETICAL MODEL AND METHOD OF SIMULATION

We describe the energy states of the independent electron (hole) in the $In_cGa_{1-c}As/GaAs$ SAQRs under the external magnetic ($\mathbf{B}$) and electric ($\mathbf{F}$) fields using the effective one conduction (valence) band Hamiltonian with the position-dependent effective mass[47,59]

$$\hat{\mathrm{H}}_{e(h)} = \hat{\Pi}_{e(h)} \frac{1}{2m_{e(h)}(\mathbf{r})} \hat{\Pi}_{e(h)} + q_{e(h)}\mathbf{F}\cdot\mathbf{r} + V_{e(h)}(\mathbf{r}), \tag{1}$$

where $\mathbf{r} = (x, y, z)$ is the three-dimensional radius vector, $\hat{\Pi}_{e(h)} = -i\hbar\nabla_{\mathbf{r}} - q_{e(h)}\mathbf{A}(\mathbf{r})$ is the electron (hole) momentum operator, $\nabla_{\mathbf{r}}$ stands for the spatial gradient, $q_{e(h)} = -(+)e$, $\mathbf{A}(\mathbf{r})$ is the vector potential of the magnetic field $\mathbf{B} = \nabla_{\mathbf{r}} \times \mathbf{A}(\mathbf{r})$, and $V_{e(h)}(\mathbf{r})$ refers to the particle's confinement potential. We use a gauge-origin-independent definition for the vector potential $\mathbf{A}(\mathbf{r}) = \mathbf{B}\times(\mathbf{r} - \bar{\mathbf{r}}^m_{e(h)})/2$ (see Ref. 47 and references therein), where $\bar{\mathbf{r}}^m_{e(h)}$ can be defined according to to the principle of the minimal magnetic coupling.[60,61] For the Hamiltonian (1) this principle leads to

$$\bar{\mathbf{r}}^m_{e(h)} = \frac{\left\langle \mathbf{r}\cdot m^{-1}_{e(h)}(\mathbf{r})\right\rangle_{e(h)}}{\left\langle m^{-1}_{e(h)}(\mathbf{r})\right\rangle_{e(h)}} \approx \bar{\mathbf{r}}_{e(h)} = \langle \mathbf{r}\rangle_{e(h)}, \tag{2}$$

where $\langle f\rangle_{e(h)}$ is for the expectation value of a quantity f in the particle's ground state with the wave function $\psi_{e(h)}$:

$$\langle f\rangle_{e(h)} = \int \psi^*_{e(h)}(\mathbf{r})\, f(\mathbf{r})\, \psi_{e(h)}(\mathbf{r})\, d\mathbf{r}, \tag{3}$$

and $\bar{\mathbf{r}}_{e(h)} = (\bar{x}_{e(h)}, \bar{y}_{e(h)}, \bar{z}_{e(h)})$ is the particle's effective position in the ground state. Using this gauge we can write the diamagnetic coefficient for the electron (hole) confined in the SAQR (when the magnetic field is applied along the z direction, see Fig. 1) as[47]

$$\alpha_{e(h)} = \frac{e^2}{8}\left\langle \frac{\left(\mathbf{r}_\perp - \bar{\mathbf{r}}_{\perp,e(h)}\right)^2}{m_{e(h)}(\mathbf{r})}\right\rangle\Bigg|_{B=0}, \tag{4}$$

where $\mathbf{r}_\perp = (x, y)$ is the two-dimensional radius vector in the x-y plane.

Now we map all position dependent geometrical and material parameters according to the realistic geometry of the SAQR.[47] First, we model the SAQR shape profile by a function $h(x,y)$. The function reproduces the local height of the ring (along the z direction) at the actual position on the x-y plane. For the wobbled SAQRs, the function $h(x, y)$ is described in details in Refs. 25, 31, and 47 (see Fig. 1). In our model, the reflection asymmetry in the shape of the ring with respect to the reflection in the y-z plane (difference in the ring hills' heights) is controlled by a unitless parameter d_h as[47]

$$d_h = \frac{h(x_p,0)|_{d_h\neq 0} - h(x_p,0)|_{d_h=0}}{h(x_p,0)|_{d_h=0}}, \tag{5}$$

where $(x_p, 0)$ is the position of the ring's profile maximum on the positive x semi-axis (see Fig. 1).

