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Estimating Achievable Capacity Index based on Multiple Samples

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Abstract

This paper investigates the profitability which measures for the newsboy-type product and develops a new index "Achievable Capacity Index", denoted by I_A. It can accurately measure the profitability of newsboy-type product with normally distributed demand. Furthermore, the interrelationship between profitability and I_A is also performed. An unbiased and effective estimator is derived to estimate I_A. Practically, market information regarding demand is obtained from multiple samples rather than single sample. Then we estimate I_A based on multiple samples. Finally, a numerical result is presented to show the probability density function of the unbiased estimator of $\hat{I}_A$ under different groups and sample sizes.

Keywords: Newsboy, estimating, achievable capacity index, multiple samples.

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1. Introduction

In the traditional newsboy problem, it usually focused on short shelf-life products with common applications such as daily newspapers, milks, seasonal products, fresh food and many others. Since the surplus products are subject to storage for a short period of time, we may require additional costs to dispose these items. Generally, the demand presented in the newsboy problem is unknown and assumed to be a random variable with a known probability distribution. Consequently, the determination of the ordering quantity (or manufacturing quantity) is critical for achieving certain objective function in the newsboy problem.

There is an excellent survey of the literature on the various objective functions such as minimizing the expected cost (Nahmias, 1993), maximizing the expected profit (Khouja, 1995), maximizing the expected utility (Ismail and Louderback, 1979; Lau, 1997), and maximizing the probability of achieving a target profit (Ismail and Louderback, 1979; Shih, 1979; Lau, 1980; Sankarasubramanian and Kumaraswamy, 1983). However, no researches have tried to study the maximum expected profit and probability of achieving a target profit, which can be used to measure a product's profitability. This motivated us to develop an index combined with profitability requirement (Tseng, 2009).

In this paper, we consider a newsboy-type product with a normally distributed demand, and define product's profitability as the probability of achieving the target profit under optimal ordering condition (Tseng, 2011). Then we develop a new index which can express profitability simplify, and we call it an "achievable capacity index", denoted by I_A . As the related costs (excess, shortage, disposal and purchasing), selling price, and target profit are given, I_A depends on demand mean μ and demand standard deviation σ . In order to make the problem more relevant and applicable in practice, we assumed that the demand mean and demand standard deviation are unknown. Note that these unknown parameters can be obtained from samples. In practice, the demand data is collected from multiple samples rather than single sample. We thereby investigate I_A based on multiple samples. Then, we derive an unbiased and effective estimator to estimate I_A . Finally, a numerical result is presented to show the probability density function of the unbiased estimator of $\hat{I}_A$ under different groups and sample sizes.

2. Achievable Capacity Index

We consider a newsboy-type product. The demand, D , follows a normal distribution, $N(\mu,\sigma^2)$, and satisfies that the coefficient of variation (cv) is below 0.3 for neglecting the negative tail, i.e., $f(D<0)=\Phi(-\mu/\sigma)=\Phi(-1/cv)<\Phi(-1/0.3)\approx 0$. In addition, the profitability is defined as the probability of achieving the target profit, $k>0$, under the optimal ordering policy, in which the target profit is set according to the product property and the sales experience.

If the selling price and related cost (shortage, excess and purchasing/manufacturing costs per unit) are given, the optimal ordering quantity and the level of profitability depend on μ and σ. Therefore, we develop a new index, which is a function of μ and σ to express the product's profitability, and so-called "Achievable Capacity Index (ACI)". It is defined as follows:

$$I_A = \frac{\mu - \dfrac{k}{p-c}}{\sigma} = \frac{\mu - T}{\sigma},$$

where

- p the selling price per unit, $p > 0$.
- c the purchasing/manufacturing cost per unit, $c > 0$.
- T the target demand which is the minimal demand required for achieving profit, i.e., $T = k/(p-c) > 0$.

The numerator of I_A provides the difference between demand mean and target demand. The denominator gives demand standard deviation. Obviously, I_A is getting larger as σ increases or μ departs from T. It is desirable to have an I_A as large as possible.

Interrelationship between Profitability and I_A

Based on Sankarasubramanian and Kumaraswamy (1983), the profit Z depends on the demand D and the ordering quantity Q, which are formulated as follows:

$$Z = \begin{cases} pD - c_d(Q-D) - cQ = (c_p + c_e)D - c_eQ, & 0 \le D \le Q, \\ pQ - c_s(D-Q) - cQ = -c_sD + (c_p + c_s)Q, & D > Q, \end{cases}$$

Where

- c_p the net profit per unit (i.e., $c_p = p - c > 0$).
- c_d the disposal cost for a surplus product, $c_d > 0$.
- c_e the excess cost per unit (i.e., $c_e = c_d + c > 0$).
- c_s the shortage cost per unit, $c_s > 0$.

