

# New Nonlinear Algorithms for Estimating and Suppressing Narrowband Interference in DS Spread Spectrum Systems

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**Abstract**—It has been shown that the narrow-band (NB) interference suppression capability of a direct-sequence (DS) spread spectrum system can be enhanced considerably by processing the received signal via a prediction error filter. The conventional approach to this problem makes use of a linear filter. However, the binary DS signal, that acts as noise in the prediction process, is highly non-Gaussian. Thus, linear filtering is not optimal. Vijayan and Poor [11] first proposed using a nonlinear approximate conditional mean (ACM) filter of the Masreliez type and obtained significant results. This paper proposes a number of new nonlinear algorithms. Our work consists of three parts. 1) We develop a decision-directed Kalman (DDK) filter, that has the same performance as the ACM filter but a simpler structure. 2) Using the nonlinear function in the ACM and the DDK filters, we develop other nonlinear least mean square (LMS) filters with improved performance. 3) We further use the nonlinear functions to develop nonlinear recursive least squares (RLS) filters that can be used independently as predictors or as interference identifiers so that the ACM or the DDK filter can be applied. Simulations show that our nonlinear algorithms outperform conventional ones.

## I. INTRODUCTION

**S**PREAD-SPECTRUM techniques provide an effective way to cope with narrow-band (NB) interference in communication systems [1]. The basic idea behind these techniques is to spread the bandwidths of transmitting signals so that they are much greater than the information rate. In a direct sequence (DS) spread spectrum system, bandwidth spreading is realized by modulating the transmitting signal with a pseudo-noise (PN) signal. At the receiver, the signal is de-spread by correlating it with the same PN sequence. This system can reject interference whose bandwidth is small compared to the spread signal. It has been shown [2]–[11] that the capability of the DS spread spectrum systems to reject NB interference can be further improved by processing the received signal. This is because there exists strong correlation in the NB signals. Thus, the interference can be predicted from its past values. However, the spread signal is white and cannot be

predicted. Once the prediction of the interference is obtained, an error signal is computed by subtracting the estimate from the received signal. This error signal is then used as the input of the correlator.

In [2]–[5], Hsu and Giodano, Keltchum and Proakis, Iltis and Milstein applied linear transversal filters to the problem of NB interference rejection. Masry gave closed-form expressions for the signal-to-noise ratio (SNR) improvement [6], [7]. Linear least mean square (LMS) estimation techniques offer an adaptive way of prediction. One distinct advantage of the LMS algorithm is its simplicity. Li and Milstein [8] applied the algorithm to the NB interference rejection problem. Iltis and Milstein then analyzed the performance in [9]. Though the linear filtering approach has yielded fruitful results, one problem has been overlooked. In the prediction of interference, the DS signal acts as noise. Since noise degrades the performance of the LMS algorithm, direct use of the algorithm cannot produce optimal results. To remedy this problem, we can filter the received signal before it enters the transversal filter. This approach will be elaborated in this paper.

Another approach is to obtain the state-space representation of the system so the well-known Kalman filter can be applied. Since the DS sequence, which can be modeled as an independent, identically distributed (i.i.d.) binary sequence, is non-Gaussian, the Kalman filter is not optimal. The optimal filter in this case is nonlinear. To solve this problem, Vijayan and Poor [11] modeled the NB interference as an AR process and applied a Masreliez-type approximate conditional mean (ACM) filter. The performance of this approach is significantly better than the Kalman filter. However, the problem with this approach is the difficulty of identifying the AR parameters. Since the poles of the AR model are close to the unit circle, standard recursive identification schemes cannot work satisfactorily. Therefore, Vijayan and Poor used the nonlinear function in the ACM filter to develop adaptive nonlinear LMS type of algorithms that do not need the system model. They showed that nonlinear LMS algorithms perform appreciably better than linear ones. These algorithms were extended by Rusch and Poor [12] to suppress NB interference in CDMA spread spectrum systems.

In this paper, we propose new nonlinear algorithms to suppress NB interference. Our contribution has three parts. First, to simplify the computational complexity of the nonlinear functions in the ACM filter, we propose a decision-directed Kalman (DDK) filter. While the DDK filter retains the

Paper approved by J. S. Lehnert, the Editor for Modulation and Signal Design of the IEEE Communications Society. Manuscript received May 10, 1994; revised March 28, 1995 and August 21, 1995. This paper was presented in part at the 1995 International Symposium on Communications, Taipei, Taiwan, December 27–29, 1995. This work was supported by the National Science Council, Taiwan, ROC 83-0404-E-009-105.

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 Publisher Item Identifier S 0090-6778(96)03142-X.

performance of the ACM filtering, it requires less computation. Second, we develop nonlinear LMS types of algorithms by employing estimation theory. As mentioned above, the input to the LMS algorithm is contaminated by non-Gaussian noise. Better results can be obtained by filtering the noisy input. Since the noise is non-Gaussian, the LMS algorithm becomes nonlinear. This concept of "input filtering" can be applied to the recursive least squares (RLS) algorithm, resulting in nonlinear RLS algorithms. This leads to the third part of our work. We propose to employ the nonlinear RLS algorithm to identify the AR parameters of interference and then apply the ACM or the DDK algorithm to perform filtering in the state space domain. We show that nonlinear RLS algorithms perform well even when the poles of the AR model are close to the unit circle.

