# Observer-Based Direct Adaptive Fuzzy-Neural Control for Nonaffine Nonlinear Systems

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***Abstract*—In this paper, an observer-based direct adaptive fuzzy-neural control scheme is presented for nonaffine nonlinear systems in the presence of unknown structure of nonlinearities. A direct adaptive fuzzy-neural controller and a class of generalized nonlinear systems, which are called nonaffine nonlinear systems, are instead of the indirect one and affine nonlinear systems given by Leu *et al.* By using implicit function theorem and Taylor series expansion, the observer-based control law and the weight update law of the fuzzy-neural controller are derived for the nonaffine nonlinear systems. Based on strictly-positive-real (SPR) Lyapunov theory, the stability of the closed-loop system can be verified. Moreover, the overall adaptive scheme guarantees that all signals involved are bounded and the output of the closed-loop system will asymptotically track the desired output trajectory. To demonstrate the effectiveness of the proposed method, simulation results are illustrated in this paper.**

***Index Terms*—Direct adaptive control, fuzzy-neural control, nonaffine nonlinear systems, output feedback control.**

## I. Introduction

ADAPTIVE control theory has been an active area of research for at least a quarter of a century [1]–[11]. For linear systems, there have been some researches on stability analysis of adaptive control systems, design of adaptive observers, and adaptive control of plants [2], [3]. Also, many researchers focus on robust adaptive control that guarantees signal boundedness in the presence of modeling errors and bounded disturbances [4]–[6]. For nonlinear systems, some adaptive control schemes via feedback linearization have been reported [7]–[11]. The fundamental ideal of feedback linearization is to transform a nonlinear system into a linear one. Then, linear control techniques are employed to acquire desired performance.

Recently, since neural networks [12] and fuzzy logic [13] are universal approximators, some adaptive control schemes of nonlinear systems via fuzzy logic and/or neural networks [14]–[19], [27]–[29], [37]–[40] have been proposed. Likewise, for a class of nonlinear continuous-time systems, adaptive control using neural networks has been proposed in [20] by feedback linearization. A dynamic recurrent neural-network-based adaptive observer for a class of nonlinear systems has been presented in [21]. In [22], [23], and [37], the output feedback controllers have been developed based on a high-gain observer used to estimate the time derivatives of the system output by using neural networks. By using the high gain observer, the closed system may exhibit a peaking phenomenon in the transient behavior [23], [37]. The controller saturates to prevent peaking [23], [37]. In this paper, an observer-based adaptive fuzzy-neural controller is proposed and derived for avoiding the high gain observer and preventing the peaking phenomenon in the transient behavior.

More recently, applications of fuzzy logic incorporated into neural networks in function approximation, decision systems and nonlinear control systems have been proposed in [24]–[31], [34]–[36]. In [28], the observer-based indirect adaptive fuzzy-neural controller for affine nonlinear systems has been proposed. Most of them deal with the control problem of the affine nonlinear systems. However, in practice, the control methods of affine nonlinear systems do not always hold and the control methods of the nonaffine nonlinear systems are necessary. In [28], the adaptive fuzzy-neural control systems are derived from the *indirect* adaptive control method, which uses fuzzy-neural networks as function approximators to estimate nonlinear functions of the nonlinear systems. Since nonlinear functions in nonaffine nonlinear systems are implicit functions with respect to the controller, the indirect adaptive control method in [28], which uses fuzzy-neural networks to estimate the nonlinear functions, cannot be employed to the nonaffine nonlinear systems again. On the other hand, direct adaptive fuzzy-neural controllers, in which fuzzy-neural networks are used to directly be controllers rather than to be nonlinear functions, are suitable for the nonaffine nonlinear systems. Theoretical justification on the use of the direct adaptive fuzzy controllers in [14] using a state feedback approach is valid if all of the system states are available for measurement. In practice, however, the state feedback control does not always hold because system states are not always available. Estimations of states from the system output for output feedback control design of the direct adaptive fuzzy-neural controller is required.

The goal of this paper is to develop an observer-based adaptive fuzzy-neural control scheme that extends the design method in [28] by using direct adaptive control instead of indirect is presented for the nonaffine nonlinear system in the presence of unknown structure of nonlinearities. By using implicit function theorem, and Taylor series expansion, the output feedback control law and the update laws are derived. Moreover, the overall adaptive scheme guarantees that all

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signals involved are bounded and the output of the closed-loop system will asymptotically track the desired output trajectory.

The paper is organized as follows. In Section II, the problem is formulated and a brief description of fuzzy-neural networks is presented. Design methodology of the direct adaptive fuzzy-neural controller is included in Section III. In Section IV, simulation results are presented to confirm the effectiveness and applicability of the proposed method. Finally, Section V concludes the paper.

## II. Problem Formulation and Fuzzy-Neural Network

Consider the single-input–single-output (SISO) nonaffine nonlinear system of the form

$$\begin{aligned}\dot{\mathbf{z}} &= \mathbf{F}(\mathbf{z}, u)\\ y &= h(\mathbf{z})\end{aligned} \tag{1}$$

where $\mathbf{z} \in R^n$ is a vector of states, and $u \in R$ and $y \in R$ are the control input and system output, respectively. $\mathbf{F}(\cdot,\cdot)$ and $h(\cdot)$ are unknown smooth vector functions. In nonaffine systems, the nonlinear functions $\mathbf{F}(\cdot,\cdot)$ are implicit functions with respect to the controller $u$.

Suppose that the nonaffine nonlinear system possesses a strong relative degree. Then, the system can be transformed into the following form [23]:

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}q(\mathbf{x}, u)\\ y &= \mathbf{C}^T\mathbf{x}\end{aligned} \tag{2}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0\\ 0 & 0 & 1 & \dots & 0\\ \dots & \dots & \dots & \dots & \dots\\ 0 & 0 & 0 & \dots & 1\\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0\\ 0\\ \vdots\\ 0\\ 1 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 1\\ 0\\ \vdots\\ 0\\ 0 \end{bmatrix}$$

$\mathbf{x} = [x, \dot{x}, \dots, x^{(n-1)}]^T = [x_1, x_2, \dots, x_n]^T \in R^n$ is a vector of the transformed states, and $q(\cdot,\cdot)$ is a smooth function. Here, we assume that the structure of $q(\cdot,\cdot)$ is unknown. Without losing generality, we also assume that $\partial q(\mathbf{x}, u)/\partial u > 0$. In addition, only the system output $y$ is assumed to be measurable. The control objective is to design an observer-based direct adaptive fuzzy-neural controller such that the system output $y$ follows a given bounded smooth signal $y_m$, and all signals involved are bounded.

First, define the reference vector $\mathbf{y}_m = \left[y_m, \dot{y}_m, \dots, y_m^{(n-1)}\right]^T$, the output tracking error $e_1 = y_m - y$, the tracking error vector $\mathbf{e} = \mathbf{y}_m - \mathbf{x}$, and $\zeta = \mathbf{K}_c^T\mathbf{e}^T + y_m^{(n)}$, where $\mathbf{K}_c = [k_n^c \quad k_{n-1}^c \quad \dots \quad k_1^c]^T$ is the feedback gain vector, chosen such that the characteristic polynomial of $\mathbf{A} - \mathbf{B}\mathbf{K}_c^T$ is Hurwitz because $(\mathbf{A}, \mathbf{B})$ is controllable. Next, we describe the existence of the control solution for the system (1). According to implicit function theorem [32], there exists a unique ideal implicit feedback control for the system (1). Equation (2) can be rewritten as

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}(q(\mathbf{x}, u) + \zeta - \zeta)\\ y &= \mathbf{C}^T\mathbf{x}.\end{aligned} \tag{3}$$

From the definitions of $\zeta$ and $\mathbf{e}$, we obtain

$$\begin{aligned}\dot{\mathbf{e}} &= (\mathbf{A} - \mathbf{B}\mathbf{K}_c^T)\mathbf{e} + \mathbf{B}(\zeta - q(\mathbf{x}, u))\\ e_1 &= \mathbf{C}^T\mathbf{e}.\end{aligned} \tag{4}$$

