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Influence of an external electric field on high- T_c superconductivity

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Abstract

The surface superconductivity induced by applying an electric field perpendicular to the surface of a high- $T_{\rm c}$ superconductor covered by an insulator and a gate electrode is studied theoretically. Taking the idea into account that the change of the surface carrier density confined in a thin screening layer modulates the transport properties of superconductors, we develop a simple method based on the Chen and Yang model to study the $T_{\rm c}$ shift dependence on the applied voltage in the frameworks of a Thomas–Fermi approximation and BCS theory. In such a case a novel boundary condition is introduced to modify the G–L equations. Solving the modified one, the $T_{\rm c}$ shift derived by the present model is similar in qualitative behavior to the Shapiro result, but much simpler in form. In addition, it is shown to be proportional to the applied field (the square of applied field) for thin film (thick film) for a given material. Based on our simple results, the critical current density $J_{\rm c}$ and nucleation field $H_{\rm c3}$ as functions of the applied voltage are easily derived.

1. Introduction

The surface superconductivity induced by an electric field is a very important subject [1] for studies in the hope that the transport properties of superconductors are to be controlled by applying an electric field. For more than thirty years, since the first experiment on the electric-field effect in superconductors was made by Glover and Sherill as early as 1960 [2], there have been many efforts [3–10] to attempt to build a superconducting field-effect transistor for this purpose. However, the observed field effects for conventional superconducting metals were so small

Assuming that the BCS theory can apply to the present high- T_c oxide superconductor [11]. Shapiro et al. have calculated the critical temperature in the presence of an electric field. However, their results

as to be neglected. This is ascribed to the long coherence length ξ and the large concentration of the charge carriers, which leads to the small electric-field penetration length $l_{\rm D}$ and the small ratio of $l_{\rm D}$ to ξ . Recently the discovery of high- $T_{\rm c}$ superconductors was expected to open up new possibilities to study the electric-field effects because of the relatively low mobile carrier density and anomalously short coherence length. Further, their small coherence lengths are helpful for dealing with the technological difficulties in the fabrication of ultrathin superconducting films in which the total carrier density can be modulated by a substantial proportion.

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are complicated in form and thus the numerical calculations are essential to understand the influence of the external electric field on superconductivity. In order to obtain a simple analytic solution, in this paper we reexamine the electric-field effect and attempt to develop a new model. Now, we consider the superconducting transport properties to depend sensitively on the carrier density according to the BCS model and take them to be possibly modulated by applying an electric field because the carrier density is changed [12] on the surface characterized by the Debye screening length l_D . High- T_c oxide superconductors have a low carrier concentration, giving $l_{\rm D}$ $\sim 5-10$ Å and a very small coherence length. Thus, the critical temperature on the surface called T_{cs} is different from the intact T_{co} of the bulk material. Although the situation is similar in outward appearance to the assumptions made by Chen and Yang [13], it should be emphasized here that it is different in nature. Namely, from the technical point of view it is valuable to either suppress or enhance T_{cs} by controlling the direction of the external electric field [7], but not in the Chen and Yang [13] case, which is based on the theoretical results found by Valls et al. [14,15], that is, the pair potential is not depleted but enhanced up to 25% near the surface. To calculate the voltage dependences of the superconducting properties, the changes of the critical temperature resulting from the external electric field are only considered in our analysis, not against the background of the Chen and Yang model. However, it is worth mentioning here that in this work, the T_c shift, J_c and H_{c3} are derived by the three-dimensional free-electron model and Thomas-Fermi approximation [9-11].

2. Theory

For a discussion of potential device applications such as high- T_c effect transistors (the basic configuration is a structure with a superconducting film of thickness L separated from a gate electrode by an insulating barrier of thickness d), now there is a common point of view [8,16] that the electric-field effect is due to the energy band bending and depletion or enhancement of charge carriers near the interface in just the same way as in semiconductors.