The organization of this paper is as follows. Section II provides a brief review of the LMS algorithm and related nonlinear algorithms in [11] and [12]. In Section III, new nonlinear filters are derived. Simulations are described in Section IV. Section V presents our concluding remarks.

## II. LMS AND ACM FILTERING

### A. System Model

We follow the model described in [11]. The low-pass equivalent of a DS spread spectrum modulation waveform is given by

$$m(t) = \sum_{k=0}^{N_c-1} c_k q(t - k\tau_c) \quad (1)$$

where  $N_c$  is the number of PN chips per message bit,  $\tau_c$  is the chip interval,  $c_k$  is the  $k$ th chip of the PN sequence, and  $q(t)$  is a rectangular pulse of duration  $\tau_c$ . The transmitted signal is expressed as

$$s(t) = \sum_k b_k m(t - kT_b) \quad (2)$$

where  $\{b_k\}$  is the binary information sequence and  $T_b = N_c\tau_c$  is the bit duration. The received signal is defined by

$$z(t) = \alpha s(t - \tau) + n(t) + i(t) \quad (3)$$

where  $\alpha$  is an attenuation factor,  $\tau$  is a delay offset,  $n(t)$  is wideband Gaussian noise, and  $i(t)$  is narrow-band interference. Here, we assume that  $n(t)$  is bandlimited and becomes white after sampling. For simplicity, let  $\tau$  be zero and  $\alpha = 1$ . Suppose that the received signal is chip-matched and sampled at the chip rate of the PN sequence. We thus have

$$z_k = s_k + n_k + i_k \quad (4)$$

where  $\{s_k\}$ ,  $\{n_k\}$ , and  $\{i_k\}$  are the discrete-time sequences from  $\{s(t)\}$ ,  $\{n(t)\}$ , and  $\{i(t)\}$ , respectively.  $\{s_k\}$ ,  $\{n_k\}$ , and  $\{i_k\}$  are assumed to be mutually independent. Since the PN sequence is random, we can consider  $\{s_k\}$  to be a sequence of i.i.d. random variables taking values of  $+1$  or  $-1$  with equal probability.

### B. LMS Algorithm

Let the tap weights of a transversal filter be  $\{a_1, \dots, a_L\}$ . The tap-weight update equations of the LMS algorithm can be expressed as

$$\Theta_k = \Theta_{k-1} + \mu \epsilon_k X_k \quad (5)$$

where  $\Theta_k \triangleq [a_{1,k} \ a_{2,k} \ \dots \ a_{L,k}]^T$  is the estimated tap weight at time  $k$ ,  $X_k \triangleq [z_{k-1} \ z_{k-2} \ \dots \ z_{k-L}]^T$  is the input vector, and  $\epsilon_k = z_k - X_k^T \Theta_{k-1}$  is the prediction error. For the suppression of NB interference, it has been shown that better results can be obtained by using a two-sided LMS filter. For this filter,  $\Theta_k$  and  $X_k$  are changed to  $\Theta_k \triangleq [a_{-N,k} \ a_{-N+1,k} \ \dots \ a_{-1,k} \ a_{1,k} \ \dots \ a_{N,k}]^T$  and  $X_k \triangleq [z_{k+N} \ z_{k+N-1} \ \dots \ z_{k+1} \ z_{k-1} \ \dots \ z_{k-N}]$ . To make the step size invariant with respect to the input power, a normalized LMS algorithm is frequently used. This algorithm is defined as

$$\Theta_{k+1} = \Theta_k + \frac{\mu}{r_k} \epsilon_k X_k \quad (6)$$

where  $r_k$  is an estimate of the input power, that can be obtained by

$$r_k = \beta r_{k-1} + (1 - \beta) z_k^2 \quad (7)$$

$\beta$ ,  $0 < \beta < 1$ , is a forgetting factor chosen to yield a compromise between the prediction accuracy and the tracking capability.

### C. Approximate Conditional Mean (ACM) Filter

Assume that the NB interference  $\{i_k\}$  is a Gaussian AR process of order  $p$ , i.e.,

$$i_k = \sum_{j=1}^p \phi_j i_{k-j} + d_k \quad (8)$$

where the AR parameters  $\{\phi_1, \dots, \phi_p\}$  are known to the receiver and  $\{d_k\}$  is a white Gaussian process of zero mean and variance  $\sigma_e^2$ , i.e.,  $p_{e_k}(x) \sim N_{\sigma_e^2}(x)$ , where  $N_{\sigma^2}(x) = 1/\sqrt{2\pi\sigma} \exp\{-x^2/2\sigma^2\}$ . From (8), we can construct a state space representation for the received signal and the interference

$$X_k = \Phi X_{k-1} + w_k \quad (9)$$

$$z_k = H X_k + v_k \quad (10)$$

where  $X_k = [i_k \ i_{k-1} \ \dots \ i_{k-p+1}]^T$ ,  $w_k = [d_k \ 0 \ \dots \ 0]^T$ ,  $H = [1 \ 0 \ \dots \ 0]$ ,  $v_k = s_k + n_k$ , and

$$\Phi = \begin{bmatrix} \phi_{1,k} & \phi_{2,k} & \dots & \phi_{p-1,k} & \phi_{p,k} \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix} \quad (11)$$

