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Cost Analysis of a vacation machine repair model

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Abstract

This paper investigates a machine repair problem with homogeneous machines and standbys available, in which multiple technicians are responsible for supervising these machines and operate a (R, V, K) synchronous vacation policy. With such a policy, if any V idle technicians exist in the system, these V (V < R) technicians would take a synchronous vacation. Upon returning from vacation, they would take another vacation if there is no broken machine waiting in the queue. This pattern continues until at least one failed machine arrives. It is assumed that the number of teams/groups on vacation is less than or equal to K ($0 \leqq KV < R$). The matrix analytical method is employed to obtain a steady-state probability and the closed-form expression of the system performance measures. Efficient approaches are performed to deal with the optimization problem of the discrete / continuous variables while maintaining the system availability at a specified acceptable level.

Keywords: machine repair models; synchronous multiple vacation; standby;[1]

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1. Introduction

In many industrial processes, production machines are unreliable and may have a breakdown. When a machine fails, it is sent to a maintenance facility and repaired by a group of technicians (servers). In order to achieve the production quota and reduce the loss of production capacity, the plant usually keeps standby machines that can substitute for a failed machine. In this paper, a machine repair problem, which includes M identical machines, S standby machines, and R technicians with synchronous multiple vacation policy is investigated. There are numerous researches on the machine repair problem or the multi-server queueing system with various vacation policies.

The objectives of this paper are as follows: 1) provide a matrix-analytical computational algorithm to develop the steady-state probability vectors; 2) derive the steady-state availability, and other system performance measures; 3) construct a cost model to determine the optimal number of technicians (servers), the optimal vacation policy, the optimal service rate, and the optimal vacation rate; 4) conduct numerical study on the effect of parameters on the system characteristics.

2. The system

This paper considers a multi-server machine repair problem with a synchronous multiple vacation policy and standby. There are M operating machines, S standby machines, and R technicians (servers) in this system. The detailed descriptions and assumptions of this model are given as follows:

1. M operating machines are required for the function of the system. In other words, the system is short only if $S+1$ (or more) machines fail.
2. Operating machines are subject to breakdowns, according to an independent Poisson process, with rate λ. When an operating machine breaks down, it is immediately backed up by an available standby.
3. Each of the standby machines fails independently of the others with Poisson rate α, where ($0 \le \alpha \le \lambda$). When a standby machine moves into an operating state, its characteristics are the same as an operating machine.
4. Failed machines in the system form a single waiting line and receive repair in the order of their breakdown, i.e. FCFS discipline. The service time provided by each technician is an independent and identically distributed exponential random variable with rate μ.
5. When a failed machine is repaired, it enters into a standby state unless the system is short, then the repaired machine would be sent back to an operating state.
6. Each technician can repair only one failed machine at a time, and a failed machine arriving at the repair facility where all technicians are busy or on vacation must wait in the queue until a technician is available.
7. When there are any V idle technicians, they take a synchronous multiple vacation. Upon returning from the vacation, they would take a vacation again if there are no fail machines

waiting in the queue. The number of teams/groups on synchronous vacation is restricted no more than K ($1 \le K \le \lceil R/V \rceil - 1$) at any time.

8. The vacation time of each team/group has an exponential distribution with parameter θ. The various stochastic processes involved in this system are independent of each other.

It should be noted that the inequality equation, $1 \le K \le \lceil R/V \rceil - 1$, means that it is not allowed to have all technicians (servers) on vacation at any time. Therefore, the vacation policy introduced by this study, the (R, V, K) synchronous multiple vacation policy, is a vacation policy without exhausting the servers, which is different from the vacation polices in literature, but closer to practical use than past studies.