So the surface charges induced by the gate voltage $V_{\rm g}$ is given by

$$\Delta \sigma = \varepsilon (V_{g} - \phi_{s})/(4\pi d)$$

$$= e\Delta N, \qquad (1)$$

where ε is the dielectric constant of the insulating barrier, ϕ_s is the electric potential at the surface of superconductors, and ΔN is the induced surface carrier density. Here the high- T_c superconductor is regarded as a three-dimensional free-electron system because in general the field effects observed are in agreement with the most simple free-carrier prediction [12,17,18]. According to the Poisson equation in the framework of the Thomas–Fermi approximation, the potential $\phi(x)$ of the field penetrating the sample is described by

$$\phi(x) = \phi_s \exp(-x/l_D), \tag{2}$$

$$l_{\rm D}^2 = \varepsilon_0 E_{\rm F} / \left(6\pi n_0 e^2\right),\tag{3}$$

where $l_{\rm D}$ is the Debye screening length, defining the region where the field-induced effect occurs, n_0 is the carrier density far from the surface, $E_{\rm F}$ is the Fermi energy, and ε_0 is the dielectric constant of the superconductor. Thus the difference in mobile charges can be expressed by

$$\delta n(x) = n(x) - n_0$$

= $3n_0 e\phi(x)/2E_F$. (4)

Based on conservation of charge, the whole induced charge of the system is zero. This means that

$$\Delta \sigma = \int_0^L e \delta n(x) dx = 3n_0 e^2 \phi_s l_D / 2E_F.$$

So, we have

$$\phi_{\rm s} = V_{\rm g} / \left(1 + \frac{\varepsilon_0 d}{\varepsilon l_{\rm D}} \right) \propto V_{\rm g}$$

and

$$\Delta N/N_0 = 3e\phi_{\rm s}l_{\rm D}/2E_{\rm F}L,$$

where $N_0 = n_0 L$ is the surface carrier density without applying electric field. L is assumed to be much larger than l_D . In fact, this assumption is in general satisfied with the experimental sample dimensions.

Because the critical temperature is sensitive to the carrier density n and n is nearly the same in the normal and superconducting states [17], the surface

parameter $T_c[n(x)]$ induced by the external electric field is a function of coordinate x and can be expressed as [19]

$$T_{c}(n(x))$$

$$= T_{c}(n_{0}) \left[1 + (\partial \ln T_{c}/\partial \ln n)_{n=n_{0}} (\delta n/n_{0}) \right]$$
(5)

Although the mechanism of superconductivity for high- $T_{\rm c}$ oxide superconductors is still unclear, for simplicity, the BCS theory is taken into account in this paper. In the BCS model, superconductivity is based upon the existence of a net attractive interaction V between the electrons in a narrow energy range near the Fermi surface. This produces a critical temperature given by

$$T_{\rm co}[=T_{\rm c}(n_0)] = 1.14\omega_{\rm D} \exp(-1/g_0),$$
 (6)

where $T_{\rm co}$ is the bulk critical temperature, $\omega_{\rm D}$ is Debye frequency and $g_0 = N(0)V$ is the intrinsic electron—phonon coupling constant depending on the materials. Here we assume that V is independent of the electric field, and the density of states N(0) far from the surface is related to the carrier density in the three-dimensional free-electron model. That is to say, N(0) is proportional to $n^{1/3}$. Thus,

$$T_{\rm c}[n(x)] = T_{\rm co}[1 + (e\phi_{\rm s}/2E_{\rm F}g_0) \exp(-x/l_{\rm D})].$$
 (7)

This indicates that the change of the local transition temperature is only within the screening length $l_{\rm D}$. Thus, it is reasonable to treat the superconducting film as consisting of two component parts: one is the surface layer affected by the applied field and the other is the intact bulk. In such a case, we naturally introduce the parameter T_{cs} representing the average critical temperature on the surface characterized by $l_{\rm D}$. Here it is worth noting that in spite of the fact that $l_{\rm D}$ is comparable to the coherence length $\xi_0(T)$ = 0 K) of high- T_c superconductors; for example [1], $l_{\rm D} = 5$ Å, $\xi_0 = 5$ Å for YBaCuO, and $l_{\rm D} = 5$ Å, $\xi_0 = 3$ Å for BiBaSrCuO, the order parameter within the screening length from the interface is nearly a constant due to the divergence of the Ginzburg-Landau coherence length $\xi(T)$ at the critical temperature in the mean-field approximation. Thus, the calculation of the parameter T_{cs} , hereafter called the

surface critical temperature, is easily done by simple averaging $T_{\rm c}[n(x)]$ over $l_{\rm D}$. It is expressed as

$$T_{cs} - T_{co} = \int_{0}^{L} \{ T_{c} [n(x)] - T_{co} \} dx / l_{D}$$

$$\approx (e\phi_{s} / 2E_{F}g_{0}) T_{co}.$$
(8)

The last expression indicates the order of magnitude. In such a case, to obtain the $T_{\rm c}$ shift of the whole superconductor, according to our analysis of the effect of the electric field, a novel boundary condition introduced to modify the G-L equations should be considered, [13], as in the theory proposed by Chen et al. Here it should be emphasized again that in the following paragraphs our consideration is valid under the condition that both the sample size L and the GL coherence length $\xi(T)$ are much larger than $I_{\rm D}$.

