# Acceleration of Iterative Tomographic Image Reconstruction by Reference-based Back Projection

Chang-Chieh Cheng, Ping-Hui Li, and Yu-Tai Ching

Department of Computer Science, National Chiao Tung University, Hsinchu, Taiwan

## ABSTRACT

The purpose of this paper is to design and implement an efficient iterative reconstruction algorithm for computational tomography. We accelerate the reconstruction speed of *algebraic reconstruction technique* (ART), an iterative reconstruction method, by using the result of *filtered backprojection* (FBP), a wide used algorithm of analytical reconstruction, to be an initial guess and the reference for the first iteration and each back projection stage respectively. Both two improvements can reduce the error between the forward projection of each iteration and the measurements. We use three methods of quantitative analysis, *root-mean-square error* (RMSE), *peak signal to noise ratio* (PSNR), and *structural content* (SC), to show that our method can reduce the number of iterations by more than half and the quality of the result is better than the original ART.

**Keywords:** Computational tomography, filtered backprojection, algebraic reconstruction technique

## 1. INTRODUCTION

According the Radon transform, a sequence of X-ray projections taken over angular ranges of 0° to 180° can be reconstructed to a tomographic image.[1] All tomographic reconstruction methods can be classified into two major categories, analytical reconstruction and iterative reconstruction.[2] A wide used algorithm of analytical reconstruction is the filtered backprojection (FBP)[3] that has fast performance but could have strike artifacts in the constructed result. Iterative reconstruction can be implemented by the algebraic reconstruction technique (ART), an algorithm based the Kaczmarz's method, requires several iterations to estimate better results than the analytical reconstruction.[4] However, the ART take much longer time than FBP even a parallel computation implementation is applied. In this study, we accelerate the ART by using the result of FBP to be an initial guess and the reference for the first iteration and each back projection stage respectively. Both two improvements can reduce the error between the forward projection of each iteration and the measurements. We use three methods of quantitative analysis, root-mean-square error (RMSE), peak signal to noise ratio (PSNR), and structural content (SC), to show that our method can reduce the number of iterations by more than half and the quality of the result is better than the original ART.

The rest of the paper is organized as follows. The algorithms of ART and FBP are described in Section 2 and 3 respectively. Section 4 represents the proposed method. The results of the proposed method are shown in Section 5. Section 6 represents the conclusions.

## 2. FILTERED BACK PROJECTION

The Radon transform[1] expresses that using an apparatus of parallel-beam projection to gather the intensity of X-ray. Let $f(x, y)$ be a two-dimensional function to represent a slice of an object. Given a projection angle $\theta$, The Radon transform is defined as follows.

$$p_\theta(x') = \int_{-\infty}^{\infty} f(y' \sin\theta + x' \cos\theta, -y' \cos\theta + x' \sin\theta) dy'. \tag{1}$$

$p_\theta(x')$ can be expressed a $\theta - x$ plane that is called a sinogram.

Further author information: (Send correspondence to Yu-Tai Ching)
Yu-Tai Ching: E-mail: ytc@cs.nctu.edu.tw, Telephone: +886-3-5712121 ext. 31547
Chang-Chieh Cheng.: E-mail: chengchc@cs.nctu.edu.tw, Telephone: +886-3-5712121 ext. 59290

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The FBP algorithm is based on the Fourier slice theorem[3] that can estimate a tomographic image from a sinogram. The following equation represents the algorithm of FBP.

$$f(x,y) = \int_0^{\pi} q_\theta(x\cos\theta + y\sin\theta)d\theta, \tag{2}$$

where

$$q_\theta(x') = \int_{-\infty}^{\infty} |w|F(w,\theta)e^{w2\pi x' i}dw. \tag{3}$$

$q_\theta(x')$ is the inverse Fourier transform of a filtered line segment which passes though the origin of frequency domain with $\theta$ angle. Then, a ramp waveform, $|w|$, is multiplied to this line segment. The following steps describe the implementation of Eq. 2

1. Apply one-dimensional Fourier trasnform, $\mathcal{F}_1$, to transform each row of the sinogram $p_\theta(x')$, $P_\theta(w) = \mathcal{F}_1\{p_\theta(x')\}$.
2. Multiply the ramp filter, $|w|$, to $P_\theta(w)$.
3. Calculate the filtered projections, $q_\theta$, by applying $\mathcal{F}_1^{-1}$ to the filtered $P_\theta(w)$.
4. Back project $q_\theta$ for all $\theta$ to the output image.