3. Steady-state results

For the multi-server machine repair model, with a (R, V, K) synchronous multiple vacation policy and standby machines, the state of the system can be described by the pairs $\{(i,n): i=R, R-V, R-2V,..., R-KV$, and $n=\max\{i-V+1,0\},...,M+S\}$, where i denotes the number of operating (not on vacation) technicians in the system, and n represents the number of failed machines in the system. The mean failure rate λ_n and mean repair rate μ_n for this system are given by:

$$\lambda_n = \begin{cases} M\lambda + (S-n)\alpha, & 0 \le n \le S; \\ [M-(n-S)]\lambda, & S \le n \le M+S; \\ 0 & \text{, otherwise,} \end{cases}$$

and

$$\mu_n = \begin{cases} n\mu, & 1 \le n \le R; \\ 0 & \text{, otherwise.} \end{cases}$$

In the steady-state, the following notations are used:

$P_{i,n} \equiv$ probability that there are n failed machines in the system when there are i operating technicians in the system ($R-i$ technicians are on vacation).

where $i=R, R-V,..., R-(K-1)V, R-KV$, $n=\max\{i-V+1,0\},...,M+S$.

Steady-state equations

Applying Markov process, the steady-state equations for multiple-server machine repair problems, with standby under a (R, V, K) synchronous multiple vacation policy, are obtained as follows.

(1) $i=R-KV$

$$\lambda_0 P_{R-KV,0} = \mu_1 P_{R-KV,1} \tag{1}$$

$$(\lambda_n+\mu_n)P_{R-KV,n}=\lambda_{n-1}P_{R-KV,n-1}+\mu_{n+1}P_{R-KV,n+1},\ 1\le n\le R-KV-1 \tag{2}$$

$$(\lambda_{R-V}+\mu_{R-V})P_{R-KV,R-V}=\lambda_{R-KV-1}P_{R-KV,R-V-1}+\mu_{R-KV}P_{R-KV,R-KV+1} +\mu_{R-KV+1}P_{R-(K-1)V,R-KV+1} \tag{3}$$

$$(\lambda_n+\mu_n+K\theta)P_{R-KV,n}=\lambda_{n-1}P_{R-KV,n-1}+\mu_{R-KV}P_{R-KV,n+1},\ R-KV+1\le n\le M+S-1 \tag{4}$$

$$(\mu_{R-KV}+K\theta)P_{R-KV,M+S} = \lambda_{M+S-1}P_{R-KV,M+S-1} \tag{5}$$

(2) $R-(K-1)V\le i\le R-V$

$$(\lambda_{R-(i+1)V+1}+\mu_{R-(i+1)V+1})P_{R-iV,R-(i+1)V+1} = (i+1)\theta P_{R-(i+1)V,R-(i+1)V+1} +\mu_{R-(i+1)V+2}P_{R-iV,R-(i+1)V+2} \tag{6}$$

$$(\lambda_n+\mu_n)P_{R-iV,n}=(i+1)\theta P_{R-(i+1)V,n}+\lambda_{n-1}P_{R-iV,n-1}+\mu_{n+1}P_{R-iV,n+1},\quad R-(i+1)V+2\le n\le R-iV-1 \tag{7}$$

$$(\lambda_{R-iV}+\mu_{R-iV})P_{R-iV,R-iV}=(i+1)\theta P_{R-(i+1)V,R-iV}+\lambda_{R-iV-1}P_{R-iV,R-iV-1} +\mu_{R-iV}P_{R-iV,R-iV+1}+\mu_{R-iV+1}P_{R-(i-1)V,R-iV+1} \tag{8}$$

$$(\lambda_n+\mu_{R-iV}+i\theta)P_{R-iV,n}=(i+1)\theta P_{R-(i+1)V,n}+\lambda_{n-1}P_{R-iV,n-1}+\mu_{R-iV}P_{R-iV,n+1}\quad R-iV+1\le n\le M+S-1 \tag{9}$$

$$(\mu_{R-iV}+i\theta)P_{R-iV,M+S} = (i+1)\theta P_{R-(i+1)V,M+S}+\lambda_{M+S-1}P_{R-iV,M+S-1} \tag{10}$$