Deviations from the reflection symmetry in the particle's confinement potential $V_{e(h)}(\mathbf{r})$ is presented by a unitless parameter d_V which can be written as:[47]

$$d_V \approx \frac{V_{e(h)}(\mathbf{r}_p)|_{d_V=0} - V_{e(h)}(\mathbf{r}_p)|_{d_V\neq 0}}{\Delta E_{e(h)}}, \tag{6}$$

where $\Delta E_{e(h)} = E^{\rm out}_{c(v)} - E^{\rm in}_{c(v)}$ is the overall conduction (valence) band offset between the outer and inner semiconductor materials in the $\mathrm{In}_c\mathrm{Ga}_{1-c}\mathrm{As/GaAs}$ heterostructure, subscripts "out" and "in" denote the actual material parameters outside and inside the ring. and $\mathbf{r}_p = (x_p, 0, z_p)$ refers to the position of the appropriate potential valley. Variations of the parameters d_V generate the material content asymmetry at the ring's hill locations (this was discussed in details in Ref. 47). Fig. 2 demonstrates projections of the actual confinement potentials for the electron and hole which is used in our simulation.

The mapping function can be written as[36,47]

$$M(\mathbf{r}) = -\frac{V_{e(h)}(\mathbf{r})}{\Delta E_{e(h)}}. \tag{7}$$

Therefore, the position-dependent effective mass $m_{e(h)}(\mathbf{r})$ of the electron (hole) is defined as $m_{e(h)} = m^{\rm in}_{e(h)} M(\mathbf{r}) + m^{\rm out}_{e(h)}[1 - M(\mathbf{r})]$ in the structure. The confinement potentials and position-dependent effective masses we use to simulate and analyze the electron and hole energy states, wave functions, expectation value of the particle's position, and diamagnetic coefficient in SAQR with deviations from the reflection symmetry (different d_h and d_V). To observe clearly the suppression (in reflectional asymmetric rings) and recovery of the Aharnow-Bohm oscillations (under application of the lateral electric field) we study the differential diamagnetic coefficient $A_{e(h)}(B)$ for the independent electron (hole).

III. SIMULATION RESULTS AND DISCUSSION

We assume that the $\mathrm{In}_c\mathrm{Ga}_{1-c}\mathrm{As/GaAs}$ SAQR geometry and material parameters are close to the rings discussed in Refs. 31 and 32. Therefore, the function $h(x, y)$ is characterized by the set of parameters those are relevant to the type-A ring in the Ref. 47 with $c = 0.55$. The material parameters are taken from Refs. 62 and 63 and fitted according to the composition and strain inside the rings.[36,47,64]

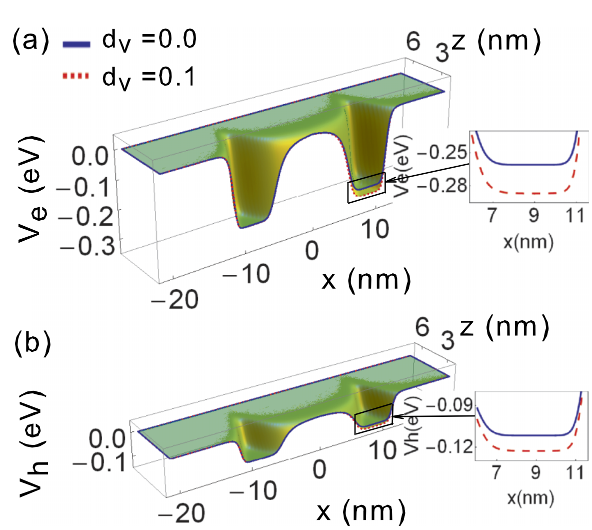

FIG. 2. Projections of the electron (a) and hole (b) confinement potentials on the $(x, 0, z)$ plane for different values of the asymmetry parameter d_V ($d_h = 0.0$). Zoomed profiles of the confinement potentials along the $[x, 0, z_p]$ direction near $x = x_p$ are shown. See description in the text.