By the assumption of $\partial q(\mathbf{x}, u)/\partial u > 0$ for all $(\mathbf{x}, u) \in R^{n+1}$ and the fact $\partial\zeta/\partial u = 0$, we have

$$\frac{\partial}{\partial u}[q(\mathbf{x}, u) - \zeta] \neq 0 \tag{5}$$

for all $(u, \bar{\mathbf{e}}) \in R^{2n+2}$, where $\bar{\mathbf{e}} = \left[\mathbf{x}^T, \mathbf{y}_m^T, \zeta\right]^T \in R^{2n+1}$. According to implicit function theorem [32], for each $\bar{\mathbf{e}}$ there exists a unique solution $\bar{u}(\bar{\mathbf{e}})$ such that $q(\mathbf{x}, \bar{u}) - \zeta = 0$.Under the control $\bar{u}, (\bar{\mathbf{e}})$, (4) can be rewritten as

$$\begin{aligned}\dot{\mathbf{e}} &= \left(\mathbf{A} - \mathbf{B}\mathbf{K}_c^T\right)\mathbf{e}\\ e_1 &= \mathbf{C}^T\mathbf{e}.\end{aligned} \tag{6}$$

Because the characteristic polynomial of $\mathbf{A} - \mathbf{B}\mathbf{K}_c^T$ is Hurwitz, we have

$$\lim_{t\to\infty} \mathbf{e}(t) = 0. \tag{7}$$

By using Taylor series expansion of the nonlinear system (2) around $\bar{u}(\bar{\mathbf{e}})$, we obtain

$$\dot{x}_n = q(\mathbf{x}, \bar{u}) + g(\bar{\mathbf{e}})(u - \bar{u}) + d_h \tag{8}$$

where $g(\bar{\mathbf{e}}) = \partial q(\mathbf{x}, u)/\partial u|_{u=\bar{u}(\bar{\mathbf{e}})}$, and $d_h$ stands for higher order term. Suppose a control input $u$ is

$$u = u_f + u_s \tag{9}$$

where $u_f$ is designed to approximate the control $\bar{u}$, and the control term $u_s$ is employed to compensate the modeling error. From (4), (8), and (9), we have

$$\begin{aligned}\dot{\mathbf{e}} &= \mathbf{A}\mathbf{e} - \mathbf{B}\mathbf{K}_c^T\hat{\mathbf{e}} + \mathbf{B}[g(\bar{\mathbf{e}})\bar{u} - g(\bar{\mathbf{e}})u_f - g(\bar{\mathbf{e}})u_s + \hat{\zeta} - d]\\ e_1 &= \mathbf{C}^T\mathbf{e}\end{aligned} \tag{10}$$

where $\hat{\zeta} = \mathbf{K}_c^T\hat{\mathbf{e}}^T + y_m^{(n)}$, $d = q(\mathbf{x}, \bar{u}) + d_h$ and $\hat{\mathbf{e}}$ denotes the estimate of $\mathbf{e}$. According to (10), the control objective is to design a state observer for estimating the state vector $\mathbf{e}$ in (10) in order to regulate $e_1$ to zero.

In addition, the configuration of the fuzzy-neural network shown in Fig. 1 consists of fuzzy logic and neural network. The fuzzy logic system can be divided into two

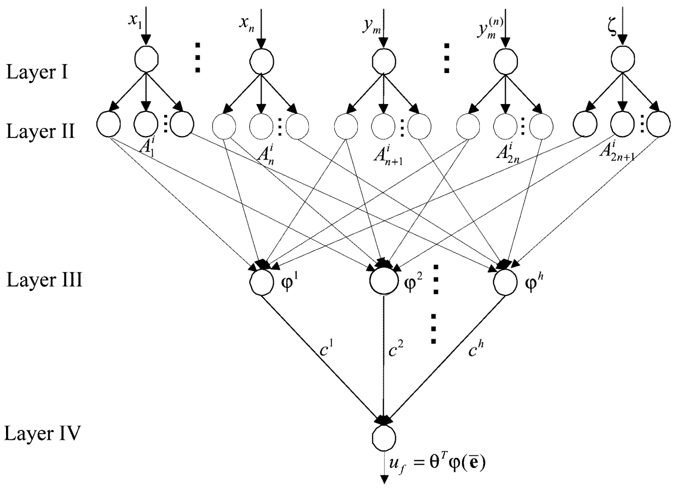


Fig. 1. Configuration of a fuzzy-neural approximator.

parts: some fuzzy IF–THEN rules and a fuzzy inference engine. The fuzzy inference engine uses the fuzzy IF–THEN rules to perform a mapping form an input linguistic vector $\bar{\mathbf{e}} = \left[\mathbf{x}^T, \mathbf{y}_m^T, \zeta\right]^T = [\bar{e}_1 \quad \bar{e}_2 \quad \ldots \quad \bar{e}_{2n+1}]^T \in R^{2n+1}$ to an output linguistic variable $u_f \in R$. The $i$th fuzzy IF–THEN rule is written as

$$\begin{aligned}&\text{If } x_1 \text{ is } A_1^i \text{ and } \ldots \text{ and } x_n \text{ is } A_n^i \text{ and } y_m \text{ is } A_{n+1}^i\\ &\text{and } \ldots \text{ and } y_m^{(n)} \text{ is } A_{2n}^i\\ &\text{and } \zeta \text{ is } A_{2n+1}^i \text{ then } u_f \text{ is } B^i \end{aligned} \tag{11}$$

where $A_1^i, A_2^i, \ldots, A_{2n+1}^i$ and $B_i$ are fuzzy sets [14], [15]. By using product inference, center-average and singleton fuzzifier, the output of the fuzzy-neural network can be expressed as

$$u_f = \frac{\sum_{i=1}^{h} c^i \left[\prod_{j=1}^{2n+1} \mu_{A_j^i}(\bar{e}_j)\right]}{\sum_{i=1}^{h} \left[\prod_{j=1}^{2n+1} \mu_{A_j^i}(\bar{e}_j)\right]} = \boldsymbol{\theta}^T p(\bar{e}) \tag{12}$$

where $\mu_{A_j^i}(\bar{e}_j)$ is the membership function value of the fuzzy variable, $h$ is the total number of the IF-THEN rules, $c^i$ is the point at which $\mu_{B^i}(c^i) = 1$, $\boldsymbol{\theta} = [c^1 \quad c^2 \quad \ldots \quad c^h]^T$ is an adjustable parameter vector, and $\varphi = [\varphi^1 \quad \varphi^2 \quad \ldots \quad \varphi^h]^T$ is a fuzzy basis vector, where $\varphi^i$ is defined as

$$\varphi^i(\bar{\mathbf{e}}) = \frac{\left[\prod_{j=1}^{2n+1} \mu_{A_j^i}(\bar{e}_j)\right]}{\sum_{i=1}^{h} \left[\prod_{j=1}^{2n+1} \mu_{A_j^i}(\bar{e}_j)\right]}. \tag{13}$$

When the inputs are given into the fuzzy-neural network shown in Fig. 1. The truth value $\varphi^i$ (layer III) of the antecedent part of the $i$th implication is calculated by (13). Among the commonly used deffuzzification strategies, the output (layer IV) of the fuzzy-neural network is expressed as (12). The fuzzy logic approximator based on the neural network can be established [25], [27]. Fig. 1 shows the configuration of the fuzzy-neural function approximator. The approximator has four layers. At layer I, nodes, which are input ones, stand for the input linguistic variables. At layer II, nodes represent the values of the membership function value. At layer III, nodes are the values of the fuzzy basis vector $\varphi$. Each node of layer III performs a fuzzy rule. The links between layer III and layer IV are full connected by the weighting vector. $\boldsymbol{\theta}$, i.e., the adjusted parameters. At layer IV, the output stands for the value of $u_f$.