(3) $i=R$

$$(\lambda_{R-V+1}+\mu_{R-V+1})P_{R,R-V+1} = \theta P_{R-V,R-V+1}+\mu_{R-V+2}P_{R,R-V+2} \tag{11}$$

$$(\lambda_n+\mu_n)P_{R,n}=\theta P_{R-V,n}+\lambda_{n-1}P_{R,n-1}+\mu_{n+1}P_{R,n+1},\ R-V+2\le n\le R-1 \tag{12}$$

$$(\lambda_n+\mu_R)P_{R,n}=\theta P_{R-V,n}+\lambda_{n-1}P_{R,n-1}+\mu_R P_{R,n+1},\ R\le n\le M+S-1 \tag{13}$$

$$\mu_R P_{R,M+S} = \theta P_{R-V,M+S}+\lambda_{M+S-1}P_{R,M+S-1} \tag{14}$$

There is no way of solving (1)-(14) in a recursive manner in order to develop explicit expressions for steady-state probabilities. In the next section, this study provides a matrix-analytic method to address this problem.

Matrix-analytical solutions

To analyze the resulting system of linear equations (1)-(14), a matrix-analytic approach is used. Following concepts by Neuts (1981), in order to represent the steady-state equations in a matrix-form, the transition rate matrix $\mathbf{Q}$ (the coefficient matrix) of this Markov chain can be partitioned as follows:

$$\mathbf{Q}=\begin{pmatrix} \mathbf{A}_K & \mathbf{B}_{K-1} & & & & & \\ \mathbf{C}_K & \mathbf{A}_{K-1} & \mathbf{B}_{K-2} & & & & \\ & \mathbf{C}_{K-1} & \mathbf{A}_{K-2} & \mathbf{B}_{K-3} & & & \\ & & \ddots & \ddots & \ddots & & \\ & & & \mathbf{C}_3 & \mathbf{A}_2 & \mathbf{B}_1 & \\ & & & & \mathbf{C}_2 & \mathbf{A}_1 & \mathbf{B}_0 \\ & & & & & \mathbf{C}_1 & \mathbf{A}_0 \end{pmatrix}. \tag{15}$$

Matrix $\mathbf{Q}$ is a square matrix of order $K(M+S-R)-K(K+1)/2+(M+S+1)$, and each entry of the matrix $\mathbf{Q}$ is listed in the following:

$$\mathbf{A}_K=\begin{pmatrix} * & \lambda_0 & & & & & & \\ \mu_1 & * & \lambda_1 & & & & & \\ & \mu_2 & * & \lambda_2 & & & & \\ & & \ddots & \ddots & \ddots & & & \\ & & & \mu_{R-KV} & * & \lambda_{R-KV} & & \\ & & & & \ddots & \ddots & \ddots & \\ & & & & & \mu_{R-KV} & * & \lambda_{M+S-1} \\ & & & & & & \mu_{R-KV} & * \end{pmatrix}_{(M+S+1)\times(M+S+1)} . \tag{16}$$

For $i=1,2,\ldots,K-1$,

$$\mathbf{A}_i=\begin{pmatrix} * & \lambda_{R-(i+1)V+1} & & & & & & \\ \mu_{R-(i+1)V+2} & * & \lambda_{R-(i+1)V+2} & & & & & \\ & \mu_{R-(i+1)V+3} & * & \lambda_{R-(i+1)V+3} & & & & \\ & \ddots & \ddots & \ddots & & & & \\ & & \mu_{R-iV} & * & \lambda_{R-iV} & & & \\ & & & \ddots & \ddots & \ddots & & \\ & & & & \mu_{R-iV} & * & \lambda_{M+S-1} & \\ & & & & & \mu_{R-iV} & * & \end{pmatrix}_{(M+S-R+(i+1)V)\times(M+S-R+(i+1)V)}, \tag{17}$$

and

$$\mathbf{A}_0=\begin{pmatrix} * & \lambda_{R-V+1} & & & & & & \\ \mu_{R-V+2} & * & \lambda_{R-V+2} & & & & & \\ & \mu_{R-V+3} & * & \lambda_{R-V+3} & & & & \\ & & \ddots & \ddots & \ddots & & & \\ & & & \mu_R & * & \lambda_R & & \\ & & & & \ddots & \ddots & \ddots & \\ & & & & & \mu_R & * & \lambda_{M+S-1} \\ & & & & & & \mu_R & * \end{pmatrix}_{(M+S-R+V)\times(M+S-R+V)} . \quad (18)$$