The estimate of  $X_k$  based on observations until time  $k$  is referred to as the filtering problem, and the estimate of  $X_k$  based on observations until time  $k-1$  is referred to as the prediction problem. It is known that the optimums of such estimates, in the minimum mean square error sense, are

the conditional expectations  $\hat{X}_k = E\{X_k|Z^k\}$  and  $\bar{X}_k = E\{X_k|Z^{k-1}\}$ , where  $Z^k = \{z_1 \ z_2 \ \dots \ z_k\}$ . When  $\{v_k\}$  and  $\{w_k\}$  are Gaussian processes, the optimal estimates can be recursively obtained from the well-known Kalman filter. However,  $v_k$  is the sum of two independent random variables; one is Gaussian distributed and the other takes on values  $+1$  or  $-1$  with equal probability, i.e.,

$$p_{v_k}(v) = \frac{1}{2}[N_{\sigma_n^2}(v-1) + N_{\sigma_n^2}(v+1)]. \quad (12)$$

Such a sequence is highly non-Gaussian. The optimal filter for such a sequence is nonlinear. In [14], Masreliez developed an ACM filter to estimate the state of a linear system with non-Gaussian observation noise. If we assume that the state prediction density  $p(X_k|Z^{k-1})$  is Gaussian with mean  $\bar{X}_k$  and covariance  $M_k$ , the conditional mean  $\hat{X}_k$  and its covariance  $P_k$  can be recursively calculated as

$$\hat{X}_k = \bar{X}_k + M_k H^T g_k(z_k) \quad (13)$$

$$P_k = M_k - M_k H^T G_k(z_k) H M_k \quad (14)$$

$$M_{k+1} = \Phi P_k \Phi^T + Q_k \quad (15)$$

$$\bar{X}_{k+1} = \Phi \hat{X}_k \quad (16)$$

where

$$g_k(z_k) = \left[ \frac{\partial p(z_k|Z^{k-1})}{\partial z_k} \right] [p(z_k|Z^{k-1})]^{-1} \quad (17)$$

$$G_k(z_k) = \frac{\partial g_k(z_k)}{\partial z_k} \quad (18)$$

and  $Q_k = E\{w_k w_k^T\}$ . The function  $g(\cdot)$  is called the score function. Vijayan and Poor employed this algorithm to solve the NB interference suppression problem in the DS spread spectrum system [11]. By the model in (9) and (10), the score function and its derivative turn out to be

$$g_k(z_k) = \frac{1}{HM_k H^T + \sigma_n^2} \cdot \left[ z_k - H\bar{x}_k - \tanh\left(\frac{z_k - H\bar{x}_k}{HM_k H^T + \sigma_n^2}\right) \right] \quad (19)$$

and

$$G_k(z_k) = \frac{1}{HM_k H^T + \sigma_n^2} \left[ 1 - \frac{1}{(HM_k H^T + \sigma_n^2)} \cdot \operatorname{sech}^2\left(\frac{z_k - H\bar{x}_k}{HM_k H^T + \sigma_n^2}\right) \right]. \quad (20)$$

It can be seen that the ACM filter has a structure similar to that of the standard Kalman filter. The time update equations (15) and (16) are identical to those in the Kalman filter. The measurement update in (13) involves the correction of the predicted value by a nonlinear score function. The nature of the nonlinearity is determined by the density function of the observation noise.

#### D. Adaptive Nonlinear LMS Prediction Filter

Vijayan and Poor also developed an adaptive nonlinear LMS algorithm based on the ACM filtering algorithm. Let the tap

weights of the transversal filter be  $[a_1, a_2, \dots, a_L]^T$ . The algorithm is described as follows:

$$\Theta_k = \Theta_{k-1} + \frac{\mu}{r_k} \epsilon_k \hat{X}_k \quad (21)$$

where

$$\Theta_k \triangleq [a_{1,k} \ a_{2,k} \ \dots \ a_{L,k}]^T \quad (22)$$

$$\hat{X}_k \triangleq [\hat{z}_{k-1} \ \hat{z}_{k-2} \ \dots \ \hat{z}_{k-L}]^T \quad (23)$$

$$\begin{aligned} \bar{z}_k &= \sum_{i=1}^L a_{i,k-1} \hat{z}_{k-i} \\ &= \sum_{i=1}^L a_{i,k-1} [\bar{z}_{k-i} + \rho(\epsilon_{k-i})]. \end{aligned} \quad (24)$$

$\epsilon_k = z_k - \bar{z}_k$ , and  $r_k = \beta_1 r_{k-1} + (1 - \beta_1) z_k^2$ .  $\rho(\cdot)$  is a nonlinear function of  $\epsilon_k$  defined by

$$\rho(\epsilon_k) = \epsilon_k - \tanh\left(\frac{\epsilon_k}{\sigma_k^2}\right) \quad (25)$$

and  $\sigma_k^2$  can be estimated by

$$\hat{\sigma}_k^2 = \Delta_k - 1 \quad (26)$$

where  $\Delta_k$  is a sample estimate of the prediction error variance that can be obtained by  $\Delta_k = \beta_2 \Delta_{k-1} + (1 - \beta_2) \epsilon_k^2$ .  $\beta_1$  and  $\beta_2$  are forgetting factors with values between zero and one. In [12], Rusch and Poor modified (21) into

$$\Theta_k = \Theta_{k-1} + \frac{\mu}{r_k} \rho(\epsilon_k) \hat{X}_k \quad (27)$$

showing that (27) can provide better results.