Fig. 1 shows the algorithm of FBP.

## 3. ALGEBRAIC RECONSTRUCTION TECHNIQUE

ART has two differences between with FBP. First, all estimation processes of ART is in spatial domain rather than in frequency domain. Second, the ART uses several iterations to correct the errors to reconstruct the image. In most cases, the ART can produce better tomographic images than the results of FBP. However, the computational cost of ART is much larger than the FBP.

Fig. 2 shows the concept of ART. Assume that the size of output image is $N$ pixels, and the size of sinogram is $M$ projections. Let the reconstructed image and the pixel areas covered by $j$th ray respectively be two $N$-dimensional vector, $\vec{\mathbf{f}} = [f_1, f_2, \cdots, f_N]$ and $\vec{\mathbf{w}}_j = [w_1^j, w_2^j, \cdots, w_N^j]$. The projection measured from the $j$th ray, $p_j$, then can be expressed as follows.

$$p_j = \vec{\mathbf{w}}_j \cdot \vec{\mathbf{f}}. \tag{4}$$

Assume that $\vec{\mathbf{f}}^k$ is the result of $k$-th iteration. The line $\overline{OU}$ in Fig. 2 is the normalized vector of $\vec{\mathbf{w}}_j$,

$$\overline{OU} = \frac{\vec{\mathbf{w}}_j}{\sqrt{\vec{\mathbf{w}}_j \cdot \vec{\mathbf{w}}_j}}, \tag{5}$$

and

$$|\overline{OF}| = \vec{\mathbf{f}}^{k-1} \cdot \overline{OU}.$$

The lengths of $\overline{OA}$ and $\overline{HG}$ then can be obtained by the following equations.

$$|\overline{OA}| = \overline{OU} \cdot \overline{OC} = \frac{p_j}{\sqrt{\vec{\mathbf{w}}_j \cdot \vec{\mathbf{w}}_j}} \tag{6}$$

$$|\overline{HG}| = \overline{OF} - \overline{OA} = \frac{\mathbf{w}_j \cdot \vec{\mathbf{f}}^{k-1} - p_j}{\sqrt{\vec{\mathbf{w}}_j \cdot \vec{\mathbf{w}}_j}} \vec{\mathbf{w}}_j. \tag{7}$$