## III. Design of Direct Adaptive Fuzzy-Neural Controller Via Output Feedback

In this section, our primary task is to design an observer that estimates the state vector $\mathbf{e}$ in (10), to use the fuzzy-neural network to approximate the control $\bar{u}$ and to the develop direct adaptive output-feedback update law to adjust the parameters of the fuzzy-neural network in order to achieve the control objective.

First, we replace $u_f$ in (9) by the output of the fuzzy-neural network, $\boldsymbol{\theta}^T\varphi\left(\hat{\bar{\mathbf{e}}}\right)$ in (12), i.e.

$$u_f\left(\hat{\bar{\mathbf{e}}}|\boldsymbol{\theta}\right) = \boldsymbol{\theta}^T\varphi\left(\hat{\bar{\mathbf{e}}}\right) \tag{14}$$

where $\hat{\bar{\mathbf{e}}} = \left[\hat{\mathbf{x}}^T, \mathbf{y}_m^T, \hat{\zeta}\right]^T = \left[\mathbf{y}_m^T - \hat{\mathbf{e}}^T, \mathbf{y}_m^T, \mathbf{K}_c^T\hat{\mathbf{e}}^T + y_m^{(n)}\right]^T$.

Next, consider the following observer that estimates the state vector $\mathbf{e}$ in (10):

$$\begin{aligned}\dot{\hat{\mathbf{e}}} &= \mathbf{A}\hat{\mathbf{e}} - \mathbf{B}\mathbf{K}_c^T\hat{\mathbf{e}} + \mathbf{B}(g(\bar{\mathbf{e}})v - g(\bar{\mathbf{e}})u_s) + \mathbf{K}_o(e_1 - \hat{e}_1)\\ \hat{e}_1 &= \mathbf{C}^T\hat{\mathbf{e}}\end{aligned} \tag{15}$$

where $\mathbf{K}_o = [k_1^o \quad k_2^o \quad \ldots \quad k_n^o]^T$ is the observer gain vector, chosen such that the characteristic polynomial of $\mathbf{A} - \mathbf{K}_o\mathbf{C}^T$ is strictly Hurwitz because $(\mathbf{C}, \mathbf{A})$ is observable. The control term $v$ is employed to compensate the modeling error.

Although the state observer (15) includes the unknown function $g$, a significant part of our design is that $u_s = v$ in Theorems 1 and 2, and so we can eliminate the unknown function $g$ from the state observer (15). Therefore, the proposed state observer (28) in Theorem 2 of this paper does not require the function $g$, which is assumed as an unknown function.

Define the observation errors as $\tilde{\mathbf{e}} = \mathbf{e} - \hat{\mathbf{e}}$ and $\tilde{e}_1 = e_1 - \hat{e}_1$. Subtracting (15) from (10), we have

$$\begin{aligned}\dot{\tilde{\mathbf{e}}} &= \left(\mathbf{A} - \mathbf{K}_o\mathbf{C}^T\right)\tilde{\mathbf{e}}\\ &\quad + \mathbf{B}\left[g(\bar{\mathbf{e}})\bar{u} - g(\bar{\mathbf{e}})u_f\left(\hat{\bar{\mathbf{e}}}|\boldsymbol{\theta}\right) - g(\bar{\mathbf{e}})v + w\right]\\ \tilde{e}_1 &= \mathbf{C}^T\tilde{\mathbf{e}}\end{aligned} \tag{16}$$

where $w = \hat{\zeta} - d$. Besides, the output error dynamics of (16) can be given as

$$\tilde{e}_1 = H(s)\left[g(\bar{\mathbf{e}})\bar{u} - g(\bar{\mathbf{e}})u_f\left(\hat{\bar{\mathbf{e}}}|\boldsymbol{\theta}\right) - g(\bar{\mathbf{e}})v + w\right] \tag{17}$$

where $s$ is the Laplace variable, and $H(s) = \mathbf{C}^T(I - s(\mathbf{A} - \mathbf{K}_o\mathbf{C}^T))^{-1}\mathbf{B}$ is the transfer function of (16).

In order to derive the direct adaptive output feedback update law, the following assumption must be required.

*Assumption 1 [33]:* Let $\bar{\mathbf{e}}$ and $\hat{\bar{\mathbf{e}}}$ belong to compact sets $U_{\bar{\mathbf{e}}} = \{\bar{\mathbf{e}} \in \Re^{2n+1} : \|\bar{\mathbf{e}}\| \le m_{\bar{\mathbf{e}}} < \infty\}$ and $U_{\hat{\bar{\mathbf{e}}}} = \left\{\hat{\bar{\mathbf{e}}} \in \Re^{2n+1} : \left\|\hat{\bar{\mathbf{e}}}\right\| \le m_{\hat{\bar{\mathbf{e}}}} < \infty\right\}$, respectively, where $\hat{\bar{\mathbf{e}}}$ denotes the estimate of $\bar{\mathbf{e}}$ and $m_{\bar{\mathbf{e}}}$ and $m_{\hat{\bar{\mathbf{e}}}}$ are the upper bounds of $\bar{\mathbf{e}}$ and $\hat{\mathbf{e}}$, respectively. It is known that the optimal parameter vector $\boldsymbol{\theta}^* = \arg\min_{\boldsymbol{\theta}\in M_{\boldsymbol{\theta}}} \left[\sup_{\bar{\mathbf{e}}\in U_{\bar{\mathbf{e}}},\hat{\mathbf{e}}\in U_{\hat{\mathbf{e}}}} \left|\bar{u}(\bar{\mathbf{e}}) - u_f(\hat{\mathbf{e}}|\boldsymbol{\theta})\right|\right]$ lies in some convex region $M_{\boldsymbol{\theta}} = \{\boldsymbol{\theta} \in \Re^n : \|\boldsymbol{\theta}\| \leq m_{\boldsymbol{\theta}}\}$, where the radius $m_{\boldsymbol{\theta}}$ is a design parameter. ♦

According to Assumption 1, (16) can be rewritten as

$$\begin{aligned}\dot{\tilde{\mathbf{e}}} &= \left(\mathbf{A} - \mathbf{K}_o\mathbf{C}^T\right)\tilde{\mathbf{e}} \\ &\quad + \mathbf{B}\left[g(\bar{\mathbf{e}})u_f\left(\hat{\mathbf{e}}|\boldsymbol{\theta}^*\right) - g(\bar{\mathbf{e}})u_f\left(\hat{\mathbf{e}}|\boldsymbol{\theta}\right)\right. \\ &\qquad \left. g(\bar{\mathbf{e}})v + w_m + w\right] \\ \tilde{e}_1 &= \mathbf{C}^T\tilde{\mathbf{e}}\end{aligned} \tag{18}$$

where $w_m = g(\bar{\mathbf{e}})\left[\bar{u} - u_f\left(\hat{\mathbf{e}}\,|\boldsymbol{\theta}^*\right)\right]$ is an approximation error. According to (14), (18) can be rewritten as

$$\begin{aligned}\dot{\tilde{\mathbf{e}}} &= \left(\mathbf{A} - \mathbf{K}_o\mathbf{C}^T\right)\tilde{\mathbf{e}} \\ &\quad + \mathbf{B}\left[g(\bar{\mathbf{e}})\tilde{\boldsymbol{\theta}}^T\varphi\left(\hat{\mathbf{e}}\right) - g(\bar{\mathbf{e}})v + w_m + w\right] \\ \tilde{\mathbf{e}}_1 &= \mathbf{C}^T\tilde{\mathbf{e}}\end{aligned} \tag{19}$$

where $\tilde{\boldsymbol{\theta}} = \boldsymbol{\theta}^* - \boldsymbol{\theta}$. Since only the output $\tilde{e}_1$ in (19) is assumed to be measurable, we use the strictly-positive-real (SPR) Lyapunov design approach to analyze the stability of (19) and generate the direct adaptive output-feedback update law for $\boldsymbol{\theta}$. Equation (19) can be rewritten as