### III. NEW NONLINEAR ALGORITHMS

#### A. Decision-Directed Kalman (DDK) Filter

From the last section, we know that the performance of the ACM filter will be nearly optimal. The computation of the score function and its derivative, (19) and (20), however, involves the  $\tanh(\cdot)$  and  $\operatorname{sech}(\cdot)$  functions which are not efficient. To improve the computational efficiency, we now propose a DDK filter. The density of the observation noise is a Gaussian mixture, i.e.,

$$p(v_k) = \frac{1}{2}[N_{\sigma_n^2}(v_k - 1) + N_{\sigma_n^2}(v_k + 1)]. \quad (28)$$

We can construct hypotheses equivalent to (28) as follows:

$$H_0: p(v_k) = N_{\sigma_n^2}(v_k - 1) \quad (29)$$

$$H_1: p(v_k) = N_{\sigma_n^2}(v_k + 1) \quad (30)$$

and  $p(H_0) = p(H_1) = 0.5$ . Given  $H_0$  or  $H_1$ , the optimal state estimator is just the standard Kalman filter. Let

$$p(x_k|Z^{k-1}) = N_{M_k}(x_k - \bar{x}_k) \quad (31)$$

where  $\bar{x}_k$  is the mean of  $p(x_k|Z^{k-1})$ . We then have

$$\hat{x}_k = \bar{x}_k + M_k H^t (HM_k H^t + \sigma_n^2)^{-1} (z_k - H\bar{x}_k - 1) \quad (32)$$

$$P_k = M_k - M_k H^t (HM_k H^t + \sigma_n^2)^{-1} H M_k \quad (33)$$

if  $H_0$  is given and

$$\hat{x}_k = \bar{x}_k + M_k H^t (H M_k H^t + \sigma_n^2)^{-1} (z_k - H \bar{x}_k + 1) \quad (34)$$

$$P_k = M_k - M_k H^t (H M_k H^t + \sigma_n^2)^{-1} H M_k \quad (35)$$

if  $H_1$  is given. Note that the error covariance matrix is the same in both cases. If we can determine which hypothesis is true, we can apply the corresponding Kalman filter. This is the standard hypotheses testing problem. Thus, we have to evaluate  $p(H_i|Z^k)$

$$\begin{aligned} p(H_i|Z^k) &= \frac{p(z_k|Z^{k-1}, H_i)p(H_i|Z^{k-1})}{p(z_k|Z^{k-1})} \\ &= \frac{p(z_k|\bar{x}_k, H_i)p(H_i)}{p(z_k|Z^{k-1})} \end{aligned} \quad (36)$$

where  $i = 0$  or  $1$ . Since  $p(H_0) = p(H_1) = 0.5$  and  $p(z_k|Z^{k-1})$  is a constant,  $p(H_i|Z^k)$  is proportional to  $p(z_k|\bar{x}_k, H_i)$ . It is simple to show that

$$p(z_k|\bar{x}_k, H_0) = N_{H M_k H^t + \sigma_n^2}(z_k - H \bar{x}_k - 1) \quad (37)$$

$$p(z_k|\bar{x}_k, H_1) = N_{H M_k H^t + \sigma_n^2}(z_k - H \bar{x}_k + 1). \quad (38)$$

Thus, the optimal decision rule is

$$\text{If } p(z_k|\bar{x}_k, H_0) \geq p(z_k|\bar{x}_k, H_1), \quad \text{Choose } H_0. \quad (39)$$

$$\text{If } p(z_k|\bar{x}_k, H_0) < p(z_k|\bar{x}_k, H_1), \quad \text{Choose } H_1. \quad (40)$$

Observing (37) and (38), we find the decision is simple to carry out, i.e.,

$$\text{If } z_k - H \bar{x}_k \geq 0, \quad \text{Choose } H_0. \quad (41)$$

$$\text{If } z_k - H \bar{x}_k < 0, \quad \text{Choose } H_1. \quad (42)$$

Summarizing the results developed above, we have obtained the DDK filter described as follows:

$$\begin{aligned} \hat{X}_k &= \bar{X}_k + M_k H^T (H M_k H^T + \sigma_n^2)^{-1} \\ &\quad \cdot [z_k - H \bar{X}_k - \text{sgn}(z_k - H \bar{X}_k)] \end{aligned} \quad (43)$$

$$P_k = M_k - M_k H^T (H M_k H^T + \sigma_n^2)^{-1} H M_k \quad (44)$$

$$\bar{X}_{k+1} = \Phi \hat{X}_k \quad (45)$$

$$M_{k+1} = \Phi P_k \Phi^T + Q_k \quad (46)$$

where

$$\text{sgn}(x) = \begin{cases} 1 & x \geq 0 \\ -1 & x < 0. \end{cases}$$

Except for (43), the other equations are the same as the standard Kalman equations. The measurement update in (43) is a nonlinear function of  $z_k - H \bar{X}_k$ . The nonlinearity is determined only by the sign of the prediction error. The computational complexity is then greatly reduced. It is interesting to consider the  $\text{sgn}(\cdot)$  function as an approximation of the  $\tanh(\cdot)$  function. From the simulation results in Section IV, we find that the performance of the DDK filter is almost the same as that of the ACM filter.