Note that if the surplus products can be salvaged, the value of c_d is negative and redefine into salvage price. It is well known that in order to possibly achieve the target profit, the ordering quantity must be greater than target demand, i.e., $Q \ge T$. For any $Q \ge T$, Z is strictly increasing in $D \in [0, Q]$ and strictly decreasing in $D \in [Q, \infty)$, and has a maximum at point $D = Q$. The maximum value of Z is equal and higher than k, i.e., $Z = pD - cQ = c_pD = c_pQ \ge c_pT = k$. The target profit will be realized when D is equal to either $LAL(Q)$ or $UAL(Q)$, so the target profit will be achieved in $D\in[LAL(Q), UAL(Q)]$, where

$$LAL(Q)=\frac{c_eQ+k}{c_p+c_e} \text{ and } UAL(Q)=\frac{(c_p+c_s)Q-k}{c_s}$$

are the lower and upper achievable limits, respectively. It is noted that both are the functions of Q. Under the assumption that the demand is normally distributed, the probability of achieving the target profit is:

$$\Pr(Z\geq k)=\Phi\left(\frac{UAL(Q)-\mu}{\sigma}\right)-\Phi\left(\frac{LAL(Q)-\mu}{\sigma}\right), \tag{1}$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution. Before calculating the profitability, we first find the optimal ordering quantity to maximize $\Pr(Z\geq k)$. We take the first-order of $\Pr(Z\geq k)$ with respect to Q, and obtain

$$\frac{d\Pr(Z\geq k)}{dQ}=\frac{1}{\sqrt{2\pi}}\left[\frac{c_p+c_s}{c_s}e^{-\frac{1}{2}\left(\frac{UAL(Q)-\mu}{\sigma}\right)^2}-\frac{c_e}{c_p+c_e}e^{-\frac{1}{2}\left(\frac{LAL(Q)-\mu}{\sigma}\right)^2}\right].$$

It is well known that the necessary condition for Q to be optimal must satisfy the equation $\mathrm{d}\Pr(Z\geq k)/\mathrm{d}Q=0$, which implies

$$\mu=\frac{UAL(Q)+LAL(Q)}{2}-\frac{\omega\sigma^2}{UAL(Q)-LAL(Q)}, \tag{2}$$

where $\omega=\ln[1+c_pA/c_sc_e]$ and $A=c_p+c_e+c_s$. For $Q\geq T$, we solve Eq. (2), and obtain the unique optimal ordering quantity

$$Q^*=T+\frac{c_s(c_p+c_e)(c_p\mu-k)}{c_p(c_pA+2c_ec_s)}+\sqrt{\left[\frac{c_s(c_p+c_e)(c_p\mu-k)}{c_p(c_pA+2c_ec_s)}\right]^2+\frac{2c_s^2(c_p+c_e)^2\omega\sigma^2}{c_pA(c_pA+2c_ec_s)}}>T. \tag{3}$$

Additionally, the sufficient condition is given by

$$\left.\frac{\mathrm{d}^2\Pr(Z\geq k)}{\mathrm{d}Q^2}\right|_{Q=Q^*}=-\frac{(c_p+c_s)}{\sqrt{2\pi}\sigma^3c_s^2(c_p+c_e)}\exp\left[-\frac{1}{2}\left(\frac{UAL(Q^*)-\mu}{\sigma}\right)\right]\times\left\{\frac{\left[UAL(Q^*)-LAL(Q^*)\right](c_pA+2c_ec_s)}{2}+\frac{c_pA\omega\sigma^2}{UAL(Q^*)-LAL(Q^*)}\right\}<0.$$

It shows that the stationary point Q^* is a global maximum. By using Eq. (2) and substituting Eq. (3) into Eq. (1), the profitability, Ω, can be obtained as follows:

$$\Omega=\Phi\left(G+\frac{\omega}{2G}\right)-\Phi\left(-G+\frac{\omega}{2G}\right),$$

where

$$G=\frac{UAL(Q^*)-LAL(Q^*)}{2\sigma}=M\left(\frac{\mu-T}{\sigma}\right)+\sqrt{M^2\left(\frac{\mu-T}{\sigma}\right)^2+M\omega}=MI_A+\sqrt{M^2I_A^2+M\omega}>0,$$

and

$$M=\frac{c_pA}{2(c_pA+2c_ec_s)}>0\,.$$

It is easy to see that Ω is a function of I_A. Taking the first-order derivative of $\Omega(I_A)$ with respect to I_A, and we have

$$\frac{\mathrm{d}\Omega(I_A)}{\mathrm{d}I_A}=\frac{MG}{\sqrt{2\pi(MI_A{}^2+M\omega)}}\left[e^{\omega}+1+\frac{\omega}{2G^2}\left(e^{\omega}-1\right)\right]e^{-\frac{1}{2}\left(G+\frac{\omega}{2G}\right)^2}>0\,.$$

As a result, $\Omega(I_A)$ is a strictly increasing function of I_A. Therefore, we can express the product's profitability according to the value of I_A.