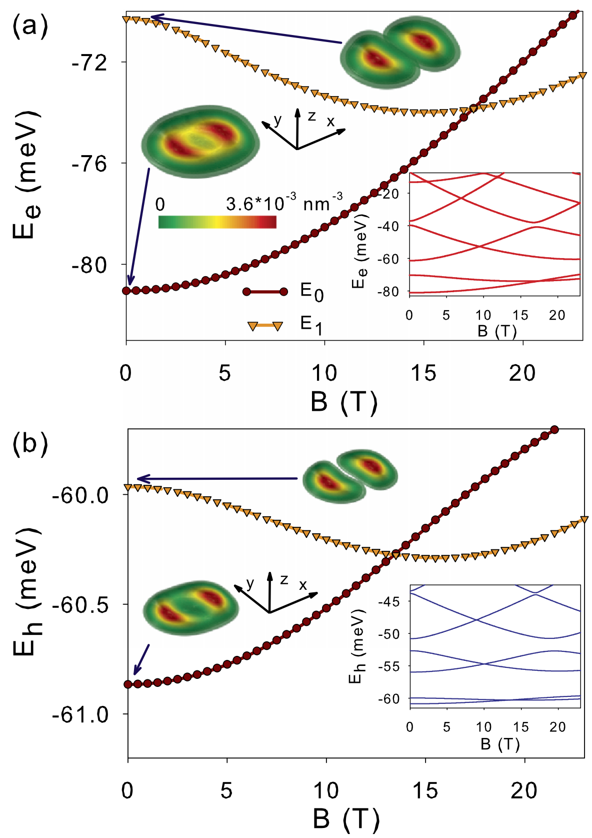

FIG. 3. Two lowest energy-levels as functions of the magnetic field for the electron (a) and hole (b) in the reflection symmetrical ring when $F = 0$. Three-dimensional contour plots of the spatial probability distribution of the particles at $B = 0$ are shown. Insets: Several electron and hole energy levels vs magnetic field.

In this paper we concentrate on the principal possibility to recover by the external electric field the Aharonow-Bohm oscillations in reflection asymmetrical rings. Therefore, utilizing our previous findings[47] we keep the d_h and d_V parameters' variations within certain bars and study the impacts of the d_h and d_V deviations on the ring diamagnetic properties only independently and non-cumulatively. Eigenenergies and eigeinfunctions of the Hamiltonian (1) for the electron and hole confined in the SAQR we obtain numerically using the iterative method[65] and the COMSOL multiphysics package.[66] We employ the particle's energies and wave functions to obtain the expectation value of the particle's position, diamagnetic coefficient in the weak field limit, and differential diamagnetic coefficient $A_{e(h)}(B)$ for the SAQRs with and without reflection symmetry under the external electric field.

In Fig. 3 we present parts of the the electron and hole energy spectra as functions of the magnetic field for the reflection symmetrical ring ($d_h = d_V = 0$) when no electric field is applied ($F = 0$). The first crossing between two lowest-energy levels (at $B_{Ce} = 17.5$ T for the electron and $B_{Ch} = 13.5$ T for the hole) is evident and it is manifesting the first Aharonov-Bohm oscillation. In

TABLE I. Electron and hole characteristic lengths (all in nanometers).[a]

$\{d_h; d_V\}$	$\bar{x}_e$	$\bar{x}_h$	ρ_e	ρ_h
{0.0; 0.0}	0.0	0.0	8.7	8.7
{0.1; 0.0}	2.9	6.9	8.2	5.4
{0.0; 0.1}	4.6	7.6	7.4	4.5

[a] $F = 0$; for all configurations $\bar{y}_{e(h)} = 0.0$.