$$\tilde{e}_1 = H(s)\left[g(\bar{\mathbf{e}})\tilde{\boldsymbol{\theta}}^T\varphi\left(\hat{\mathbf{e}}\right) - g(\bar{\mathbf{e}})v + w_m + w\right] \tag{20}$$

where $H(s) = \mathbf{C}^T(s\mathbf{I} - (\mathbf{A} - \mathbf{K}_o\mathbf{C}^T))^{-1}\mathbf{B}$ is a known stable transfer function. In order to employ the SPR-Lyapunov design approach, (20) can be written as

$$\tilde{e}_1 = H(s)L(s)\left[\tilde{\boldsymbol{\theta}}^T\varphi\left(\hat{\mathbf{e}}\right) - v_f + w_f\right] \tag{21}$$

where $w_f = L^{-1}w_T$, $w_T = w_m + w + g(\bar{\mathbf{e}})\tilde{\boldsymbol{\theta}}^T\varphi(\hat{\mathbf{e}}) - L(s)\tilde{\boldsymbol{\theta}}^T\varphi\left(\hat{\mathbf{e}}\right)$, and $v_f = L^{-1}g(\bar{\mathbf{e}})v$. $L(s)$ is chosen so that $L^{-1}(s)$ is a proper stable transfer function and $H(s)L(s)$ is a proper SPR transfer function. Suppose that $L(s) = s^m + b_1s^{m-1} + b_2s^{m-2} + \cdots + b_m$, where $m = n-1$, such that $H(s)L(s)$ is a proper SPR transfer function. The state–space realization of (21) can be written as

$$\begin{aligned}\dot{\tilde{\mathbf{e}}} &= \mathbf{A}_c\tilde{\mathbf{e}} + \mathbf{B}_c[\tilde{\boldsymbol{\theta}}^T\varphi\left(\hat{\mathbf{e}}\right) - v_f + w_f] \\ \tilde{e}_1 &= \mathbf{C}_c^T\tilde{\mathbf{e}}\end{aligned} \tag{22}$$

where $\mathbf{A}_c = \left(\mathbf{A} - \mathbf{K}_o\mathbf{C}^T\right) \in R^{n\times n}$, $\mathbf{B}_c^T = [1\, b_1 \quad b_2 \quad \ldots \quad b_m] \in R^n$ and $\mathbf{C}_c^T = [1 \quad 0 \quad \ldots \quad 0] \in R^n$. For the purpose of stability analysis of the observer-based direct adaptive fuzzy-neural controller, the following assumptions and lemma must be required.

*Lemma 1 [14], [27]:* Suppose that the adaptive laws are chosen as (23), shown at the bottom of the page, where the projection operator [14] is given as

$$\Pr\left(\gamma\tilde{e}_1\varphi\left(\hat{\mathbf{e}}\right)\right) = \gamma\tilde{e}_1\varphi\left(\hat{\mathbf{e}}\right) - \gamma\frac{\tilde{e}_1\boldsymbol{\theta}^T\varphi\left(\hat{\mathbf{e}}\right)}{\|\boldsymbol{\theta}\|^2}\boldsymbol{\theta}.$$

Then $\|\boldsymbol{\theta}\| \leq m_{\boldsymbol{\theta}}$ and $\|\tilde{\boldsymbol{\theta}}\| \leq 2m_{\boldsymbol{\theta}}$.

*Assumption 2:* The unknown function $g(\bar{\mathbf{e}})$ is bounded by

$$\beta_1 \leq \|g(\bar{\mathbf{e}})\| \leq \beta_2 \tag{24}$$

where $\beta_1$ and $\beta_2$ positive constants.

*Assumption 3:* $w_T$ is assumed to satisfy

$$|w_T| \leq \varepsilon \tag{25}$$

where $\varepsilon$ is a positive constant.

*Remark 1:* Due to (6) and (7), and the existence of $\bar{u}(\bar{\mathbf{e}})$ in [32], the assumption of boundedness of $\bar{\mathbf{e}} = \left[\mathbf{x}^T, \mathbf{y}_m^T, \zeta\right]^T$ in Assumption 1 is reasonable. Since $\hat{\mathbf{e}} = \left[\hat{\mathbf{x}}^T, \mathbf{y}_m^T, \hat{\zeta}\right]^T$ denotes the estimation of $\bar{\mathbf{e}}$, the assumption of boundedness of $\hat{\mathbf{e}}$ is also reasonable. The boundedness of $w = \hat{\zeta} - d = \mathbf{K}_c^T\hat{\mathbf{e}} + y_m^{(n)} - q(\mathbf{x}, \bar{u}) - d_h$ follows that of $\hat{\zeta}$ (or $\hat{\mathbf{e}}$). The $\varphi\left(\hat{\mathbf{e}}\right)$ would be differentiable, if we choose differentiable functions, for example, exponential functions, to be the fuzzy membership functions. From Lemma 1, the vector $\boldsymbol{\theta}$ is adjustable and differentiable. Therefore, $\tilde{\boldsymbol{\theta}}^T\varphi\left(\hat{\mathbf{e}}\right)$ is differentiable. Since $w_f$ is bounded, $w_T$ is bounded. Therefore, Assumption 3 is reasonable. The goal of this paper is to develop a controller $u = u_f + u_s$, where $u_f$ is designed to approximate the controller $\bar{u}(\bar{\mathbf{e}})$ in [32] and the control term $u_s$ is employed to compensate the modeling error, to guarantee that all signals involved are bounded and the output of the closed-loop system will asymptotically track the desired output trajectory.

On the basis of the previous discussions, the following theorems can be obtained.

*Theorem 1:* Consider the system (22) that satisfies Assumptions 1–3. Let $\boldsymbol{\theta}$ be adjusted by the update law (23), and let $v$ be given as

$$v = \begin{cases}\rho, & \text{if } \tilde{e}_1 \geq 0 \\ -\rho, & \text{if } \tilde{e}_1 < 0\end{cases} \tag{26}$$

where $\rho \geq (\varepsilon/\beta_1)$. Then $\tilde{e}_1(t)$ converges to zero as $t \to \infty$.

*Proof:* Given in the Appendix.

*Theorem 2:* Consider the nonlinear system (1) that satisfies Assumptions 1–3. Suppose that the control law is

$$u = u_f(\hat{\mathbf{e}}|\boldsymbol{\theta}) + u_s \tag{27}$$

$$\dot{\boldsymbol{\theta}} = \begin{cases}\gamma\tilde{e}_1\varphi\left(\hat{\mathbf{e}}\right), & \text{if } \|\boldsymbol{\theta}\| < m_{\boldsymbol{\theta}} \text{ or } \left(\|\boldsymbol{\theta}\| = m_{\boldsymbol{\theta}} \text{ and } \tilde{e}_1\boldsymbol{\theta}^T\varphi\left(\hat{\mathbf{e}}\right) \geq 0\right) \\ \Pr\left(\gamma\tilde{e}_1\varphi\left(\hat{\mathbf{e}}\right)\right), & \text{if } \|\boldsymbol{\theta}\| = m_{\boldsymbol{\theta}} \text{ and } \tilde{e}_1\,\boldsymbol{\theta}^T\varphi\left(\hat{\mathbf{e}}\right) < 0\end{cases} \tag{23}$$

with the adaptive law (23). Let $u_s = v$. The state observer (15) becomes

$$\begin{aligned}\dot{\hat{\mathbf{e}}} &= \mathbf{A}\hat{\mathbf{e}} - \mathbf{B}\mathbf{K}_c^T\hat{\mathbf{e}} + \mathbf{K}_o(e_1 - \hat{e}_1)\\ \hat{e}_1 &= \mathbf{C}^T\hat{\mathbf{e}}.\end{aligned} \tag{28}$$

Then all signals in the closed-loop system are bounded, and $e_1(t)$ converges to zero as $t \to \infty$. ♦

*Proof:* Given in the Appendix.