3. Estimating I_A based on Multiple Samples

The historical data of the demand ought to be collected in order to estimate the actual I_A due to unknown μ and σ. For multiple samples of m groups each of size n is given as $\{x_{i1}, x_{i2},..., x_{in}\}$, where $i=1,2,...,m$, let $\bar{x}_i=\sum_{j=1}^{n}x_{ij}/n$ and $s_i{}^2=\sum_{j=1}^{n}(x_{ij}-\bar{x}_i)^2/(n-1)$ be the ith sample mean and sample standard deviation, respectively. We first consider the natural estimator $\hat{I}_A$ which is obtained by replacing the μ and σ by their unbiased estimators $\bar{\bar{x}}=\sum_{i=1}^{m}\bar{x}_i/m$ and $s_p=[\sum_{i=1}^{m}s_i{}^2/m]^{1/2}$ i.e.,

$$\hat{I}_A=\frac{\bar{\bar{x}}-T}{s_p}\,.$$

Furthermore, we rewrite the natural estimator, $\hat{I}_A$, in the following:

$$\hat{I}_A=\frac{\bar{\bar{x}}-T}{s_p}=\frac{1}{\sqrt{mn}}\times\frac{\dfrac{\bar{\bar{x}}-\mu}{\sigma/\sqrt{mn}}+\dfrac{\mu-T}{\sigma/\sqrt{mn}}}{\sqrt{\dfrac{m(n-1)s_p{}^2/\sigma^2}{m(n-1)}}}=\frac{1}{\sqrt{mn}}\times\frac{Z+\sqrt{mn}I_A}{\sqrt{\dfrac{W}{m(n-1)}}}$$

$$=\frac{1}{\sqrt{mn}}\times\frac{Z_A}{\sqrt{\dfrac{W}{m(n-1)}}},$$

where $Z_A=Z+\sqrt{mn}I_A\sim N(\sqrt{mn}I_A,1)$, $Z\sim N(0,1)$, $W=m(n-1)s_p{}^2/\sigma^2\sim\chi^2_{m(n-1)}$. Since Z_A and W are independent, the estimator $\hat{I}_A$ is distributed as $(mn)^{-1/2}t_{m(n-1)}(\theta)$, where $t_{m(n-1)}(\theta)$ is a non-central t distribution with $m(n-1)$ degree of freedom and the non-centrality parameter $\theta=(mn)^{1/2}I_A$. Since

$$E(\hat{I}_A)=\frac{[m(n-1)/2]^{1/2}\Gamma[(m(n-1)-1)/2]}{\Gamma[m(n-1)/2]}\times I_A\neq I_A\,,$$

the natural estimator $\hat{I}_A$ is biased. To tackle this problem, we add a correction factor as follows

$$b=\frac{[2/m(n-1)]^{1/2}\,\Gamma[m(n-1)/2]}{\Gamma[(m(n-1)-1)/2]}.$$

Thus, we obtain the unbiased estimator $b\hat{I}_A$, which is denoted by $\tilde{I}_A$. Since $\tilde{I}_A$ is based solely on the complete and sufficient statistics $(\bar{\bar{x}}, s_p{}^2)$, we conclude that the estimator $\tilde{I}_A$ is the uniformly minimum variance unbiased estimator (UMVUE) of I_A based on multiple samples. The probability density function of $\tilde{I}_A=R$ can be derived as follows:

$$f_R(r) = \frac{\sqrt{2mn}\left(\frac{m(n-1)}{2}\right)^{\frac{m(n-1)}{2}}}{b\sqrt{\pi}\,\Gamma\left(\frac{m(n-1)}{2}\right)} \int_0^\infty v^{m(n-1)} \exp\left\{-\frac{1}{2}\left[\frac{(vr-bI_A)^2}{b^2/mn}+m(n-1)v^2\right]\right\}dv, -\infty < r < \infty.$$

Numerical Results

Fig. 1 plots the probability density function of R, $I_A=1.0, 1.5, 2.0,$ $n=3, 4, 5,$ and $m=10, 25, 40$ (from bottom to top in plots). From Fig. 1, one can easily see that (1) for fixed sample sizes m and n, the variance of $\tilde{I}_A=R$ increases as I_A increases; (2) for a fixed n and I_A, the variance of $\tilde{I}_A=R$ decreases as m increases; (3) for a fixed m and I_A, the variance of $\tilde{I}_A=R$ decreases as n increases.