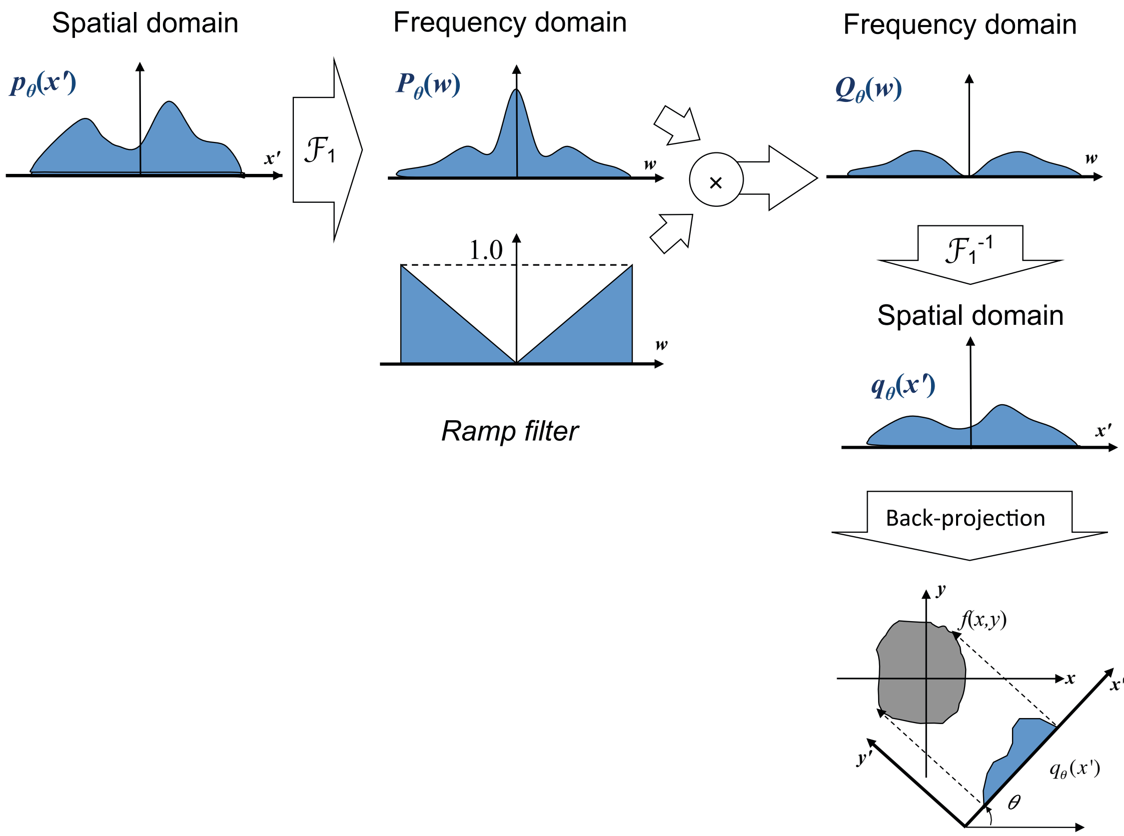


Figure 1. **The FBP algorithm.**

Finally, the result of $k$-th round, $\vec{\mathbf{f}}^k$, then can be obtained by the following equation.

$$\vec{\mathbf{f}}^k = \vec{\mathbf{f}}^{k-1} + \frac{p_i - \vec{\mathbf{w}}_j \cdot \vec{\mathbf{f}}^{k-1}}{\vec{\mathbf{w}}_j \cdot \vec{\mathbf{w}}_j}\vec{\mathbf{w}}_j. \tag{8}$$

Eq. 8 is the recursion for the ART. The implementation of ART consists of four steps as described as follows. First, calculate the $\vec{\mathbf{w}}_j$ for the $j$-th ray. The second step is called forward-projection that estimates a projection value by inner product of $\vec{\mathbf{f}}^{k-1}$ and $\vec{\mathbf{w}}_j$. The correction then can be obtained by subtraction of $p_j$ and $\vec{\mathbf{w}}_j \cdot \vec{\mathbf{f}}^{k-1}$. Finally, back project this correction to $\vec{\mathbf{f}}^{k-1}$. These four steps are iteratively executed until the correction is smaller than a threshold or the number iteration reaches to a constant.

## 4. COMBINING FBP AND ART

The ART algorithm can be described by the Kaczmarz's method[5] as follows.

$$f_i^k = f_i^{k-1} + r\frac{p_j - \vec{\mathbf{w}}_j \cdot \vec{\mathbf{f}}^{k-1}}{\vec{\mathbf{w}}_j \cdot \vec{\mathbf{w}}_j}\vec{\mathbf{w}}_j, \tag{9}$$

where $r$ is a relaxation parameter.

Given an image $\vec{\mathbf{g}} = [g_1, g_2, \ldots, g_n]^T$ reconstructed by FBP. We replace the relaxation parameter in Eq. 9 by $\vec{\mathbf{g}}$. Thus, Eq. 8 can be rewritten as follows.