According to the previous theorems, the design algorithm of the direct adaptive fuzzy-neural controller is described as the following.

Design Algorithm

Step 1) Select the feedback and observer gain vectors $\mathbf{K}_c$, $\mathbf{K}_o$ such that the matrices $\mathbf{A} - \mathbf{B}\mathbf{K}_c^T$ and $\mathbf{A} - \mathbf{K}_o\mathbf{C}^T$ are Hurwitz matrices, respectively.

Step 2) Choose appropriate values $\rho$ in (26), $\gamma$ and $m_{\boldsymbol{\theta}}$ in (23). In order to remedy the control chattering, (26) can be modified as

$$v = \begin{cases} \rho & \text{if } \tilde{e}_1 \geq 0 \text{ and } |\tilde{e}_1| > \alpha, \\ -\rho & \text{if } \tilde{e}_1 < 0 \text{ and } |\tilde{e}_1| > \alpha, \\ \frac{\rho\tilde{e}_1}{\alpha} & \text{if } |\tilde{e}_1| < \alpha \end{cases}$$

where $\alpha$ is a positive constant.

Step 3) Solve the state observer in (28).

Step 4) Construct fuzzy sets for $\hat{\hat{\mathbf{e}}}(t)$. From (13), compute the fuzzy basis vector $\varphi$.

Step 5) Obtain the control law (27), and the update law (23).

*Remark 2:* The initial values of $\boldsymbol{\theta}(0)$ should be determined before solving the adaptive laws in (23). The value of $\gamma$ in (23) is obtained by trial and error according to the values of $\boldsymbol{\theta}(0)$. In addition to compute the controller $u = u_f + u_s$ in (27), we need to decide $\rho$. The chosen value of $\rho$ is obtained by trial and error such that $\rho \geq (\varepsilon/\beta_1)$ and based on Assumptions 2–3, without using any adaptive tuning procedure in this paper. Larger $\rho$ results in larger control input according to (27). From (26), we see that the absolute value of the control term $v$ is the value of $\rho$. The control term $v$ is employed to compensate for external disturbance and modeling error.

*Remark 3:* Regarding Step 4) of the design algorithm, the number of fuzzy rules depends on the number of inputs and the number of fuzzy sets of each input. For example, we can generate $h = m^{2n+1}$ fuzzy rules for $2n+1$ inputs, in which each input has $m$ fuzzy sets. The input vector is $\hat{\hat{\mathbf{e}}} = [\hat{\hat{e}}_1 \quad \hat{\hat{e}}_2 \quad \ldots \quad \hat{\hat{e}}_{2n+1}] = \left[\hat{\mathbf{x}}^T, \mathbf{y}_m^T, \hat{\zeta}\right]^T$. The membership function of each fuzzy set can be a bell-shaped form or others. Then, we can compute the values of the fuzzy bases from (13).

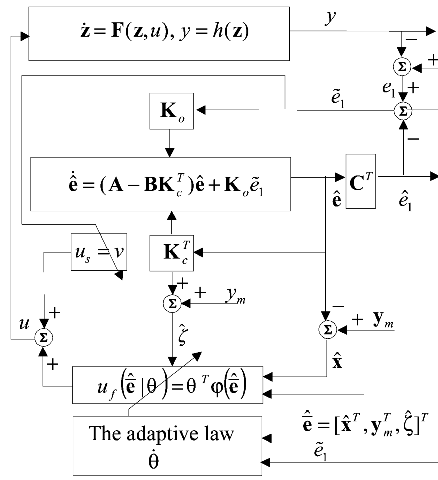


Fig. 2. Overall scheme of the proposed direct adaptive fuzzy-neural control.

To summarize, Fig. 2 shows the overall scheme of the observer-based direct adaptive fuzzy-neural control proposed in this paper.

## IV. Illustrative Examples

This section presents the simulation results of the proposed direct adaptive fuzzy-neural controller to illustrate that the stability of the closed-loop system is guaranteed, and all signal involved are bounded.

*Example 1:* Consider the nonlinear system

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= 0.2(1 + e^{x_1 x_2})(2 + \sin(x_2))(u + e^u - 1)\\ y &= x_1.\end{aligned} \tag{29}$$

The control objective is to control the state $x_1$ of the system to track the reference trajectory $y_m = 1.5 - e^{-0.5t}$ (case 1) and $y_m = \sin(0.5t) + \cos(t)$ (case 2). The design parameters are selected as $\gamma = 0.5 \times 10^3$, $\rho = 5$, and $m_\theta = 1.36 \times 10^3$. The feedback and observer gain vectors are given as $\mathbf{K}_c = [144 \quad 24]^T$ and $\mathbf{K}_o = [60 \quad 900]^T$, respectively. The filter $L^{-1}(s)$ is given as $L^{-1}(s) = 1/(s+2)$. The membership functions for $\hat{\hat{e}}_j$, $j = 1, 2, \ldots, 5$ are given as

$$\begin{aligned}\mu_{A_j^1}(\hat{\hat{e}}_j) &= \frac{1}{(1 + \exp(5 \times (\hat{\hat{e}}_j + 3)))}\\ \mu_{A_j^2}(\hat{\hat{e}}_j) &= \exp(-(\hat{\hat{e}}_j + 2)^2)\\ \mu_{A_j^3}(\hat{\hat{e}}_j) &= \exp(-(\hat{\hat{e}}_j + 1)^2)\\ \mu_{A_j^4}(\hat{\hat{e}}_j) &= \exp(-(\hat{\hat{e}}_j)^2)\\ \mu_{A_j^5}(\hat{\hat{e}}_j) &= \exp(-(\hat{\hat{e}}_j - 1)^2)\\ \mu_{A_j^6}(\hat{\hat{e}}_j) &= \exp(-(\hat{\hat{e}}_j - 2)^2)\\ \mu_{A_j^7}(\hat{\hat{e}}_j) &= \frac{1}{(1 + \exp(-5 \times (\hat{\hat{e}}_j)))}.\end{aligned}$$

The initial states are chosen to be $\mathbf{x}(0) = [0.1 \ 0]^T$(case 1), $\mathbf{x}(0) = [0 \ 0]^T$(case 2), $\hat{\hat{\mathbf{e}}}(0) = [0 \ 0 \ 0.5 \ 0.5 \ 0.25]^T$(case 1), and $\hat{\hat{\mathbf{e}}}(0) = [0 \ 0.1 \ 0.5 \ -1]^T$(case 2). The computer simulation results are shown in Figs. 3–8. From Figs. 4 and 7, it is observed

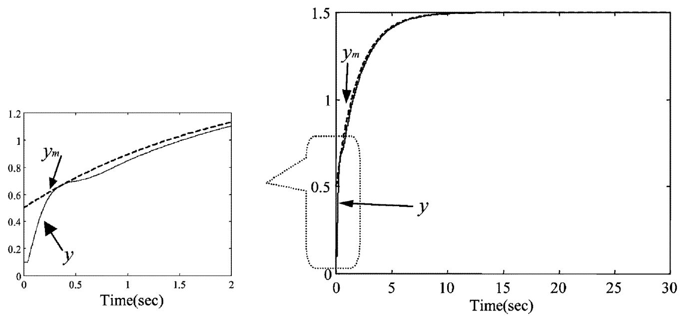


Fig. 3. Trajectories of $y(t)$ and $y_m(t)$ (case 1) in Example 1.

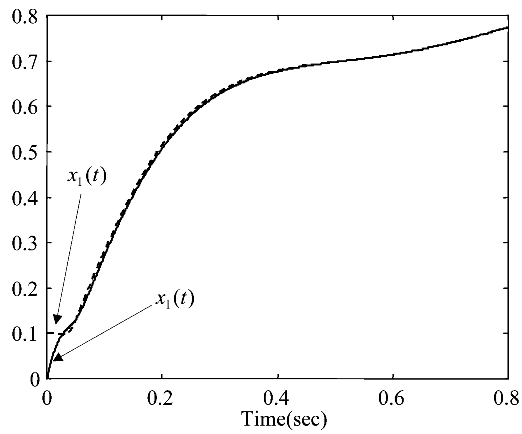


Fig. 4. Trajectories of $x_1(t)$ and $\hat{x}_1(t)$ (case 1) in Example 1.