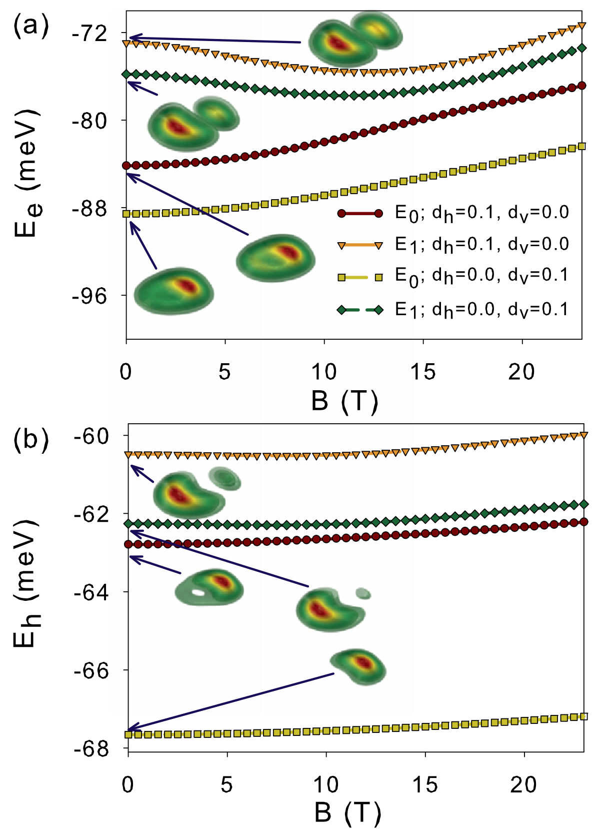

FIG. 4. Two lowest energy-levels as functions of the magnetic field for the electron (a) and hole (b) in reflection asymmetrical rings with different configurations when $F = 0$. Three-dimensional contour plots of the spatial probability distribution of the particles at $B = 0$ are shown.

addition, in this figure we plot the probability distribution $|\psi_n(\mathbf{r})|^2$ of the electron and hole wave functions in the lowest-energy states at $B = 0$. The probability distribution is reflection ringlike symmetrical (extended around the ring volume). When $d_h = d_V = 0$ and $F = 0$, the electron (hole) effective position (at zero magnetic field) in the ground state is in the center of the ring (see Table I). In addition, we present in Table I the effective lateral electron (hole) radius at zero magnetic field

$$\rho_{e(h)} = \sqrt{\left\langle\left(\mathbf{r}_\perp - \bar{\mathbf{r}}_{\perp,e(h)}\right)^2\right\rangle_{e(h)}}.$$

Clearly, in the reflection symmetrical ring the effective lateral radius is close to the ring's rim radius value for the electron (hole) and $\bar{x}_{e(h)} = \bar{x}^m_{e(h)} = 0.0$.

When two parameters (d_h or d_V) change from 0.0 (reflectional symmetrical SAQR) to 0.1 (reflectional asymmetrical SAQR) at zero lateral electric field the electron and hole energy spectra change as it is shown in Fig. 4. The lowest energy-levels do not cross. Some minor oscillations of the electron ground state energy remain (the oscillations are less evident for the holes). In the reflection asymmetrical rings the electron and hole wave functions are localized in one of the potential valleys (dotlike localization) which causes substantial changes in the particles' effective positions and radii (see Table I). For all reflection asymmetrical configurations $\bar{x}_{e(h)}$ differs from $\bar{x}^m_{e(h)}$ by less than 1% which confirms the approximation made in Eq. (2). In general (for the electron and hole), the energy spectra, probability distribution, and characteristic lengths are more sensitive to the variations of the parameter d_V. For the hole they are more receptive to the imbalance in the reflection symmetry than for the electron.

In order to see the effect of the broken reflection symmetry more clear, in Fig. 5 we plot the differential diamagnetic coefficient $A_{e(h)}(B)$. For the reflection symmetrical ring $A_{e(h)}(B)$ doubtless