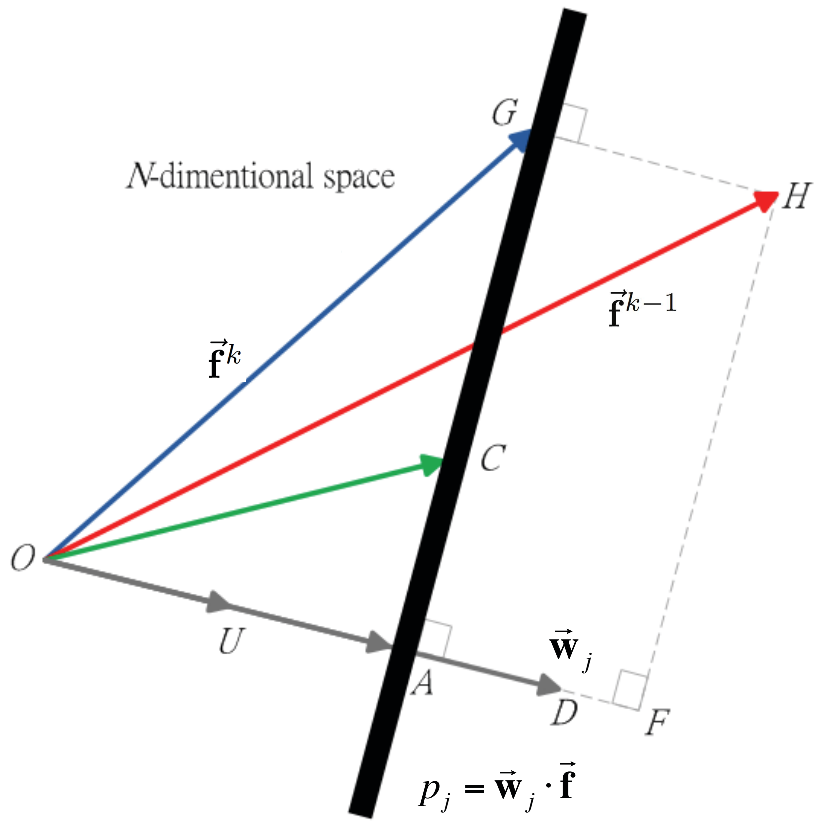


Figure 2. **The theory of ART.** The size of output image is $N$ pixels, and the size of sinogram is $M$ projections. $\vec{\mathbf{f}} = [f_1, f_2, \cdots, f_N]$ is the reconstructed image. $\vec{\mathbf{w}}_j = [w_1^j, w_2^j, \cdots, w_N^j]$ contains the pixel areas covered by $j$th ray. $p_j$ is The projection measured from the $j$th ray. $\vec{\mathbf{f}}_i$ is the reconstructed image after $i$ iteration.

$$f_i^k = f_i^{k-1} + g_i \frac{p_j - \vec{\mathbf{w}}_j \cdot \vec{\mathbf{f}}^{k-1}}{\vec{\mathbf{w}}_j \cdot \vec{\mathbf{w}}_j} \vec{\mathbf{w}}_j, \tag{10}$$

where $\vec{\mathbf{f}}^0 = \vec{\mathbf{g}}$. Eq. 10 is the core theorem of the proposed method.

## 5. RESULTS

A Shepp-Logan phantom of $256 \times 256$ pixels was tested in this study. The results reconstructed from $256 \times 256$ projections, taken over angular ranges of 0° to 180° with incremental angle 0.07°. Fig 3(a) shows the result produced by the original ART method with 20 iterations. We then used the image reconstructed by FBP as the initial guess of ART, $\vec{\mathbf{f}}^0 = \vec{\mathbf{g}}$. As shown in Fig 3(b), a result that similar to Fig 3(a) can be obtained after nine iterations. Finally, according to the Eq. 10, we used the result of FBP as the reference for each iteration and the same result can be reconstructed after eight iterations, shown in Fig 3(c).

Table 1 represents the results of three quantitative analysis methods that are root-mean-square error (RMSE), peak signal to noise ratio (PSNR), and structural content (SC), to show that our method can reduce the number of iterations by more than half and the quality of the result is better than the original ART.