### B. Nonlinear Adaptive LMS Filters

The performance of the LMS algorithm will be degraded if noise is present. For the NB interference suppression problem, the only information we have is the noisy observation  $z_k$ . The noise consists of the binary DS signal and Gaussian noise. Most previous work ignored this fact and directly used  $z_k$  as the desired signal. In [11], Vijayan and Poor first pointed out this problem and used a nonlinear processing strategy. However, it is not clear how their algorithm is derived and what it means. Here, we approach this problem with the concept of input filtering and use estimation theory to solve it. Given the observations  $Z^k$ , we assume that  $p(i_k|Z^{k-1}) \sim N_{\sigma_i^2}(i_k - \mu_k)$ . Without knowledge of the AR coefficients, computation of  $\mu_k$  and  $\sigma_i^2$  is difficult. For the time being, let us assume that at time  $k-1$ , the tap-weight vector  $\Theta_{k-1}$  represents the true AR coefficients, then, from (8), we have

$$\begin{aligned} \mu_k &= E\{i_k|Z^{k-1}\} \\ &= E\{[i_{k-1}, i_{k-2}, \dots, i_{k-L}]\Theta_{k-1} + d_k|Z^{k-1}\} \\ &= [E\{i_{k-1}|Z^{k-1}\}, \dots, E\{i_{k-L}|Z^{k-1}\}]\Theta_{k-1}. \end{aligned} \quad (47)$$

Since  $i_k$  is an NB signal, it is then reasonable to make the following approximation:

$$\begin{aligned} &[E\{i_{k-1}|Z^{k-1}\}, \dots, E\{i_{k-L}|Z^{k-1}\}] \\ &\approx [E\{i_{k-1}|Z^{k-1}\}, \dots, E\{i_{k-L}|Z^{k-L}\}] \\ &= [\hat{i}_{k-1}, \hat{i}_{k-2}, \dots, \hat{i}_{k-L}] \\ &\triangleq X_k^T. \end{aligned} \quad (48)$$

From (47) and (48), we find that if  $\hat{X}_k$  is available at time  $k$  and is used as the input to the LMS predictor, the output of the predictor  $\bar{i}_k = \hat{X}_k^T \Theta_{k-1}$  will be  $\mu_k$ . We can also show that the prediction error variance  $\sigma_p^2$  will be  $\sigma_i^2$ . We then have

$$p(i_k|Z^{k-1}) \sim N_{\sigma_p^2}(i_k - \bar{i}_k). \quad (49)$$

From (4), we have

$$z_k = i_k + v_k \quad \text{and} \quad v_k = n_k + s_k. \quad (50)$$

The prediction error is then

$$\begin{aligned} \epsilon_k &= z_k - \bar{i}_k \\ &= (i_k - \bar{i}_k) + v_k \\ &= e_k + v_k \end{aligned}$$

where  $e_k$  is the prediction error in the absence of the non-Gaussian noise  $v_k$ , i.e.,

$$p(e_k|Z^{k-1}) = N_{\sigma_p^2}(e_k). \quad (51)$$

Note that  $e_k$  is independent of  $v_k$ . Thus, the prediction error signal  $\epsilon_k$  used in the LMS contains the noise  $v_k$ , that will degrade the performance. We can remedy this problem by filtering  $\epsilon_k$ . From the Baye's law, we have

$$p(e_k|Z^k) = \frac{p(e_k|Z^{k-1})p(z_k|e_k)}{p(z_k|Z^{k-1})} \quad (52)$$

where  $p(z_k|Z^{k-1})$  is a normalized constant. The optimal estimate  $\hat{e}_k$  from  $\epsilon_k$  based on observations until time  $k$  is the approximate conditional mean. One can show that

$$\begin{aligned} \hat{e}_k &= E\{e_k|Z^k\} \\ &= \frac{\sigma_p^2}{\sigma_p^2 + \sigma_n^2} \left[ \epsilon_k - \tanh\left(\frac{\epsilon_k}{\sigma_p^2 + \sigma_n^2}\right) \right]. \end{aligned} \quad (53)$$

We can also use the decision-directed algorithm and obtain the estimate

$$\hat{e}_k = \frac{\sigma_p^2}{\sigma_p^2 + \sigma_n^2} [\epsilon_k - \text{sgn}(\epsilon_k)]. \quad (54)$$

Since  $\sigma_p^2$  is usually much larger than  $\sigma_n^2$ ,  $\sigma_p^2/(\sigma_p^2 + \sigma_n^2) \approx 1$ , (53) is reduced to

$$\begin{aligned} \hat{e}_k &\approx \rho_1(\epsilon_k) \\ &= \epsilon_k - \tanh\left(\frac{\epsilon_k}{\sigma_p^2 + \sigma_n^2}\right) \end{aligned} \quad (55)$$

and (54) is

$$\begin{aligned} \hat{e}_k &\approx \rho_2(\epsilon_k) \\ &= \epsilon_k - \text{sgn}(\epsilon_k). \end{aligned} \quad (56)$$

$\rho_1(\cdot)$  and  $\rho_2(\cdot)$  correspond to the nonlinear functions in the ACM and the DDK filter, respectively. Now, an optimal estimate of the input  $i_k$  can be easily obtained

$$\begin{aligned} \hat{i}_k &= \bar{i}_k + \hat{e}_k \\ &= \bar{i}_k + \rho_i(\epsilon_k) \end{aligned} \quad (57)$$

where  $i = 1$  or  $2$ . Since  $\hat{i}_k$  is the input to the LMS predictor, using (57), we obtain nonlinear transversal filters for the prediction of  $i_k$ :

$$\begin{aligned} \bar{i}_k &= \hat{X}_k^T \Theta_{k-1} = \sum_{j=1}^L a_{j,k-1} \hat{i}_{k-j} \\ &= \sum_{j=1}^L a_{j,k-1} [\bar{i}_{k-j} + \rho_i(\epsilon_{k-j})]. \end{aligned} \quad (58)$$