4. Conclusions

In this paper, we developed a new index, achievable capacity index, I_A, which has a simple-form to measure the profitability of the newsboy-type product with normally distributed demand. In practical situation, the demand data is collected from multiple samples rather than single sample. Hence, we considered an unbiased and effective estimator of I_A to estimate the actual I_A based on multiple samples. The probability density function of the unbiased estimator of $\hat{I}_A$ is also provided. The result is helpful in assessing the performance of the Newsboy-type problem. In the future, the evaluation testing of I_A would be discussed that deserves further investigation.

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Appendix A. An example appendix

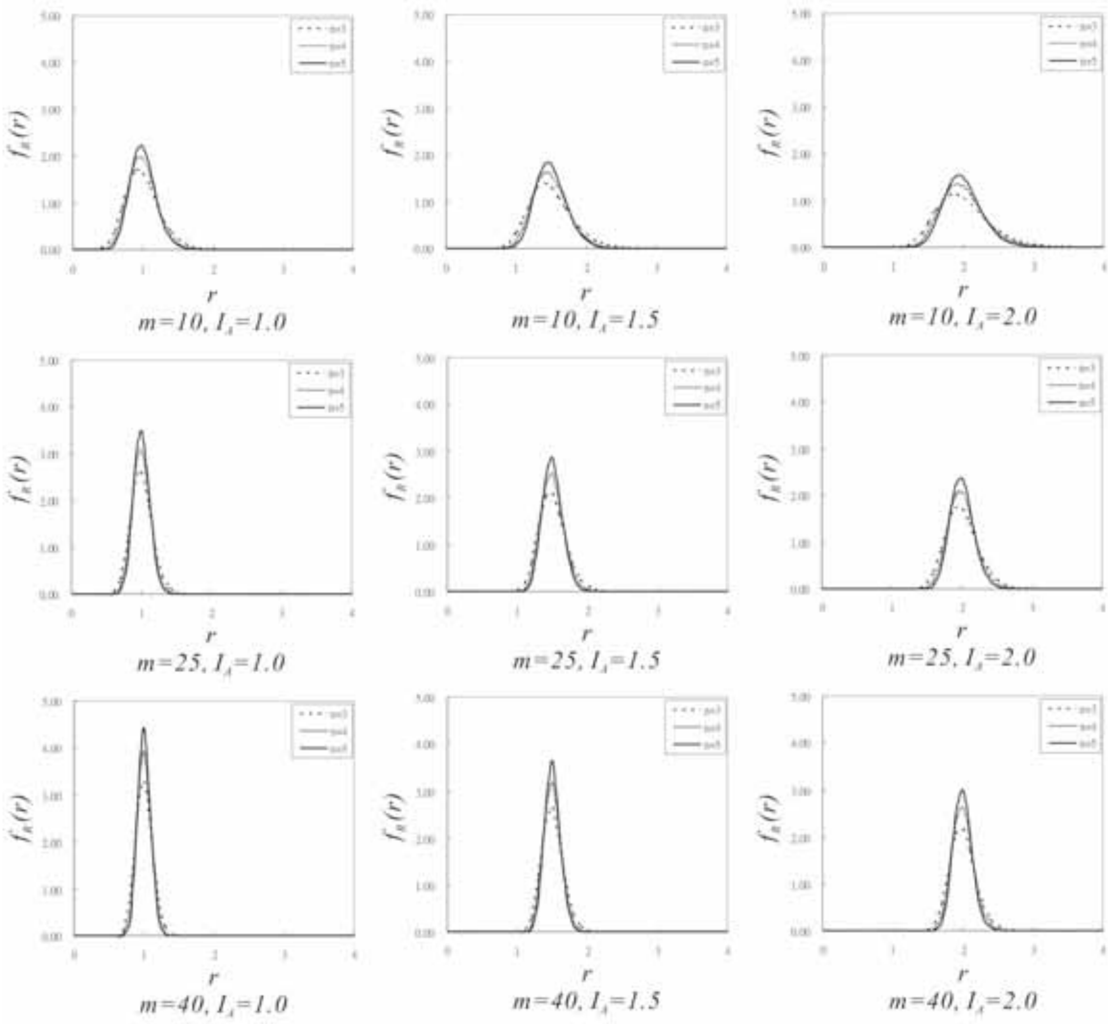

Fig. 1. PDF plots of r for $n=3, 4, 5,$ and $m=10, 25, 40$ (from bottom to top in plots).