First, we assume that the external electric field only acts on the surface layer characterized by $l_{\rm D}$ and $T_{\rm cs}$. Thus, based on the Chen and Yang model [13], the $T_{\rm c}$ shift dependence on the applied voltage can be described by

$$\frac{-\hbar^2}{2m} \frac{d^2 \psi}{dx^2} + \alpha (T - T_{\infty}) \psi + \beta \psi^3 = 0,$$
 (9)

$$\frac{1}{\psi} \frac{\mathrm{d}\psi}{\mathrm{d}x} \bigg|_{x=0} = -\frac{2m\gamma}{\hbar^2},\tag{10}$$

and

$$\frac{1}{\psi} \frac{\mathrm{d}\psi}{\mathrm{d}x} \bigg|_{x=L} = 0,\tag{11}$$

where $\gamma = l_{\rm D} \alpha (T_{\rm cs} - T_{\rm co})$, and α , β are material parameters. In the vicinity of the critical temperature $T_{\rm c}$ the order parameter ψ is very small resulting in the term ψ^3 being neglected. Thus $T_{\rm c}$ is determined by

$$\tanh \left\{ (L/\hbar) \left[2m\alpha (T_{c} - T_{co}) \right]^{1/2} \right\}$$

$$= (\gamma/\hbar) \left\{ 2m/\left[\alpha (T_{c} - T_{co}) \right] \right\}^{1/2}, \qquad (12)$$
and $\xi(T_{c}) = \hbar [2m\alpha (T_{c} - T_{co})]^{-1/2}; \text{ we have}$

$$T_{c} = T_{co} \left[1 + \left(e\phi_{s}/2E_{F}g_{0} \right)^{2} \left(l_{D}/\xi_{0} \right)^{2} \right]$$

$$= T_{co} \left[1 + \left(\frac{\Delta N}{N_{0}} \right)^{2} \left(\frac{L}{\xi_{0}} \right)^{2} \frac{1}{9g_{0}^{2}} \right]$$
for $L \gg \xi(T_{c})$ (13)

and

$$\begin{split} T_{\rm c} &= T_{\rm co} \big[1 + \big(e \phi_{\rm s} / 2 E_{\rm F} g_0 \big) (l_{\rm D} / L) \big] \\ &= T_{\rm co} \bigg[1 + \frac{1}{3 g_0} \bigg(\frac{\Delta N}{N_0} \bigg) \bigg] \qquad \text{for } L \ll \xi(T_{\rm c}) \,. \end{split}$$
(14)

However, it is well known that the framework of the GL theory is valid under the condition $(T-T_{co})/T_{co} \ll 1$. Therefore, it must constantly be kept in mind that the basic requirement for our theory to work, is that the ratio of the applied electric field to the Fermi energy, i.e. $e\phi_s/(2E_Fg_0)$, must be much smaller than 1.

On the basis of the above discussion, the T_c shift in Eqs. (13) and (14) in relation to the parameter $l_{\rm D}/\xi_0$ ($l_{\rm D}/L$) for the thick film (thin film) is physically reasonable because although the carrier density is changed only in the thin surface layer, all the Cooper pairs within the coherence length will feel the perturbation. A similar result was obtained by Kechiantz [16]; let us discuss this in detail. On the one hand, if the framework considered in this paper is valid, the T_c shift is proportional to T_{co} from Eqs. (13) and (14). On the other hand, the relations between the T_c shift and intrinsic material parameters $l_{\rm D}$, ξ_0 , $E_{\rm F}$ and g_0 would clearly point out why we have greater expectations of high- T_c superconductors than conventional superconductors for building superconducting electronics controlled by an electric field. Finally it should be noted that according to the expression of $\xi(T_c)$, if the electric field is weak enough to lead the T_c shift ΔT_c (i.e., $T_c - T_{co}$) approach zero, ΔT_c is proportional to $\Delta N/N_0$ for $L \ll \xi(T_c)$. On the other hand, if the electric field is strong enough to lead to $L \gg \xi(T_c)$, ΔT_c is proportional to $(\Delta N/N_0)^2$. Therefore, we come to the conclusions that