Thus, the weight-update algorithm for the LMS filter is

$$\Theta_k = \Theta_{k-1} + \mu \frac{\epsilon_k}{r_k} \hat{X}_k \quad (59)$$

where  $\epsilon_k$  is the prediction error and  $r_k$  is an estimate of the input power obtained as in (7). Furthermore, we can use the filtered error signal  $\rho_1(\epsilon_k)$  or  $\rho_2(\epsilon_k)$  in the weight-update equation and, thus, obtain

$$\Theta_k = \Theta_{k-1} + \mu \frac{\rho_i(\epsilon_k)}{r_k} \hat{X}_k. \quad (60)$$

Note that only when  $\hat{X}_k$  is noiseless (i.e.,  $\hat{X}_k^T = [\hat{i}_{k-1}, \dots, \hat{i}_{k-L}]$ ), will (59) and (60) converge to the true AR coefficients. On the other hand, only when  $\Theta_{k-1}$  represents the true AR coefficients, will  $\bar{i}_k$  in (58) be  $\mu_k$ . When  $k$  is small, these assumptions are not true. Through the two-step iteration scheme; input-filtering and weight-updating, however, as  $k$  increases,  $\hat{X}_k$  will become less and less noisy and  $\Theta_k$  will approach the true AR coefficients.

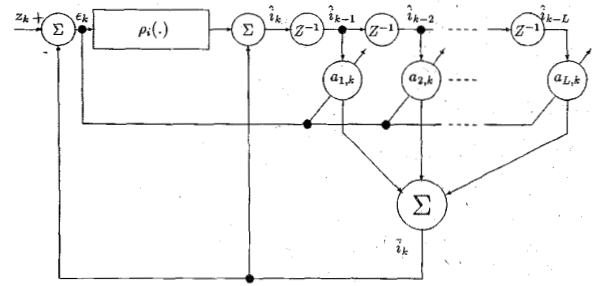


Fig. 1. The structure of the LMS algorithm with filtered input.

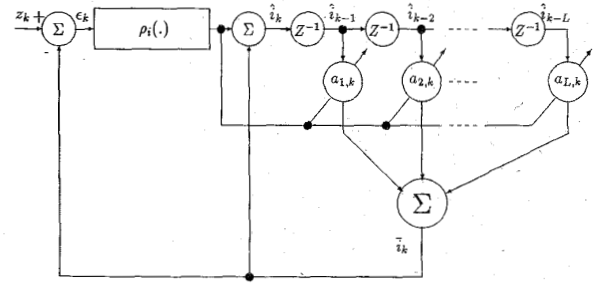


Fig. 2. The structure of the LMS algorithm with filtered input and filtered error signal.

In summary, we have the following results. For tap weight updating, we can use (59) or (60). For prediction, we can use (58). This results in four nonlinear LMS filters. Figs. 1 and 2 show the structures of the filters. For convenience, we shall use the following abbreviations in the sequel:

*Filter ALMS1:* Use (58) and (59) with  $i = 1$ .

*Filter ALMS2:* Use (58) and (60) with  $i = 1$ .

*Filter DLMS1:* Use (58) and (59) with  $i = 2$ .

*Filter DLMS2:* Use (58) and (60) with  $i = 2$ .

The ALMS1 filter is just the nonlinear LMS filter proposed by Vijayan and Poor in [11] and the ALMS2 filter is the modified filter proposed by Rusch and Poor in [12]. However, they did not give the derivation of the filters. Equation (53) is the closed form solution that did not appear in [11] either.  $(\sigma_p^2 + \sigma_n^2)$  in (55) is the term denoted by  $\sigma_k^2$  in (26) that can be recursively estimated. From (56), we see that despite the simple sign operation, the decision-directed algorithm provides another advantage, i.e., computation of  $(\sigma_p^2 + \sigma_n^2)$  is not required. Note that the ALMS2 and the DLMS2 filters do not increase the computational complexity at all. This is clear from Figs. 1 and 2.

### C. Suppression of Unknown Interference Using ACM/DDK Filter

As described in Section II, we see that the interference and the received signal can be formulated in a state space representation and the ACM or the DDK filter can be applied. However, the AR parameters of interference have to be known. In real applications, we cannot determine the parameters in advance. One possible solution is to perform parameter estimation first and then apply the ACM or DDK filter.

Vijayan and Poor mentioned this method in [11]. Since the poles of the AR model are close to the unit circle, standard recursive identification schemes such as recursive maximum likelihood (RML) algorithm [15]–[17] converge rather slowly. In addition, the performance of the ACM filter is sensitive to parameter variation. Consequently, this method was not successfully implemented in [11]. Here, we propose to use the RLS method to identify the AR parameters. It is well-known that the RLS algorithm converges faster than the LMS algorithm. However, when the observations contain noise, the performance of the standard RLS algorithm is seriously degraded. We shall use the concept of “input filtering” to combat this problem. Because the filtering mechanism is similar to that of the LMS algorithm, the results developed above are directly applicable here. The standard RLS algorithm is given as

$$K_k = \frac{\lambda^{-1} P_k X_k}{1 + \lambda^{-1} X_k^T P_{k-1} X_k} \quad (61)$$

