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An Alternative Formulation for Certain Fuzzy Set-Covering Problems

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Abstract—The fuzzy set-covering model using auxiliary 0-1 variables and a system of inequalities was developed by Hwang *et al.* [1]. By taking logarithm of the inequalities constraints and using the nature of the Boolean variables of this fuzzy set covering model, a simplified model is obtained.

Keywords—Set covering problems, Fuzzy set-covering problems, 0-1 linear programming, Natural logarithm function.

1. INTRODUCTION

Given two subsets $I = \{1, 2, \ldots, m\}$ and $J = \{1, 2, \ldots, n\}$ of integers. Let

$\tilde{P}_j = \{(i, \mu_j(i)) : i \in I\}$ be a fuzzy subset of I, and $\mu_j(i) \in [0,1]$ be the membership grade of $i \in I$, using the membership function μ_j of fuzzy set $\tilde{P}_j$. Hwang *et al.* [1] presented the following form of the fuzzy set-covering problem,

(P1)

$$\begin{aligned} &\text{Min} \quad \sum_{j=1}^{n} c_j x_j \\ &\text{s.t.} \quad 1 - \prod_{j=1}^{n} (1 - \mu_j(i) x_j) \geq \alpha, \qquad i = 1, 2, \ldots, m, \\ &\qquad x_j \in \{0, 1\}, \qquad j = 1, 2, \ldots, n, \end{aligned} \tag{1}$$

where

$$x_j = \begin{cases} 1, & \text{if } \tilde{P}_j \in \tilde{\wp}^*, \\ 0, & \text{otherwise.} \end{cases}$$

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Note that α, a given real number (level degree), represents the desired level which for each $i \in I$, the membership grade of i is no less than the level degree α. According to Theorem 1 in [1], Problem (P1) can be transformed to Problem (P2) by replacing the product of 0-1 variables in constraints (1) with auxiliary variables and a system of linear inequalities. The details can be found in [1].

(P2)

$$\begin{aligned}
\text{Min} \quad & \sum_{j=1}^{n} c_j x_j \\
\text{s.t.} \quad & \sum_{t=1}^{n} \mu_{j_t}(i)\, x_{j_t} + \sum_{k=2}^{n} \sum_{j_1<j_2<\cdots<j_k} (-1)^{k+1} \left(\prod_{t=1}^{k} \mu_{j_t}(i) \right) y_{j_1 j_2 \ldots j_t} \geq \alpha, \\
& i = 1, 2, \ldots, m, \\
& 2y_{j_1 j_2 \ldots j_t} \leq y_{j_1 j_2 \ldots j_{t-1}} + x_{j_t} \leq 1 + y_{j_1 j_2 \ldots j_t}, \qquad t = 1, 2, \ldots, k, \\
& y_{j_1 j_2 \ldots j_k} = \prod_{t=1}^{k} x_{j_t}, \\
& x_j \in \{0, 1\}, \qquad j = 1, 2, \ldots, n.
\end{aligned}$$

The above formulation is mathematically elegant, however, solutions are difficulty to find due to the fact that the number of decision variables is proportion to the number of auxiliary variables and extra constraints, which usually grows exponentially [2] as the number of variables increases.

2. TRANSFORMATION OF CONSTRAINTS

In (P1), since $x_j \in \{0,1\}$, obviously that $1 - \mu_j(i)x_j = (1 - \mu_j(i))^{x_j}$. It follows that $1 - \prod_{j=1}^{n}(1 - \mu_j(i) \cdot x_j) = 1 - \prod_{j=1}^{n}(1 - \mu_j(i))^{x_j}$. Therefore, the constraint (1) can be rewritten as

$$1 - \alpha \geq \prod_{j=1}^{n} (1 - \mu_j(i))^{x_j}. \tag{2}$$

By taking the natural logarithm function on both sides of inequality (2) and changing the signs, (2) can be transformed into a linear constraint as,

$$-\ln(1-\alpha) \leq \sum_{j=1}^{n} x_j \left[-\ln(1 - \mu_j(i)) \right]. \tag{3}$$

Notice that $\alpha = 1$, i.e., the crisp cases, is not considered here. Therefore, (P1) can be rewritten as follows.

(P3)

$$\begin{aligned}
\text{Min} \quad & \sum_{j=1}^{n} c_j x_j \\
\text{s.t.} \quad & \sum_{j=1}^{n} x_j \left[-\ln(1 - \mu_j(i)) \right] \geq -\ln(1-\alpha), \\
& i = 1, 2, \ldots, m, \\
& x_j \in \{0, 1\}, \qquad j = 1, 2, \ldots, n,
\end{aligned}$$

where $0 < \alpha < 1$.

One can easily see the number of variables, and hence, the number of inequalities, is reduced greatly after (P3) is obtained from (P2) by applying the natural logarithm to the constraints of (P2).

3. AN EXAMPLE AND CONCLUDING REMARKS

The same numerical example as in [1] is used to illustrate the advantages of the proposed method. Table 1 shows the matrix of $\mu_j(i)$, $i = 1, 2, \ldots, 5$, $j = 1, 2, \ldots, 4$.

Table 1. The matrix of $\mu_j(i)$ [1].

Means \ Goals	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$
j=1 (c_1=4)	0.4	0.1	0.5	0.7	0.8
$j = 2$ ($c_2 = 3$)	0.1	0.3	0.8	0.2	0.6
$j = 3$ ($c_3 = 5$)	0.3	0.7	0.2	0.9	0.4
$j = 4$ ($c_4 = 2$)	0.5	0.9	0.4	0.1	0.2

According to (P3), we obtain the following mathematical programming,

$$\begin{aligned}
\text{Min}\quad & 4x_1 + 3x_2 + 5x_3 + 2x_4 \\
\text{s.t.}\quad & 0.51x_1 + 0.11x_2 + 0.36x_3 + 0.69x_4 \geq 0.69, \\
& 0.11x_1 + 0.36x_2 + 1.20x_3 + 2.30x_4 \geq 0.69, \\
& 0.69x_1 + 1.61x_2 + 0.22x_3 + 0.51x_4 \geq 0.69, \\
& 1.20x_1 + 0.22x_2 + 2.30x_3 + 0.11x_4 \geq 0.69, \\
& 1.61x_1 + 0.92x_2 + 0.51x_3 + 0.22x_4 \geq 0.69,
\end{aligned}$$

$$x_j \in \{0, 1\}, \quad j = 1, 2, \ldots, 4.$$

Note that the coefficients in the constraints are obtained by taking the natural logarithm of the corresponding values in the table 1 and $\alpha = 0.5$ is set for the value of the right-hand sides of the constraints. By using software LINGO, an optimal solution is obtained,

$$x_1^* = 1, \qquad x_2^* = 0, \qquad x_3^* = 0, \qquad x_4^* = 1.$$

From the example, obviously, we can see the reduction of the formulation. With the proposed technique, the fuzzy set-covering model does not require additional 0-1 variables and constraints which complicate the formulation. Consequently, the model may be easily applied to many large-scale problems in the real world.

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