The diagonal elements of matrix $\mathbf{A}_i$ ($\mathbf{Q}$), indicated by *, are such that the sum of each row of $\mathbf{Q}$ is zero. $\mathbf{C}_K$ is a matrix of size $(M+S-R+KV)\times(M+S+1)$ with only one nonzero element $\mathbf{C}_K[1,R-KV+1]=\mu_{R-KV+1}$. For $i=1,2,...,K-1$, $\mathbf{C}_i$ is a matrix of size $(M+S-R+iV)\times[M+S-R+(i+1)V]$ with only one nonzero element $\mathbf{C}_i[1,V]=\mu_{R-iV+1}$.

$\mathbf{B}_{K-1}$ is a matrix of size $(M+S+1)\times(M+S-R+KV)$ with elements $\mathbf{B}_{K-1}[R+1-KV+n,n]=K\theta$, $n=1,2,...,(M+S-R+KV)$. For $i=0,1,...,K-2$, $\mathbf{B}_i$ is a matrix of size $[M+S-R+(i+2)V]\times[M+S-R+(i+1)V]$ with elements $\mathbf{B}_i[V+n,n]=(i+1)\theta$, $n=1,2,...,M+S-R+(i+1)V$.

Let $\mathbf{\Pi}$ denote the steady-state probability vector of $\mathbf{Q}$. Vector $\mathbf{\Pi}$ is partitioned as $\mathbf{\Pi}=[\mathbf{\Pi}_K,\mathbf{\Pi}_{K-1},...,\mathbf{\Pi}_1,\mathbf{\Pi}_0]$ where $\mathbf{\Pi}_K=[P_{R-KV,0},P_{R-KV,1},...,P_{R-KV,M+S-1},P_{R-KV,M+S}]$ denotes the steady-state probability vector that the number of teams / groups on vacation is equal to K. The sub-vectors $\mathbf{\Pi}_k=[P_{R-kV,R-(k+1)V+1},P_{R-kV,R-(k+1)V+2},...,P_{R-kV,M+S}]$ represent the steady-state probability vector that the number of teams/group on vacation is equal to k, $k=0,1,...,K-1$. The steady-state equations $\mathbf{\Pi Q}=\mathbf{0}$ are given by

$$\mathbf{\Pi}_K\mathbf{A}_K+\mathbf{\Pi}_{K-1}\mathbf{C}_K=\mathbf{0}, \quad (19)$$

$$\mathbf{\Pi}_{k+1}\mathbf{B}_k+\mathbf{\Pi}_k\mathbf{A}_k+\mathbf{\Pi}_{k-1}\mathbf{C}_k=\mathbf{0},\ k=1,2,...,K-1, \quad (20)$$

$$\mathbf{\Pi}_1\mathbf{B}_0+\mathbf{\Pi}_0\mathbf{A}_0=\mathbf{0}, \quad (21)$$

and the following normalizing equation

$$\sum_i\sum_n P_{i,n}=\sum_k \mathbf{\Pi}_k\mathbf{e}=1, \quad (22)$$

where $\mathbf{e}$ represents a column vector with suitable size and each component equal to one. After performing routine manipulations to equation (19)-(21), we obtain

$$\begin{aligned} \mathbf{\Pi}_K &= \mathbf{\Pi}_{K-1}\mathbf{C}_K(-\mathbf{A}_K)^{-1}=\mathbf{\Pi}_{K-1}\phi_K, \\ \mathbf{\Pi}_k &= \mathbf{\Pi}_{k-1}\mathbf{C}_k[-(\mathbf{\Phi}_{k+1}\mathbf{B}_k+\mathbf{A}_k)]^{-1}=\mathbf{\Pi}_{k-1}\phi_k,\ 1\le k\le K-1, \end{aligned} \quad (23)$$

and

$$\mathbf{\Pi}_0\phi_1\mathbf{B}_0+\mathbf{\Pi}_0\mathbf{A}_0=\mathbf{0}. \tag{24}$$