$$\epsilon_k = z_k - \Theta_{k-1}^T X_k \quad (62)$$

$$\Theta_k = \Theta_{k-1} + K_k \epsilon_k \quad (63)$$

$$P_k = \lambda^{-1} P_{k-1} - \lambda^{-1} K_k X_k^T P_{k-1} \quad (64)$$

where  $\Theta_k$  is the tap weight vector of the filter,  $\lambda$  is the forgetting factor, and  $X_k = [z_{k-1} \cdots z_{k-L}]^T$  is the input. Thus,  $X_k$  can be replaced by its filtered version  $\hat{X}_k$  as defined in (48) and (57). The error signal  $\epsilon_k$  in (63) can also be replaced by  $\rho_i(\epsilon_k)$ . Then, we have the four nonlinear RLS algorithms described below

$$K_k = \frac{\lambda^{-1} P_k \hat{X}_k}{1 + \lambda^{-1} \hat{X}_k^T P_{k-1} \hat{X}_k} \quad (65)$$

$$\epsilon_k = z_k - \Theta_{k-1}^T \hat{X}_k \quad (66)$$

$$\Theta_k = \Theta_{k-1} + K_k f(\epsilon_k) \quad (67)$$

$$P_k = \lambda^{-1} P_{k-1} - \lambda^{-1} K_k \hat{X}_k^T P_{k-1} \quad (68)$$

*Filter ARLS1:* Use  $\rho_1(\cdot)$  in  $\hat{X}_k$  and  $f(\epsilon_k) = \epsilon_k$ .

*Filter ARLS2:* Use  $\rho_1(\cdot)$  in  $\hat{X}_k$  and  $f(\epsilon_k) = \rho_1(\epsilon_k)$ .

*Filter DRLS1:* Use  $\rho_2(\cdot)$  in  $\hat{X}_k$  and  $f(\epsilon_k) = \epsilon_k$ .

*Filter DRLS2:* Use  $\rho_2(\cdot)$  in  $\hat{X}_k$  and  $f(\epsilon_k) = \rho_2(\epsilon_k)$ .

ARLS2 and DRLS2 will be selected to perform the identification since they have better performance. It will be shown that both algorithms can adapt quickly to an unknown environment. For convenience, in the simulations reported below, we refer to the scheme with the ARLS2 algorithm and ACM filter as AR2A and the scheme with the DRLS2 algorithm and DDK filter as DR2D.

#### IV. SIMULATIONS

In this section, we report on simulations carried out to evaluate the performance of the proposed algorithms. We follow the examples studied in [11]. We first define our performance measure, which is the commonly used SNR improvement. We define this as follows:

$$\text{Input SNR} \triangleq 10 \log \left[ \frac{E(s_k^2)}{E(|z_k - s_k|^2)} \right] \text{ dB} \quad (69)$$

$$\text{Output SNR} \triangleq 10 \log \left[ \frac{E(s_k^2)}{E(|\epsilon_k - s_k|^2)} \right] \text{ dB} \quad (70)$$

where  $\epsilon_k$  is the prediction error. Therefore

$$\text{SNR improvement} \triangleq 10 \log \left[ \frac{E(|z_k - s_k|^2)}{E(|\epsilon_k - s_k|^2)} \right] \text{ dB} \quad (71)$$

The SNR at the input was varied by changing the power of the interfering signal. The variance of the background thermal noise was kept constant at  $\sigma_n^2 = 0.01$ . All results were obtained based on 20 trials, and for each trial, 5000 data points were computed. The number of filter taps was 10. Two kinds of NB interference were considered: AR and sinusoidal interference. We first considered the AR interference. The AR interfering signal was obtained by passing white noise through a second-order IIR filter with two poles at  $z = 0.99$ , i.e.,

$$i_k = 1.98i_{k-1} - 0.9801i_{k-2} + d_k \quad (72)$$

where  $\{d_k\}$  is white Gaussian noise. Four sets of simulations were conducted:

- 1) The AR parameters were assumed to be known; three filters were compared, namely, Kalman, ACM, and DDK.
- 2) The AR parameters were assumed to be unknown; five types of LMS filters were compared, namely, normalized two-sided LMS (TS-LMS), ALMS1, ALMS2, DLMS1, and DLMS2.
- 3) The AR parameters were assumed to be unknown; five types of RLS filters are compared, namely, RLS, ARLS1, ARLS2, DRLS1, and DRLS2.
- 4) The AR parameters were assumed to be unknown; two schemes were compared, namely, AR2A and DR2D.

Table I summarizes the results (predictions) of the four sets of simulations. It can be seen that adaptive nonlinear filtering techniques offer considerable advantages over conventional linear filters. The performance of the decision-directed algorithms is almost as good as that of ACM. In the known interference case, the linear Kalman filter performs poorly. The ACM and the DDK have the same results. This is due to the precise state prediction, by which the error is very close to +1 or -1. In this case, the sign function is almost equal to the tanh(.) function. For LMS filters, we can see that ALMS2/DLMS2 perform the best, with an SNR improvement significantly higher than that of the TS-LMS algorithm. The performance of ALMS1/DLMS1 is between that of TS-LMS and ALMS2/DLMS2. From part three of Table I, we find that the standard RLS algorithm is greatly affected by the non-Gaussian DS signal. Its performance is even worse than that of TS-LMS. The reason for this is as follows. The vector  $K_k$  in (63) can be viewed as a product of the step size and the input vector. The step size in the LMS algorithm is constant. However, it keeps decreasing in the RLS algorithm. The rate of decrease must be synchronized with the rate of convergence. When  $X_k$  is noisy,  $K_k$  is not correctly evaluated and this synchronization is destroyed. This is why the RLS performs poorly in a noisy environment. The influence of  $\epsilon_k$  is not that crucial, since  $K_k$  is small when the filter converges. Thus, the difference in the performance of