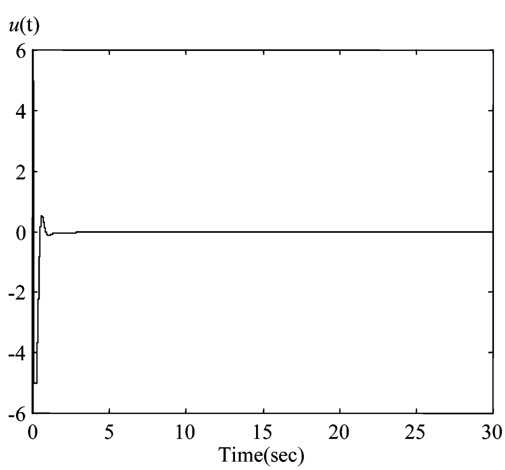


Fig. 5. Control input $u(t)$ (case 1) in Example 1.

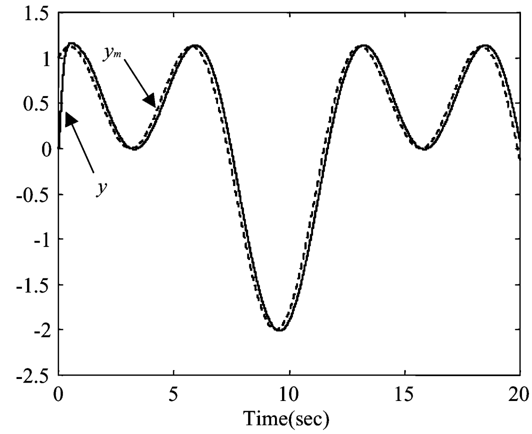


Fig. 6. Trajectories of $y(t)$ and $y_m(t)$ (case 2) in Example 1.

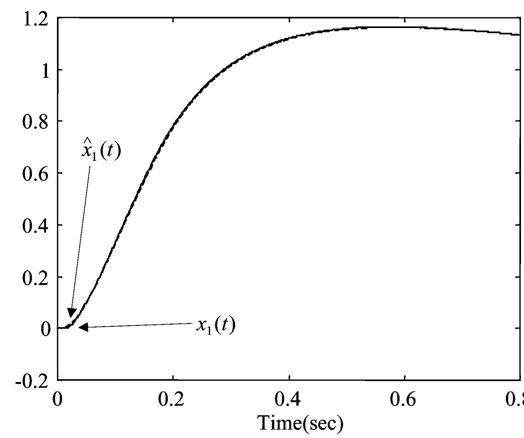


Fig. 7. Trajectories of $x_1(t)$ and $\hat{x}_1(t)$ (case 2) in Example 1.

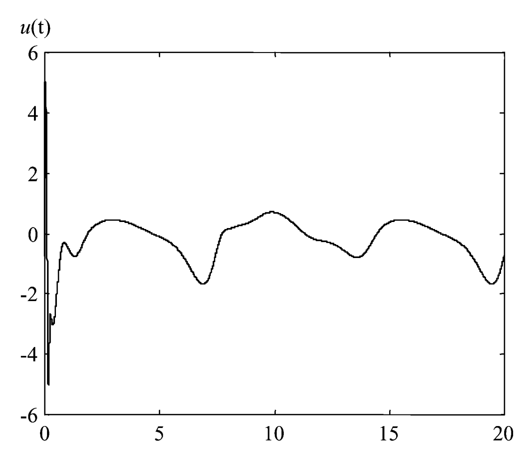


Fig. 8. Control input $u(t)$ (case 2) in Example 1.

the state observer can generate the estimated state very fast and correct. Moreover, as shown in Figs. 3 and 6, it is observed that the tracking error is small, and the convergence of tracking error is fast and well. The control signals for two cases are shown in Figs. 5 and 8. The computer simulation results show that the observer-based direct adaptive fuzzy-neural controller can perform successful control and achieve desired performance for the nonaffine nonlinear systems.

*Example 2:* Consider the nonlinear system [23]

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_1^2 + 0.15u^3 + 0.1(1+x_2^2)u + \sin(0.1u) \\ y &= x_1. \end{aligned} \tag{30}$$

The control objective is to control the state $x_1$ of the system to track the reference trajectory $y_m = \sin(t) + \cos(0.5t)$. The design parameters are selected as $\gamma = 0.5 \times 10^3$, $\rho = 5$, and $m_{\boldsymbol{\theta}} = 1.36 \times 10^3$. The feedback and observer gain vectors are given as $\mathbf{K}_c = [144 \quad 24]^T$ and $K_o = [60 \quad 900]^T$, respectively. The filter $L^{-1}(s)$ is given as $L^{-1}(s) = 1/(s+2)$. The membership functions for $\hat{\bar{e}}_j$, $j = 1, 2, \ldots, 5$ are the same as those in Example 1. The initial states are chosen to be $\mathbf{x}(0) = [0.5, 0.6]^T$ and $\hat{\bar{\mathbf{e}}}(0) = [0, 0, 1, 1, -0.25]^T$. The computer simulation results are shown in Figs. 9–11. From the simulation

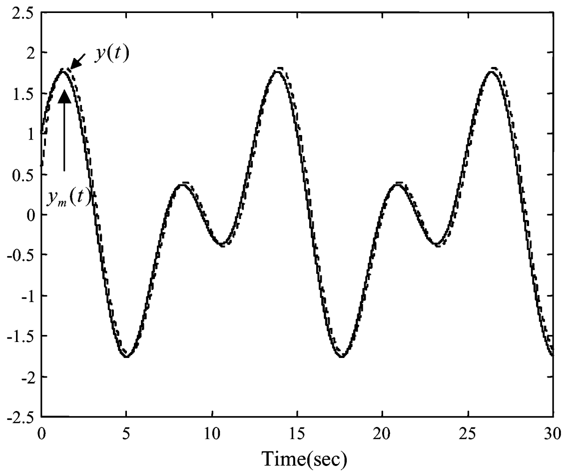


Fig. 9. Trajectories of $y(t)$ and $y_m(t)$ in Example 2.

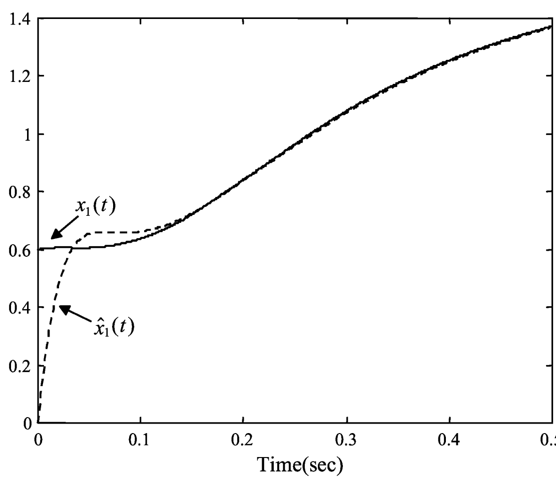


Fig. 10. Trajectories of $x_1(t)$ and $\hat{x}_1(t)$ in Example 2.