Consequently, $\mathbf{\Pi}_k$ ($1\le k\le K$) in equation (19)-(20) can be written in terms of $\mathbf{\Pi}_0$ as $\mathbf{\Pi}_k=\mathbf{\Pi}_0\mathbf{\Phi}_k$ where $\mathbf{\Phi}_k=\phi_1\phi_2\cdots\phi_k$, $1\le k\le K$. Upon the steady-state probability $\mathbf{\Pi}_0$ being obtained, the steady-state solutions $\mathbf{\Pi}=[\mathbf{\Pi}_K,\mathbf{\Pi}_{K-1},...,\mathbf{\Pi}_1,\mathbf{\Pi}_0]$ are then determined. $\mathbf{\Pi}_0$ can be solved by equation (24), with the following normalization equation

$$\sum_i\sum_n P_{i,n}=\sum_k \mathbf{\Pi}_k\mathbf{e}=\mathbf{\Pi}_0\left[\sum_{k=1}^{K}\mathbf{\Phi}_k+\mathbf{I}\right]\mathbf{e}=1, \tag{25}$$

where $\mathbf{I}$ represents an identity matrix with suitable size.

4. Performance analysis

Availability and reliability analysis

It is noted that the system fails if, and only if, $S+1$ (or more) machines fail. Hence, the steady-state availability can be calculated as

$$A.V.=P(0\le n\le S)=\sum_i \sum_{0\le n\le S} P_{i,n}. \tag{26}$$

Other system performance measures

The analysis of this study is based on the following system performance measures. Let

$E[F] \equiv$ the expected number of failed machines in the system,

$E[F_q] \equiv$ the expected number of failed machines in the queue,

$E[O] \equiv$ the expected number of operating machines in the system,

$E[S] \equiv$ the expected number of acting standby machines in the system,

$E[B] \equiv$ the expected number of busy repairmen in the system,

$E[V] \equiv$ the expected number of vacation repairmen in the system,

$E[I] \equiv$ the expected number of idle repairmen in the system,

$M.A. \equiv$ machine availability (the fraction of the total time that the machines are working),

$O.U. \equiv$ operative utilization (the fraction of busy servers).

For convenience, this study defines the symbol "$\langle f(n), n=a..b\rangle$" as denoting a column vector with dimension $(b-a+1)$, of which the n^{th} element is $f(n)$. Then, the expressions for $E[F]$, $E[F_q]$, $E[O]$, $E[S]$, $E[B]$, $E[V]$, and $E[I]$ are developed as follows:

$$
\begin{aligned}
E[F] &= \sum_i \sum_n nP_{i,n} \\
&= \sum_{k=0}^{K-1} \mathbf{\Pi}_k \langle n, n=[R-(k+1)V]..(M+S)\rangle + \mathbf{\Pi}_K \langle n, n=0..(M+S)\rangle \\
&= \mathbf{\Pi}_0 \left\{ \sum_{k=0}^{K-1} \mathbf{\Phi}_k \langle n, n=[R-(k+1)V]..(M+S)\rangle + \mathbf{\Phi}_K \langle n, n=0..(M+S)\rangle \right\}
\end{aligned} \tag{27}
$$