TABLE I  
SNR IMPROVEMENT FOR AR INTERFERENCE

Input SNR (dB)	-20	-15	-10	-5
Kalman	26.73	23.51	20.27	16.95
ACM	36.87	32.57	28.10	23.50
DDK	36.87	32.57	28.10	23.50
TS-LMS	26.88	22.30	17.63	12.94
ALMS1	31.97	27.95	23.70	19.57
DLMS1	31.95	27.62	22.65	17.47
ALMS2	36.12	32.02	17.56	23.06
DLMS2	36.41	32.07	27.50	22.92
RLS	24.48	20.90	16.85	12.32
ARLS1	36.31	31.84	27.15	22.41
DRLS1	36.28	31.84	27.15	22.41
ARLS2	36.85	32.49	27.86	23.18
DRLS2	36.85	32.49	27.86	23.18
AR2A	36.99	32.68	28.15	23.44
DR2D	36.99	32.68	28.15	23.44

ARLS1/DRLS1 and ARLS2/DRLS2 is not significant. Finally, we see that AR2A/DR2D perform satisfactorily. This indicates that ARLS2/DRLS2 did identify the interference properly. One point worth noting is that the performances of ACM, DDK, ALMS2, DLMS2, ARLS2, DRLS2, AR2A, and DR2D are all very similar, regardless of whether the interference parameters are known. It seems that the performance bound is reached and the state space formulation cannot provide additional improvement.

Another case we considered was that of sinusoidal interference. The frequency of the sinusoidal interference signal was kept constant at 0.15 radians, i.e.,

$$i(t) = A \cos(0.15t + \theta) \quad (73)$$

where  $A$  is the amplitude and  $\theta$  is a random phase with uniform distribution. Table II shows the simulation results. From the figure, we see that the performance is similar to that in the AR interference case. One difference is that since the interference in this case is deterministic, the performance of the Kalman filter is not affected by noise (only convergence speed is affected).

On the basis of the simulation results, we conclude that the nonlinear algorithms developed in this paper indeed provide good performance. We particularly recommend the DLMS2 filter, that typically provides the best performance at a computational cost as low as that of the LMS filter. For fast convergence, the DRLS2 filter is a good choice; while its convergence speed and performance are equivalent to those of the ACM filter, it is simpler to implement.

## V. CONCLUSIONS

In this paper, we have considered nonlinear prediction filters for the rejection of NB interference in the DS spread spectrum system. Conventional approaches to this problem involve the use of linear LMS filters. However, it is known that the

TABLE II  
SNR IMPROVEMENT FOR SINUSOIDAL INTERFERENCE

Input SNR (dB)	-20	-15	-10	-5
Kalman	39.76	34.79	29.80	24.80
ACM	39.66	34.89	29.91	24.91
DDK	38.44	34.25	29.68	24.74
TS-LMS	27.71	23.16	18.05	12.95
ALMS1	33.31	28.31	23.30	18.40
DLMS1	33.31	28.30	23.30	18.34
ALMS2	38.36	33.37	28.38	24.42
DLMS2	38.35	33.36	28.37	23.36
RLS	23.74	18.72	13.83	9.224
ARLS1	37.79	32.95	28.08	23.09
DRLS1	37.66	32.86	28.01	22.96
ARLS2	37.97	33.21	28.43	23.47
DRLS2	38.01	33.24	28.37	23.35
AR2A	38.52	33.63	28.72	23.77
DR2D	38.52	33.63	28.72	23.77

received signal contains a highly non-Gaussian signal. The input to the predictor is noisy. Better performance can be obtained by properly filtering the input. Since the noise is non-Gaussian, the optimal filter is nonlinear. In this paper, we have proposed nonlinear algorithms, described as follows:

- 1) We have developed a DDK filter for known interference. The DDK filter is computationally simpler than the previously proposed ACM filter, but provides the same level of performance.
- 2) We have developed nonlinear LMS filters that result in better performance and less computation.
- 3) We have developed nonlinear RLS algorithms that can be used independently, or as interference identifiers so that the ACM or the DDK filters can be applied.

Simulations show that our nonlinear filters perform significantly better than conventional ones. Among the proposed filters, DLMS2 and DRLS2 are the most useful. The proposed DLMS2 filter performs as well as filters formulated in state space, such as the ACM filter, but has a structure as simple as that of an LMS filter. The proposed DRLS2 filter offers fast convergence without involving the system model. It is mentioned in [11] that the nonlinear function of the ACM filter will become a hard limiter when  $\sigma_k^2 \rightarrow 0$ . This is different from our scheme. We derive the decision-directed algorithms based on the formulation of hypotheses testing and we do not assume  $\sigma_k^2 = 0$ . In fact,  $\sigma_k^2$  will not be zero. That's why Vijayan and Poor used the  $\tanh(\cdot)$  function as a soft limiter. Finally, our results show that the state space approach cannot provide better results in the NB interference suppression problem.

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