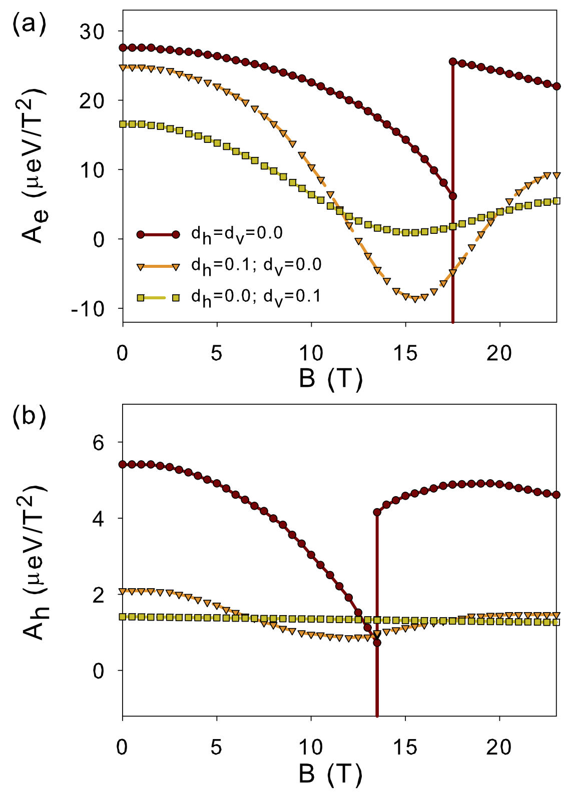

FIG. 5. Differential diamagnetic coefficient in rings with different configurations when $F = 0$.

TABLE II. Diamagnetic coefficients (all in μeV/T^2).[a]

$\{d_h; d_V\}$	α_e	α_h
{0.0; 0.0}	27.5	5.4
{0.1; 0.0}	24.6	2.1
{0.0; 0.1}	16.3	1.4

[a]For all configurations $F = 0$.

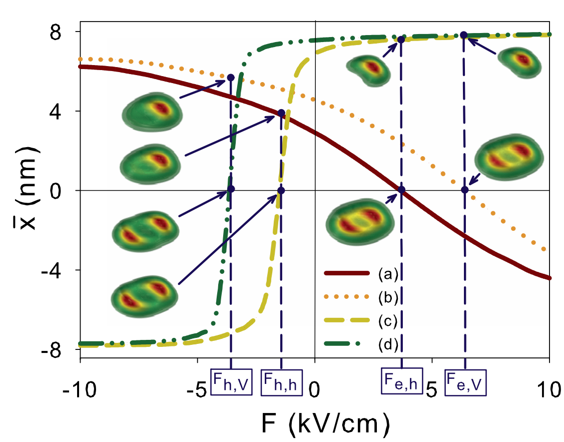

FIG. 6. Ground state effective position of the electron and hole in the reflection asymmetrical rings as functions of the lateral electric field ($B = 0$): (a) $\bar{x}_e$ in the ring with $d_h = 0.1$; $d_V = 0.0$, (b) $\bar{x}_e$ in the ring with $d_h = 0.0$; $d_V = 0.1$, (c) $\bar{x}_h$ in the ring with $d_h = 0.1$; $d_V = 0.0$, (d) $\bar{x}_h$ in the ring with $d_h = 0.0$; $d_V = 0.1$. Three-dimensional contour plots of the spatial probability distribution of the particles at $B = 0$ are shown.

TABLE III. Recovering electric fields (in kV/cm), characteristic lengths (in nanometers), and diamagnetic coefficients (in μeV/T^2).

$\{d_h; d_V\}$	F[a]	$\bar{x}_e$	$\bar{x}_h$	ρ_e	ρ_h	α_e	α_h
$\{0.1; 0.0\}$	$F_{e,h} = 3.64$	0.0	7.6	8.7	4.9	27.4	1.5
$\{0.0; 0.1\}$	$F_{e,V} = 6.36$	0.0	7.9	8.6	4.2	27.0	1.2
$\{0.1; 0.0\}$	$F_{h,h} = -1.45$	3.8	0.0	7.9	8.7	22.9	5.3
$\{0.0; 0.1\}$	$F_{h,V} = -3.64$	5.7	0.0	6.8	8.7	16.5	5.2

[a] $F_{e,h}$ ($F_{e,V}$) stands for the value of the electric field which recovers only the electron ground state wave function when $d_h = 0.1$ and $d_V = 0.0$ ($d_h = 0.0$ and $d_V = 0.1$); $F_{h,h}$ ($F_{h,V}$) stands for the value of the electric field which recovers only the hole ground state wave function when $d_h = 0.1$ and $d_V = 0.0$ ($d_h = 0.0$ and $d_V = 0.1$).

demonstrates a strong oscillation (infinite negative peak at $B_{Ce(h)}$). When the reflection symmetry of the ring is broken, due to the electron and hole wave function ringlike-dotlike transformation, the amplitude of the oscillation is significantly decreased. The amplitude is particular small (vanishing) for the hole in the SAQR with the reflection asymmetry in the potential. Fig. 5 and Table II confirm that the amplitude of the Aharonov-Bohm magnetization oscillation is obviously connected to the actual value of the diamagnetic coefficient. At the same time (as it is clearly seen from Fig. 4 and Table I), the decrease of the oscillation amplitude and diamagnetic coefficient both are direct consequences of the electron (hole) ground state wave function localization in one of the ring's potential valleys.