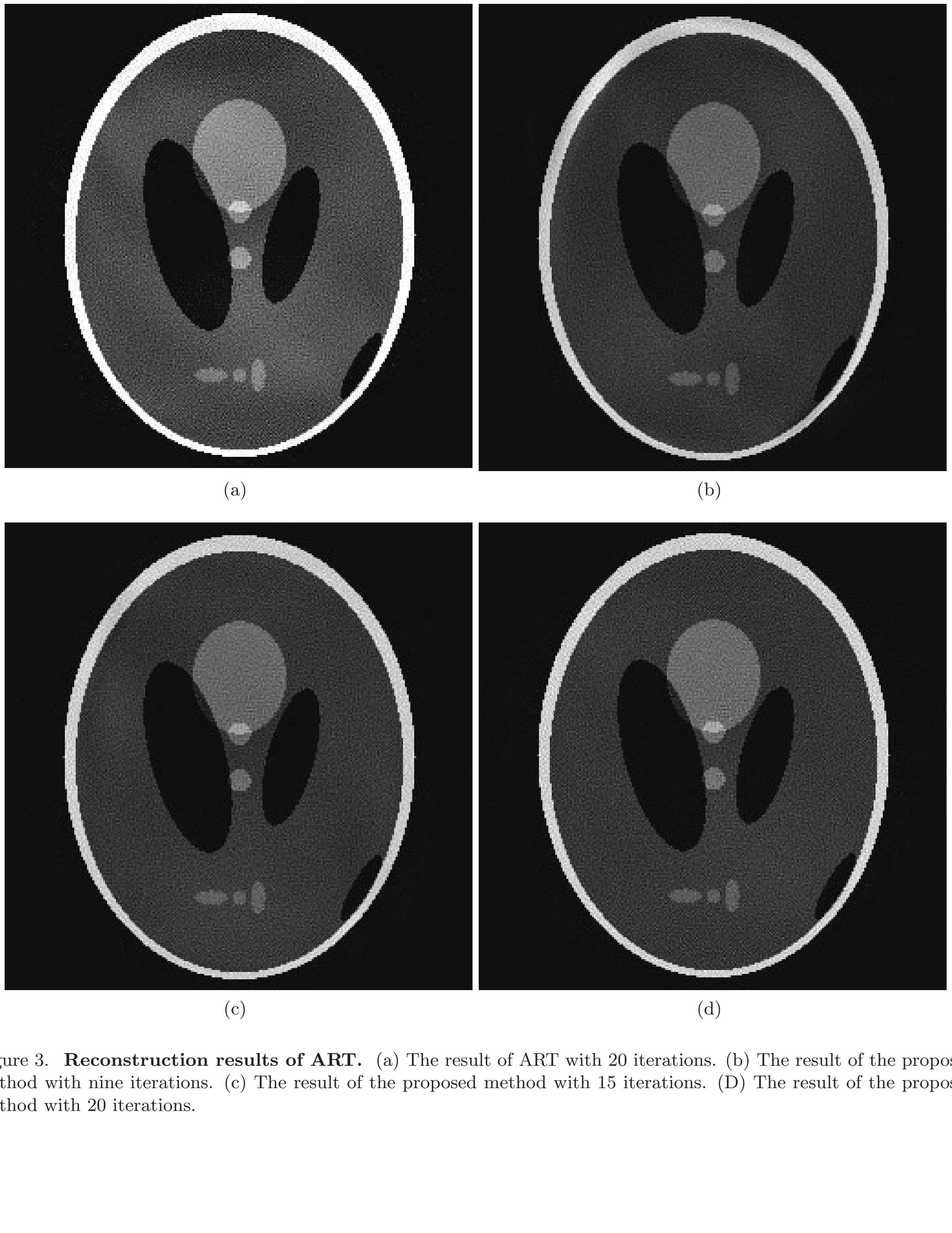


Figure 3. **Reconstruction results of ART.** (a) The result of ART with 20 iterations. (b) The result of the proposed method with nine iterations. (c) The result of the proposed method with 15 iterations. (D) The result of the proposed method with 20 iterations.

Table 1. **The comparisons of time and quality.**

| | (a) ART | (b) the proposed method | (c) the proposed method | (d) the proposed method |
|---|---|---|---|---|
| #iterations | 20 | 9 | 15 | 20 |
| Time (sec.) | 151 | 73.4 | 116 | 151 |
| RMSE | 0.075 | 0.074 | 0.074 | 0.073 |
| PSNR | 35.3 | 35.4 | 35.3 | 35.4 |
| SC | 0.693 | 0.672 | 0.672 | 0.0683 |

Using the result of FBP to be an initial guess and a reference for the first iteration and each backprojection respectively can reduce the number iterations of ART by more than half.

## 6. CONCLUSIONS

We have combined FBP and ART to accelerate the iterative reconstruction for computational tomography. We found that using the result of FBP to be an initial guess and a reference for the first iteration and each backprojection respectively can reduce the number iterations of ART by more than half. The quantitative and comparative analysis has shown that our method also can reconstruct better results than the original ART.

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