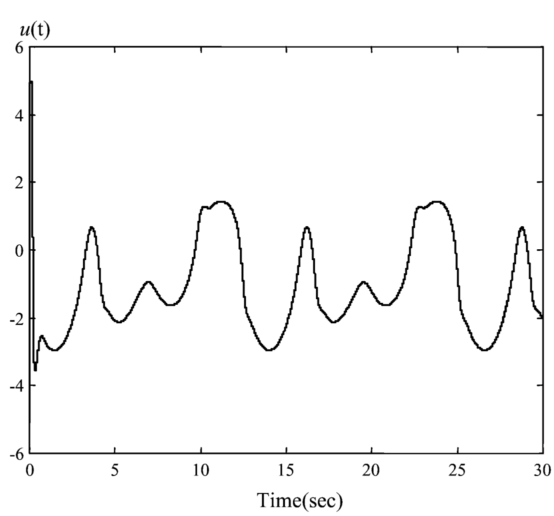


Fig. 11. Control input $u(t)$ in Example 2.

results, it is observed that the state observer can generate the estimated state very fast and correct. Moreover, it is also observed that the tracking error is small, and the convergence of tracking error is fast and well. In comparison with the control input in [23], using the high gain observer and saturation method to overcome the peaking phenomenon in the transient behavior, the proposed control input shown in Fig. 11 is without saturation and smoother than that in [23], especially during the transient period.

## V. Conclusion

In this paper, an observer-based direct adaptive fuzzy-neural control scheme is presented for nonaffine nonlinear systems in the presence of unknown structure of nonlinearities. To design the output feedback control law, no exact knowledge of structure of system nonlinearities is needed. In addition, the preliminary offline tuning of the weighting factors of the fuzzy-neural controller is not required. The overall adaptive scheme guarantees that all signals involved are bounded and the output of the closed-loop system asymptotically tracks the desired output trajectory. Finally, this method has been applied to control the nonaffine nonlinear system to track a reference trajectory. The computer simulation results show that the observer-based direct adaptive fuzzy-neural controller can perform successful control and achieve desired performance. In the future, investigation on the adaptive tuning of the design parameters and designing multi-input–multi-output (MIMO) systems will be interesting research topics in this field.

## Appendix

### A. Proof of Theorem 1

Consider the Lyapunov-like function candidate

$$V = \frac{1}{2}\tilde{\mathbf{e}}^T\mathbf{P}\tilde{\mathbf{e}} + \frac{1}{2\gamma}\tilde{\boldsymbol{\theta}}^T\tilde{\boldsymbol{\theta}} \tag{A.1}$$

where $\mathbf{P} = \mathbf{P}^T > 0$. Differentiating (A.1) with respect to time and inserting (22) in the previous equation yield

$$\dot{V} = \frac{1}{2}\tilde{\mathbf{e}}^T\left(\mathbf{A}_c^T\mathbf{P} + \mathbf{P}\mathbf{A}_c\right)\tilde{\mathbf{e}} + \tilde{\mathbf{e}}^T\mathbf{P}\mathbf{B}_c\left[\tilde{\boldsymbol{\theta}}^T\varphi - v_f + w_f\right] + \frac{1}{\gamma}\dot{\tilde{\boldsymbol{\theta}}}^T\tilde{\boldsymbol{\theta}}. \tag{A.2}$$

Because $H(s)L(s)$ is SPR, there exists $\mathbf{P} = \mathbf{P}^T > 0$ such that

$$\begin{aligned}\mathbf{A}_c^T\mathbf{P} + \mathbf{P}\mathbf{A}_c &= -\mathbf{Q}\\ \mathbf{P}\mathbf{B}_c &= \mathbf{C}_c\end{aligned} \tag{A.3}$$

where $\mathbf{Q} = \mathbf{Q}^T > 0$. By using (A.3), (A.2) becomes

$$\dot{V} = -\frac{1}{2}\tilde{\mathbf{e}}^T\mathbf{Q}\tilde{\mathbf{e}} + \tilde{e}_1\left[\tilde{\boldsymbol{\theta}}^T\varphi - v_f + w_f\right] + \frac{1}{\gamma}\dot{\tilde{\boldsymbol{\theta}}}^T\tilde{\boldsymbol{\theta}}. \tag{A.4}$$

By using Assumptions 2–3, (26) and the fact $\lambda_{\min}(\mathbf{Q})\|\tilde{\mathbf{e}}\|^2 \geq \lambda_{\min}(\mathbf{Q})|\tilde{e}_1|^2$, where $\lambda_{\min}(\mathbf{Q}) \geq 0$, we have

$$\dot{V} \leq -\frac{1}{2}\lambda_{\min}(\mathbf{Q})|\tilde{e}_1|^2 + \tilde{e}_1\left(\tilde{\boldsymbol{\theta}}^T\varphi + \frac{1}{\gamma}\dot{\tilde{\boldsymbol{\theta}}}^T\tilde{\boldsymbol{\theta}}\right). \tag{A.5}$$

Inserting (23) of Lemma 1 in (A.5) and after some manipulation yields

$$\dot{V} \le -\frac{1}{2}\lambda_{\min}(\mathbf{Q})|\tilde{e}_1|^2. \tag{A.6}$$

Equations (26) and (A.6) only guarantee that $\tilde{e}_1(t) \in L_\infty$ and $\tilde{\mathbf{e}}(t) \in L_\infty$, but do not guarantee the convergence. Because all variables in the right-hand side of (22) are bounded, $\dot{\tilde{e}}_1(t)$ is bounded, i.e., $\dot{\tilde{e}}_1(t) \in L_\infty$. Integrating both side of (A.6) and after some manipulation yields

$$\int_0^\infty |\tilde{e}_1(t)|^2 dt \le \frac{V(0) - V(\infty)}{\frac{1}{2}\lambda_{\min}(\mathbf{Q})}. \tag{A.7}$$

Since the right side of (A.7) is bounded, so $\tilde{e}_1(t) \in L_2$. Using Barbalat's lemma [3], we have $\lim_{t\to\infty} |\tilde{e}_1(t)| = 0$. This completes the proof.

### *B. Proof of Theorem 2*

First, from Theorem 1, we have $\lim_{t\to\infty} |\tilde{e}_1(t)| = 0$ and $\tilde{\mathbf{e}}(t) \in L_\infty$. Using (15) and the fact $u_s = v$, we obtain

$$\dot{\hat{\mathbf{e}}} = \left(\mathbf{A} - \mathbf{B}\mathbf{K}_c^T\right)\hat{\mathbf{e}} + \mathbf{K}_o\mathbf{C}^T\tilde{\mathbf{e}} \quad \hat{e}_1 = \mathbf{C}^T\hat{\mathbf{e}}. \tag{B.1}$$

Similarly, because $\mathbf{A} - \mathbf{B}\mathbf{K}_c^T$ is a Hurwitz matrix and $\tilde{\mathbf{e}}(t)$ is bounded, $\hat{\mathbf{e}}(t)$ is bounded. From $\tilde{\mathbf{e}} = \mathbf{e} - \hat{\mathbf{e}}$, it follows that $e_1, \mathbf{e} \in L_\infty$ and $e_1(t) \to 0$ as $t \to \infty$. From $\hat{\mathbf{e}}, \mathbf{e}, \mathbf{y}_m \in L_\infty$, it follows that $\mathbf{x}, \hat{\mathbf{x}}, \bar{\mathbf{e}}, \hat{\bar{\mathbf{e}}} \in L_\infty$. The boundedness of $y(t)$ follows that of $e_1(t)$ and $y_m(t)$. This completes the proof.

## REFERENCES


[1] R. V. Monopoli, "Model reference adaptive control with an augmented error signal," *IEEE Trans. Autom. Control*, vol. AC-19, no. 5, pp. 474–484, Oct. 1974.

[2] K. S. Narendra and A. M. Annaswamy, *Stable Adaptive Systems*. Englewood Cliffs, NJ: Prentice-Hall, 1989.

[3] S. S. Sastry and M. Bodson, *Adaptive Control: Stability, Convergence, and Robustness*. Englewood Cliffs, NJ: Prentice-Hall, 1989.

[4] P. A. Ioannou and K. S. Tsakalis, "A robust direct adaptive controller," *IEEE Trans. Autom. Control*, vol. AC-31, no. 11, pp. 1033–1043, Nov. 1986.

[5] P. A. Ioannou and A. Datta, "Robust adaptive control: A unified approach," *Proc. IEEE*, vol. 79, no. 12, pp. 1735–1768, Dec. 1991.