$$
\begin{aligned}
E[F_q] &= \sum_i \sum_n P_{i,n} \max\{n-i,0\} + \sum_n P_{R-KV,n} \max\{n-i,0\} \\
&= \sum_{k=0}^{K-1} \mathbf{\Pi}_k \langle \max\{n-(R-kV),0\}, n=[R-(k+1)V+1]..(M+S)\rangle \\
&\quad + \mathbf{\Pi}_K \langle \max\{n-(R-KV),0\}, n=0..(M+S)\rangle \\
&= \mathbf{\Pi}_0 \left\{ \sum_{k=0}^{K-1} \mathbf{\Phi}_k \langle \max\{n-(R-kV),0\}, n=[R-(k+1)V+1]..(M+S)\rangle \right. \\
&\quad \left. + \mathbf{\Phi}_K \langle \max\{n-(R-KV),0\}, n=0..(M+S)\rangle \right\}
\end{aligned} \tag{28}
$$

$$
\begin{aligned}
E[O] &= \sum_i \sum_n P_{i,n} \min\{M, M+S-n\} + \sum_n P_{R-KV,n} \min\{M, M+S-n\} \\
&= \sum_{k=0}^{K-1} \mathbf{\Pi}_k \langle \min\{M, M+S-n\}, n=[R-(k+1)V+1]..(M+S)\rangle \\
&\quad + \mathbf{\Pi}_K \langle \min\{M, M+S-n\}, n=0..(M+S)\rangle \\
&= \mathbf{\Pi}_0 \left\{ \sum_{k=0}^{K-1} \mathbf{\Phi}_k \langle \min\{M, M+S-n\}, n=[R-(k+1)V+1]..(M+S)\rangle \right. \\
&\quad \left. + \mathbf{\Phi}_K \langle \min\{M, M+S-n\}, n=0..(M+S)\rangle \right\}
\end{aligned} \tag{29}
$$

$$
\begin{aligned}
E[S] &= \sum_i \sum_n P_{i,n} \max\{0,S-n\} + \sum_n P_{R-KV,n} \max\{0,S-n\} \\
&= \sum_{k=0}^{K-1} \mathbf{\Pi}_k \langle \max\{0,S-n\}, n=[R-(k+1)V+1]..(M+S)\rangle \\
&\quad + \mathbf{\Pi}_K \langle \max\{0,S-n\}, n=0..(M+S)\rangle \\
&= \mathbf{\Pi}_0 \Big\{ \sum_{k=0}^{K-1} \mathbf{\Phi}_k \langle \max\{0,S-n\}, n=[R-(k+1)V+1]..(M+S)\rangle \\
&\quad + \mathbf{\Phi}_K \langle \max\{0,S-n\}, n=0..(M+S)\rangle \Big\}
\end{aligned} \tag{30}
$$

$$
E[V] = \sum_{k=1}^{K} kV \mathbf{\Pi}_k \mathbf{e} = \mathbf{\Pi}_0 V \sum_{k=1}^{K} k \mathbf{\Phi}_k \mathbf{e} \tag{31}
$$

$$
\begin{aligned}
E[I] &= \sum_i \sum_n P_{i,n} \max\{i-n,0\} + \sum_n P_{R-KV,n} \max\{R-KV,0\} \\
&= \sum_{k=0}^{K-1} \mathbf{\Pi}_k \langle \max\{(R-kV)-n,0\}, n=[R-(k+1)V+1]..(M+S)\rangle \\
&\quad + \mathbf{\Pi}_K \langle \max\{(R-KV)-n,0\}, n=0..(M+S)\rangle \\
&= \mathbf{\Pi}_0 \Big\{ \sum_{k=0}^{K-1} \mathbf{\Phi}_k \langle \max\{(R-kV)-n,0\}, n=[R-(k+1)V+1]..(M+S)\rangle \\
&\quad + \mathbf{\Phi}_K \langle \max\{(R-KV)-n,0\}, n=0..(M+S)\rangle \Big\}
\end{aligned} \tag{32}
$$