(1) for a given L, the linear relation between ΔT_c and $\Delta N/N_0$ is no longer valid with increasing field, (2) the larger the film thickness L is, the faster it deviates from the linear relation. This is consistent with the experimental results observed by Xi et al. [12]. Comparing the T_c shift derived by Shapiro et al. [11] with ours, we replot the Fig. 3 in Ref. [11] secundum Eqs. (13) and (14) obtained from our model and present it in Fig. 1. As shown in this

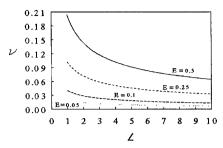


Fig. 1. Reduced critical temperature $v=((4l_{\rm D}^2\tau)/\xi_0^2)^{1/2}$ vs. reduced thickness of the film $L=L/2l_{\rm D}$ for various reduced external electric fields $E=((4l_{\rm D}^2\kappa)/(g\xi_0^2))^{1/2}$, where $\tau=(T_{\rm c}-T_{\rm co})/T_{\rm co}$, $\kappa=(3e\phi_{\rm s}/2E_{\rm F})$, $l_{\rm D}=7$ Å, $\xi_0=10$ Å, and $g_0=1$ are considered. These curves (present work) are similar in qualitative behavior to the ones obtained by Shapiro et al. in Ref. [11].

figure, an interesting result is to be noted. Although, according to our assumptions made in the beginning of our analysis, the small-dimension cases, i.e. the ones with sample thickness L comparable to $l_{\rm D}$, are physically unreasonable, our results for the $T_{\rm c}$ shift extended to $L=2\,l_{\rm D}$ are still similar at least in qualitative behavior to Shapiro's, particularly at the low-field regime. In addition, our $T_{\rm c}$ shift formulae are much simpler in form. Besides the $T_{\rm c}$ shift, both the response of the critical current density and nucleation field $H_{\rm c3}$ to the electric field are also the essential subjects of study from a practical point of view. Based on the formulae for the $T_{\rm c}$ shift obtained from our simple model, they are derived as follows. (a) Critical current density in a thin film:

If $L \ll \xi(T)$, we have the critical current density from the de Gennes theory [20,21] as

$$J_{c}(T) = 2^{5/2} \alpha^{3/2} e (T_{c} - T)^{3/2} / (3^{3/2} m^{1/2} \beta);$$
(15)

then, the ratio of the critical current density after applying the electric field to before applying it is given by

$$J_{c}(T)/J_{co}(T)$$

$$= [(T_{c} - T)/(T_{co} - T)]^{3/2}$$

$$= \{1 + (e\phi_{s}/2E_{F}g_{0})[T_{co}/(T_{co} - T)]$$

$$\times (l_{D}/L)\}^{3/2}.$$
(16)

As reported for the field effects in Refs. [22] and [23] it is consistent with Eq. (16) that by applying an

electric field, a relatively large change of the $J_{\rm c}(T)$ values in a thin film occurs in the range where the temperature T approaches the intrinsic bulk critical temperature $T_{\rm co}$, even with reduced $T_{\rm c}$ shift; i.e. $(T_{\rm c}-T_{\rm co})/T_{\rm co}$, is very small.

(b) Nucleation field H_{c3} :

The nucleation field $H_{\rm c3}$ should be modified because the surface boundary condition [24] has been changed by applying field. In this case, the G-L equation with applied magnetic field $H = H\hat{e}_z$ is given by

$$-\frac{\hbar^2}{2m} \frac{d^2 \psi}{d x^2} + \frac{1}{2} m \omega^2 (x - x_0)^2 \psi$$

= $-\alpha (T - T_{co}) \psi$, (17)

$$\frac{1}{\psi} \frac{\mathrm{d}\psi}{\mathrm{d}x} \bigg|_{x=0} = -\frac{2m\gamma}{\hbar^2},\tag{18}$$

where $\omega = 2 eH/m$ and $x_0 = \hbar k_y/(2 eH)$, and here we assume $A = (0, H_x, 0)$.