[6] P. A. Ioannou and J. Sun, *Robust Adaptive Control*. Englewood Cliffs, NJ: Prentice-Hall, 1996.

[7] A. Isidori, *Nonlinear Control System*. New York: Springer-Verlag, 1989.

[8] J. J. E. Slontine and W. Li, *Applied Nonlinear Control*. Englewood Cliffs, NJ: Prentice-Hall, 1991.

[9] S. S. Sastry and A. Isidori, "Adaptive control of linearizable systems," *IEEE Trans. Autom. Control*, vol. 34, no. 11, pp. 1123–1131, Nov. 1989.

[10] S. Monaco and D. Normand-Cyrot, "Minimum-phase nonlinear discrete-time systems and feedback stabilization," in *Proc. IEEE Conf. Decision Control*, Los Angeles, CA, 1987, pp. 979–986.

[11] B. Jakubezyk, "Feedback linearization of discrete time system," *Syst. Control Lett.*, vol. 9, pp. 411–416, 1987.

[12] K. Hornik, M. Stinchcombe, and H. White, "Multilayer feedforward networks are universal approximators," *Neural Netw.*, no. 2, pp. 359–366, 1989.

[13] L. X. Wang and J. M. Mendel, "Fuzzy basis functions, universal approximation, and orthogonal least squares learning," *IEEE Trans. Neural Netw.*, vol. 3, no. 5, pp. 807–814, Sep. 1992.

[14] L. X. Wang, *Adaptive Fuzzy Systems and Control: Design and Stability Analysis*. Englewood Cliffs, NJ: Prentice-Hall, 1994.

[15] M. Jamshidi, N. Vadiee, and T. J. Ress, *Fuzzy Logic and Control*. Englewood Cliffs, NJ: Prentice-Hall, 1993.

[16] M. Polycarpou and P. A. loannou, "Modeling, identification and stable adaptive control of continuous-time nonlinear dynamical systems using neural networks," in *Proc. Amer. Control Conf.*, 1992, pp. 36–40.

[17] E. B. Kosmatopoulos, P. A. Ioannou, and M. A. Christodoulou, "Identification of nonlinear systems using new dynamic neural network structures," in *Proc. IEEE Conf. Decision Control*, Arizona, 1992, pp. 20–25.

[18] C. A. Rovithakis and M. A. Christodoulou, "Adaptive control of unknown plants using dynamical neural networks," *IEEE Trans. Syst. Man, Cybern.*, vol. 24, no. 3, pp. 400–411, Mar. 1995.

[19] F. C. Chen and H. K. Khalil, "Adaptive control of nonlinear systems using neural networks," *Int. J. Control*, vol. 55, no. 3, pp. 1299–1317.

[20] R. M. Sanner and J. J. E. Slontine, "Gaussian networks for direct adaptive control," *IEEE Trans. Neural Netw.*, vol. 1, no. 6, pp. 837–863, Nov. 1992.

[21] Y. K. Kim, F. L. Lewis, and C. T. Abdallah, "A dynamic recurrent neural-network-based adaptive observer for a class of nonlinear systems," *Automatica*, vol. 33, no. 8, pp. 1539–1543, 1997.

[22] T. Zhang, S. S. Ge, and C. C. Hang, "Adaptive output feedback control for general nonlinear systems using multilayer neural networks," in *Proc. Amer. Control Conf.*, pp. 520–524.

[23] S. S. Ge, C. C. Hang, and T. Zhang, "Adaptive neural network control of nonlinear systems by state and output feedback," *IEEE Trans. Syst. Man, Cybern. B, Cybern.*, vol. 29, no. 6, pp. 818–828, Dec. 1999.

[24] S. Horikawa, T. Furuhashi, and Y. Uchikawa, "On fuzzy modeling using fuzzy neural networks with the back-propagation algorithm," *IEEE Trans. Neural Netw.*, vol. 3, no. 5, pp. 801–806, Sep. 1992.

[25] C. T. Lin and C. S. G. Lee, "Neural-network-based fuzzy logic control and decision system," *IEEE Trans. Comput.*, vol. 40, no. 12, pp. 1320–1336, Dec. 1991.

[26] C. H. Wang, W. Y. Wang, T. T. Lee, and P. S. Tseng, "Fuzzy B-spline membership function and its application in fuzzy-neural control," *IEEE Trans. Syst. Man, Cybern.*, vol. 25, no. 5, pp. 841–851, May 1995.

[27] Y. G. Leu, W. Y. Wang, and T. T. Lee, "Robust adaptive fuzzy-neural controller for uncertain nonlinear systems," *IEEE Trans. Robot. Automat.*, vol. 15, no. 5, pp. 805–817, Oct. 1999.

[28] Y. G. Leu, T. T. Lee, and W. Y. Wang, "Observer-based adaptive fuzzy-neural control for unknown nonlinear dynamical systems," *IEEE Trans. Syst. Man, Cybern. B, Cybern.*, vol. 29, no. 5, pp. 583–591, Oct. 1999.

[29] W. Y. Wang, M. L. Chan, C. C. Hsu, and T. T. Lee, "Tracking-based sliding mode control for uncertain nonlinear systems via an adaptive fuzzy-neural approach," *IEEE Trans. Syst. Man, Cybern. B, Cybern.*, vol. 32, no. 4, pp. 483–492, Aug. 2002.

[30] L. X. Wang, "Stable adaptive fuzzy control of nonlinear systems," *IEEE Trans. Fuzzy Syst.*, vol. 1, no. 2, pp. 146–155, May 1993.

[31] B. S. Chen, C. H. Lee, and Y. C. Chang, "$H_\infty$ Tracking design of uncertain nonlinear SISO systems: Adaptive fuzzy approach," *IEEE Trans. Fuzzy Syst.*, vol. 4, no. 1, pp. 32–43, Feb. 1996.

[32] H. K. Khalil, *Nonlinear Systems*. New York: Macmillan, 1992.

[33] K. S. Tsakalis and P. A. Ioannou, *Linear Time-Varying Systems*. Englewood Cliffs, NJ: Prentice-Hall, 1993.

[34] W.-Y. Wang and Y.-H. Li, "Evolutionary learning of BMF fuzzy-neural networks using a reduced-form genetic algorithm," *IEEE Trans. Syst. Man, Cybern. B, Cybern.*, vol. 33, no. 6, pp. 966–976, Dec. 2003.

[35] W. L. Tung and C. Quek, "Falcon: Neural fuzzy control and decision systems using FKP and PFKP clustering algorithms," *IEEE Trans. Syst. Man, Cybern. B, Cybern.*, vol. 34, no. 1, pp. 686–695, Feb. 2004.

[36] W.-Y. Wang, C.-Y. Cheng, and Y.-G. Leu, "An on-line GA-based output-feedback direct adaptive fuzzy-neural controller for uncertain nonlinear systems," *IEEE Trans. Syst. Man, Cybern. B, Cybern.*, vol. 34, no. 1, pp. 334–345, Feb. 2004.

[37] S. S. Ge and J. Zhang, "Neural-network control of nonaffine nonlinear system with zero dynamics by state and output feedback," *IEEE Trans. Neural Netw.*, vol. 14, no. 4, pp. 900–918, Jul. 2003.

[38] S. S. Ge and C. Wang, "Adaptive neural control of uncertain MIMO nonlinear systems," *IEEE Trans. Neural Netw.*, vol. 15, no. 3, pp. 674–692, May 2004.

[39] Y. Li, S. Qiang, and X. Z. O. Kaynak, "Robust and adaptive backstepping control for nonlinear systems using RBF neural networks," *IEEE Trans. Neural Netw.*, vol. 15, no. 3, pp. 693–701, May 2004.

[40] Q. Zhu and L. Guo, "Stable adaptive neurocontrol for nonlinear discrete-time systems," *IEEE Trans. Neural Netw.*, vol. 15, no. 3, pp. 653–662, May 2004.

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