$$
\begin{aligned}
E[B] &= \sum_i \sum_n P_{i,n} \min\{i,n\} \\
&= \sum_{k=0}^{K-1} \mathbf{\Pi}_k \langle \min\{(R-kV),n\}, n=[R-(k+1)V+1]..(M+S)\rangle \\
&\quad + \mathbf{\Pi}_K \langle \min\{(R-KV),n\}, n=0..(M+S)\rangle \\
&= \mathbf{\Pi}_0 \Big\{ \sum_{k=0}^{K-1} \mathbf{\Phi}_k \langle \min\{(R-kV),n\}, n=[R-(k+1)V+1]..(M+S)\rangle \\
&\quad + \mathbf{\Phi}_K \langle \min\{(R-KV),n\}, n=0..(M+S)\rangle \Big\}
\end{aligned} \tag{33}
$$

By the properties of minimum and maximum functions, it can be verified that $E[V]+E[I]+E[B]=R$. Furthermore, following Benson and Cox (1951), the machine availability and the operative utilization of servers are defined by

$$M.A.=1-\frac{E[F]}{M+S} \text{ and } O.U.=\frac{E[B]}{R}. \tag{34}$$

Finally, use Little's formula to obtain the expected waiting time in the system, $E[W]$, and in the queue $E[W_q]$, as

$$E[W]=E[F]/\lambda_e \text{ and } E[W_q]=E[F_q]/\lambda_e, \tag{35}$$

where $\lambda_e=\sum_i\sum_n\lambda_n P_{i,n}$ is the effective arrival rate into the system.

5. Cost analysis

In this section, a total expected cost function per unit time, as based on system performance measures, is constructed. A constraint on system availability is imposed on this cost model, where R, V and K are discrete decision variables. First, let

$C_h \equiv$ cost per unit time when one failed machine joins the system,

$C_e \equiv$ cost per unit time of a failed machine after all standbys are exhaused (downtime cost),

$C_s \equiv$ cost per unit time when one machine is functioning as a standby (inventory cost),

$C_b \equiv$ cost per unit time when one repairman is busy,

$C_f \equiv$ cost per unit time of each resident repairman,

$C_t \equiv$ cost per unit time of each team / group,

$\gamma \equiv$ cost per unit time of augment the size of team / group.

Using the definitions of the cost elements listed above, the total expected cost function per unit time is given by

$$T_{\text{cost}}(R,V,K) = C_h E[F] + C_e(M - E[O]) + C_s E[S] + C_b E[B] + (R-KV)C_f + (R/V)C_t + \gamma V. \tag{36}$$

An example (photolithography process problem mentioned Uzsoy et al.(1992, 1994)) is provided to perform the numerical investigation:

- There are $M=15$ stepper machines and $S=10$ standby machines in the photolithography process.
- Each operating stepper machine may be interrupted due to unpredictable accidents with Poisson breakdown rate $\lambda=1.5$.
- The standby machines are with Poisson breakdown rate $\alpha=1.0$

- In the repair facility, R technicians are responsible to provide the repair service for the failed machines. The repair time for one failed machine is exponentially with mean μ^{-1}=0.2 .
- The servers/technicians are allocated by a (R,V,K) synchronous multiple vacation policy, in which vacation time is an exponential distributed with mean θ^{-1}=2 .
- The cost elements and availability requirements are C_h=10, C_e=125, C_s=90, C_b=60, C_f=80, C_t=45, γ=30, and A=0.9

6. Conclusions

The systematic methodology provided in this paper works efficiently for a machine repair model with standbys under a synchronous vacation policy. The stationary probability vectors were obtained in terms of matrix forms using the technique of matrix partition. Firstly, we developed the steady-state solutions in matrix forms for the machine repair model by using the Markov process. These solutions were used to obtain the various system performance measures, such as the steady-state availability, the expected number of failed machines in the queue / system, the expected number of idle, busy and vacation servers, machine availability, operative utilization, etc. Next, we developed a cost model for the machine repair model to determine the optimal (R,V,K) synchronous vacation ploicy.

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