The shape and localization of the electron (hole) wave function can be controlled by the lateral electric field (the field applied along the x direction, see Fig. 1). The electric field pushes the

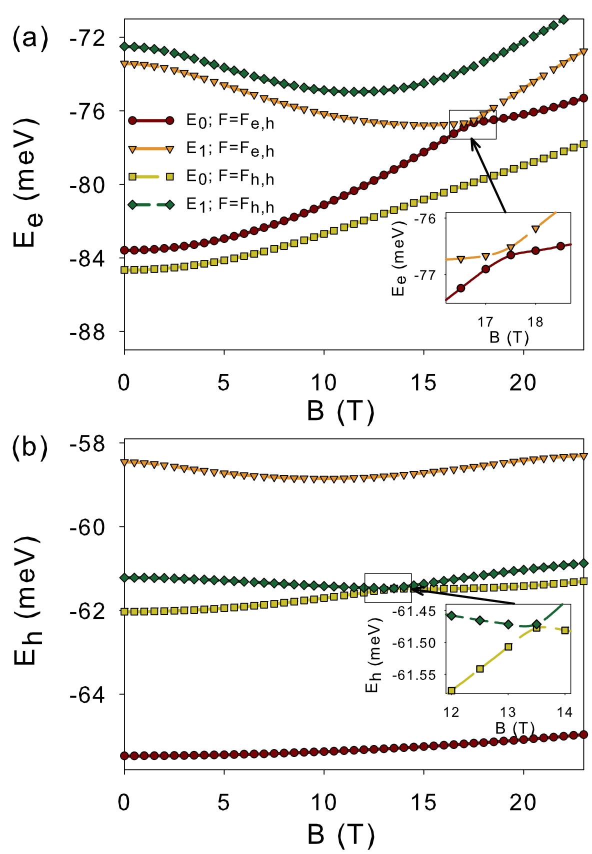

FIG. 7. Two lowest energy-levels as functions of the magnetic field for the electron (a) and hole (b) in the reflection asymmetrical ring ($d_h = 0.1; d_V = 0.0$) under application of different recovering electric fields (see Table III).

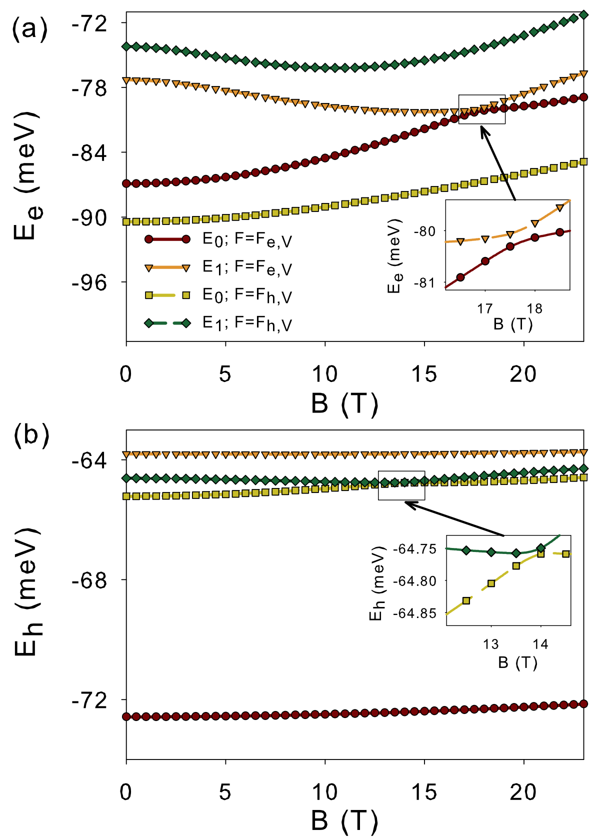

FIG. 8. Two lowest energy-levels as functions of the magnetic field for the electron (a) and hole (b) in the reflection asymmetrical ring ($d_h = 0.0; d_V = 0.1$) under application of different recovering electric fields (see Table III).