The term $\frac{1}{2}m\omega^2(x-x_0)^2$ approximates zero, while T is close to T_c . Therefore the trial function $\psi(x)$ is taken as $\exp(-\lambda x)$. Substituting this trial function into Eq. (18) and multiplying Eq. (17) by $\psi(x)dx$, then integrating over x from 0 to ∞ , we have $\lambda = 2m\gamma/\hbar^2$ and

$$-\alpha (T - T_{co})$$

$$= \left[\int_0^\infty \left[\frac{-\hbar^2}{2m} \left(\frac{2m\gamma}{\hbar^2} \right)^2 + \frac{1}{2} m \omega^2 (x - x_0)^2 \right] \right]$$

$$\times \exp \left(-\frac{4m\gamma x}{\hbar^2} \right) dx$$

$$\times \left[\int_0^\infty \exp \left(-\frac{4m\gamma x}{\hbar^2} \right) dx \right]^{-1}.$$

Varying x_0 for the minimum of $-\alpha(T-T_{co})$, the result is found that $x_0 = \hbar^2/4m\gamma$. Then

$$\omega^{2} = \frac{64m^{2}\gamma^{4} - 32m\hbar^{2}\gamma^{2}\alpha(T - T_{co})}{\hbar^{6}}$$

$$= \frac{32m\gamma^{2} \left[2m\gamma^{2} - \hbar^{2}\alpha(T - T_{co})\right]}{\hbar^{6}}$$
(19)

Let us focus on H_{c3} for the thickness of the superconducting film being much larger than $\xi(T_c)$. From Eq. (12), H_{c3} is given by

$$H_{c3} = \frac{m\alpha\phi_{s}}{\hbar E_{F}g_{0}} \left(\frac{l_{D}}{\xi_{0}}\right) \left[T_{co}(T_{c} - T)\right]^{1/2}.$$
 (20)

In calculating the nucleation field H_{c3} modified by applying the electric field, we have obtained Eq. (20) in which H_{c3} is proportional to ϕ_s (here $\phi_s \propto V_g$) limited by the condition that $e\phi_s/2E_Fg_0$ is much smaller than one. On the other hand, a special behavior of H_{c3} should be pointed out, i.e. the resulting H_{c3} is proportional to $(T_c - T)^{1/2}$ different from the result of de Gennes theory [20,21] in which H_{c3} is proportional to $(T_c - T)$. Finally, experimental confirmation of these theoretical results is necessary to support the research work for the effects of an electric field on superconductivity.

3. Conclusion

To apply an electric field perpendicular to the surface of high- T_c superconductors, the state that the surface T_{cs} is different from the bulk T_{co} assumed by Chen and Yang [13] can be achieved. Thus according to their idea, a simple model is developed by us to study field effects on superconductivity in the frameworks of BCS theory and the three-dimensional free-electron model. Each of the formulae for the T_c shift, critical current density J_c and nucleation field H_{c3} depending on the applied voltage with an analytic form is easily obtained. In addition, our formula for the $T_{\rm c}$ shift is much simpler in form than the results derived by Shapiro et al., but they are similar to each other in qualitative nature even if extended to the cases of small sample dimension. On the other hand, the evidence for our model calculations predicting the relations between the T_c shift and $\Delta N/N_0$, and the dependence of the critical current density J_c on temperature has been supplied by results observed in Refs. [12] and [22], respectively. Thus, in such a case we can draw the conclusion from the implications of our formulae that if the framework considered in this paper is valid, it should be possible to achieve higher values of T_c by designing compounds in which not only both the coherence length and carrier density should be taken as much as possible towards smaller values, but also the product of $E_{\rm F}$ by g_0 should be.

However, the mechanism of superconductivity for high- T_c materials still eludes all research. It has been commonly suggested that the BCS theory cannot apply to the present oxide superconductors, while some other people support it. On the other hand, in quasi-two-dimensional high- T_c oxide materials, the validity of the three-dimensional free-electron model for charge screening is also in question. Hence, the basic assumptions of all these discussions in this paper for field effects require further experimental investigation. Very recently, experimental results showed that T_c is proportional to n_s/m in all the planar cuprate superconductors and the carrier density is nearly the same [17] in the normal and superconducting states. Based on these experimental results and assuming that the effective mass is independent of the field, the same calculations for the T_c shift, J_c and H_{c3} can also easily be done according to our proposed model.

To sum up, it should be emphasized that in this paper the BCS theory applied to the present oxide superconductors may be inappropriate, but the method developed in this work may provide a simple and powerful tool to help the understanding of physical properties of high- $T_{\rm c}$ superconductors and the advances in materials sciences to build superconducting three-terminal devices.

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