electron and hole wave functions in opposite directions along the x axis which can result in a difference in the shapes of the wave functions. In reflection asymmetrical rings, the shape of the electron wave function under certain electric field can be transformed from the dotlike (localized in one of the potential valleys) to the ringlike (distributed equally over two potential valleys), while the hole wave function becomes more localized in one of the ring's potential valleys (see Fig. 6). The hole wave function shape also can be transformed to the ringlike by reverting of the direction of the electric field (clearly, the electron wave function becomes more localized in this case). Therefore, for each specific configuration we can firmly determine the sign and magnitude of the lateral electric field ($F_{e(h),h(V)}$) which application to the ring can recover the ringlike shape of the electron or hole wave function. We note that the hole wave function shape, effective position and radius are very sensitive to the actual value of the electric field in the neighborhood of the recovery points (near $F_{h,h}$ and $F_{h,V}$ in Fig. 6). Therefore, in the reflection asymmetrical rings the recovery of the hole's diamagnetic parameters is a more delicate task than that for the electron.

The computed values of the recovering electric field, ground state effective position, and effective lateral radius for the electron (hole) in the reflection asymmetric rings at $B = 0$ are listed in Table III (see Fig. 6 in addition). We note that according to Table III, the diamagnetic coefficient of the particle with the recovered wave function is very close to that in the reflection symmetrical ring (Table II). In Fig. 7 and Fig. 8 we plot the energy spectrum of the two lowest-energy states for the electron (hole) in the reflection asymmetrical rings (separately for the $\{d_h = 0.1; d_V = 0.0\}$ and $\{d_h = 0.0; d_V = 0.1\}$ configurations) under different recovering electric fields. It is clearly seen that when the oscillation of the ground state energy of the electron is greatly recovered the hole ground state energy demonstrates rather monotonic dependence on the magnetic field (and vice versa, see also Table III). In addition, we have to note that the overall confinement potential (even under the recovering electric fields) remains asymmetrical which results in the anti-crossing between the

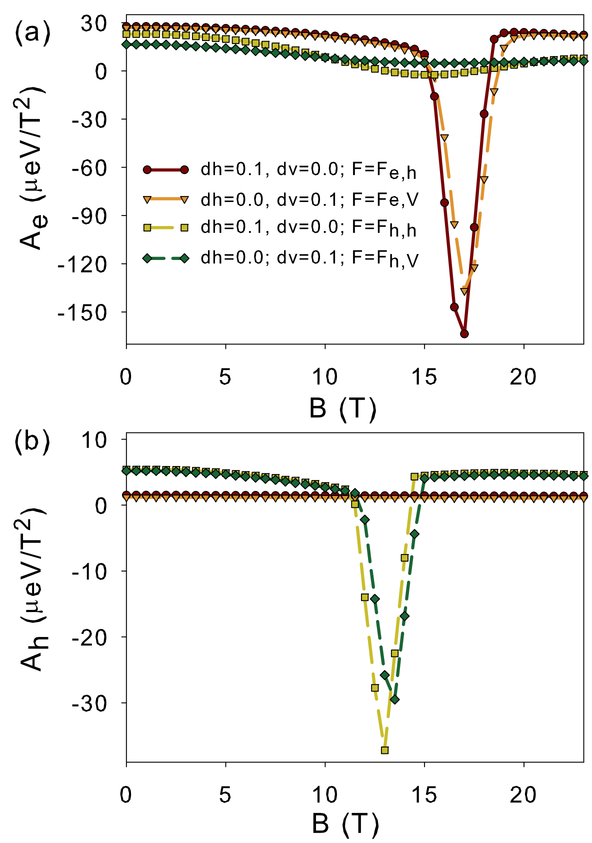

FIG. 9. Differential diamagnetic coefficient for the electron (a) and hole (b) in the rings with different configurations when the appropriate recovering electric fields are applied (see Table III).

states near B_{Ce} for the electron and B_{Ch} for the hole (unlike the crossing in the symmetrical ring) due to the asymmetry-induced hybridization of the states (see insets in Figs. 7 and 8).

Finally, Figure 9 clearly illustrates the correlation between the above described electric field manipulation of the electron (hole) wave function and recovery of the first Aharonov-Bohm oscillation of the differential diamagnetic coefficient in the reflection asymmetrical rings. A negative peak appears at $B_{Ce(h)}$ under application of the recovering electric field which is relevant to the actual configuration of the ring. The mentioned above asymmetry-induced hybridization of the electron (hole) states causes the finite amplitude and broadening of the peak. We note that the amplitude of the oscillation can be very large when the appropriate recovering electric field is applied. However, the field which recovers the electron Aharonov-Bohm diamagnetic oscillation completely suppresses that for the hole. Interestingly enough, a very small but still distinguishable oscillation of the electron differential diamagnetic coefficient can be observed for the reflection asymmetrical ring (configuration $d_h = 0.1; d_V = 0.0$) when the electric field is tuned to recover the first Aharonov-Bohm oscillation for the hole.

IV. CONCLUSION

In conclusion, we theoretically studied the impact of the broken reflection symmetry on the Aharonov-Bohm oscillation of the diamagnetic characteristics of the electron and hole in $In_cGa_{1-c}As/GaAs$ wobbled SAQRs under application of the external lateral electric field. To link the realistic geometry (topology) of a SAQR to the observation of the Aharonov-Bohm oscillations in the ring we used the effective one-band Hamiltonian, mapping method, and gauge-origin-independent definition for the magnetic vector potential. We simulated the non-interacting electron and hole energy spectra, wave functions, and diamagnetic coefficients. The ring reflection asymmetry is defined by the parameters d_h (shape asymmetry) and d_V (material content asymmetry). Therefore, the configuration $d_h = d_V = 0$ stands for the reflection symmetrical ring.

We have found that the value of the particles' diamagnetic coefficients and the amplitude of the first Aharonov-Bohm oscillation of the differential diamagnetic coefficient are associated with the actual shape of the particles' wave functions. At zero lateral electric field, when two parameters (d_h or d_V) change from 0.0 to 0.1 the shape of the electron and hole ground state wave functions change from the ringlike (distributed equally over two potential valleys) to the dotlike (localized in one of the potential valleys). This transformation causes substantial decrease of the the diamagnetic coefficient and suppression of the amplitude of the first Aharonow-Bohm oscillation.

We argue that in reflection asymmetrical rings, the shape of the electron wave function under certain electric field can be recovered from the dotlike to the ringlike. However, the hole wave function under that field becomes more localized in one of the ring's potential valleys ("more dotlike"). The hole wave function shape also can be transformed to the ringlike by reverting of the direction and adjusting the magnitude of the electric field while the electron wave function becomes more localized. The wave function transformation is followed by the recovery of the value of the diamagnetic coefficient and the first Aharonov-Bohn oscillation of the differential diamagnetic coefficient in reflection asymmetrical ring. The shape of the hole wave function is very sensitive to the actual magnitude of the electric field in the neighborhood of the recovering electric field value (substantially more sensitive than that for the electron). Therefore, in the reflection asymmetrical rings, the recovery of the hole's first Aharonow-Bohm oscillation is a more delicate task than that for the electron. In addition, the field which recovers the electron Aharonov-Bohm diamagnetic oscillation greatly suppresses that for the hole (and vice versa). Consequently, the recovering the optical Aharonow-Bohm effect in $In_cGa_{1-c}As/GaAs$ asymmetrically wobbled rings is indeed a challenging endeavor. At the same time, the observation of the recovery the Aharonov-Bohm magnetization oscillation for the individual electron even in quantum rings with broken reflection symmetry is a feasible task (with an appropriate adjustment of the magnitude and direction of the electric field).

We note that according to our results for the electrons (holes) localized in quantum rings, an appropriately large diamagnetic coefficient (when it is observed in experiments) is a reliable sign of the existence of the Aharonov-Bohm oscillations which can be confirmed by further investigations.

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