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The multi-process performance analysis chart (MPPAC) based on process capability indices has been developed to analyse the manufacturing performance for multiple processes, which conveys critical information regarding the departure of the process mean from the target value, process variability, capability levels, which provides a guideline of directions for capability improvement. Existing MPPAC researches have plotted the sample estimates of the process indices on the chart. Conclusions were then made on whether processes meet the capability requirement and directions need to be taken for further quality improvement. Such an approach is highly unreliable since the sample point estimate is a random variable with no assessment of the sampling errors. Further, existing MPPAC researches only considered one single sample. Current quality control practice is to estimate process capability using multiple groups of control chart samples rather than one single sample. In this paper, we propose the C_{pmk} MPPAC combining the accuracy index C_a to access the performance of multiple manufacturing processes. Distributions of the estimated C_{pmk} and C_a are derived based on multiple control chart samples, and accurate lower confidence bounds are calculated. The lower confidence bounds of the estimated C_{pmk} and C_a are then employed to the MPPAC to provide reliable capability grouping for those multiple processes. A real-world example is presented to illustrate the applicability of the proposed MPPAC.

Keywords: Multi-process performance analysis chart (MPPAC); Process capability index; Multiple characteristics; Lower confidence bound; Process yield

1. Introduction

Achieving customer specifications is an important job for the semiconductor manufacturing quality assurance professional. The ability to consistently deliver product within specifications often determines whether the supplier or manufacturer of semiconductor devices continue to be at the heart of customer satisfaction. Adopting statistical and engineering process control (SPC/EPC) methods (figure 1), is the first step for the supplier to achieve the goal of delivering product

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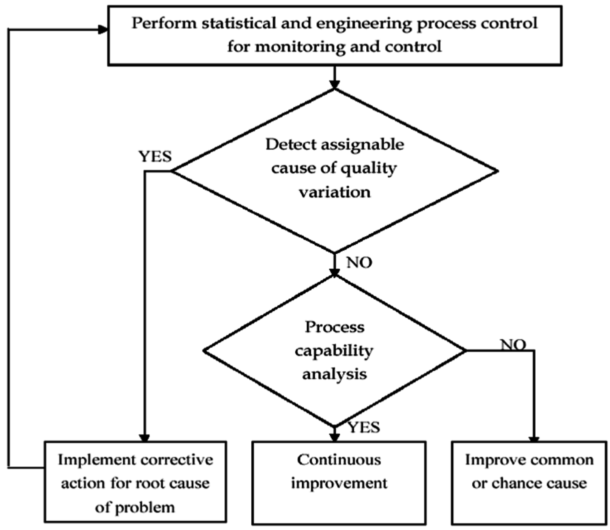


Figure 1. The analysis procedure for process performance.

within specification. A process is considered stable if there are no assignable causes or unpredictable variations and all the points on $\bar{X}$ and S control charts fall within the control limits. The stability of a process is an important property since if the process is stable in the current frame then it is likely to stay in a stable condition in the future. Thus, the output of a stable process is, in some sense, predictable. However, being in control is not sufficient to a manufacturing process since an in-control process can produce bad or non-specification products. Hence, the second component is to use process capability indices to measure how well the process meets specifications. Process capability is the repeatability and consistency ability of a manufacturing process relative to the customer requirements. The final step is that the supplier should continually reduce the level of process variation using continuous improvement techniques. This in turn improves product quality and further reduces the risk that out-of-specification products will be made. The purpose of this paper is to focus on the process capability analysis for stable processes.

Process capability indices, including C_p,C_{pk}, and C_{pm} (Kane 1986, Chan *et al.* 1988) have been proposed in the manufacturing industry to provide a quick indication of process capability with a single-number summary describing how a process has conformed to its specifications preset by manufacturers or customers. Combining the advantages of those indices, Pearn *et al.* (1992) proposed a more advanced capability index called C_{pmk}, which has been shown to be a useful capability index for processes with two-sided specification limits. Those indices are defined as:

$$C_p = \frac{USL - LSL}{6\sigma}, \quad C_{pk} = \min\left\{\frac{USL - \mu}{3\sigma}, \quad \frac{\mu - LSL}{3\sigma}\right\}$$

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}}, \quad C_{pmk} = \min\left\{\frac{USL - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - LSL}{3\sqrt{\sigma^2 + (\mu - T)^2}}\right\},$$

where USL stands for the upper specification limit, LSL stands for the lower specification limit, μ stands for the process mean, σ stands for the process standard deviation, and T stands for the target value setting to the mid-point of the specification limits ($T = (USL + LSL)/2$) predetermined by the product designer, which is quite common in practical application.

Process variation (product quality consistency), process departure, process yield, and process loss have been considered crucial benchmarks for measuring process performance. The index C_p considers the overall process variability relative to the specification tolerance, therefore it only reflects the consistency of the product quality characteristic. The index C_{pk} takes the mean of the process into consideration but it can fail to distinguish between on-target processes from off-target processes, which is a yield-based index providing lower bounds on process yield. The index C_{pm} takes the proximity of process mean from the target value into account, which is more sensitive to process departure than C_{pk}. Since the design of C_{pm} is based on the average process loss relative to the specification tolerance, the index C_{pm} provides an upper bound on the average process loss, which has been alternatively called the Taguchi index or the loss-based index. The index C_{pmk} is constructed from combining the modifications to C_p that produced C_{pk} and C_{pm}, which inherits the merits of both indices.

2. The index C_{pmk} and capability requirement

We note that a process satisfying the quality condition $C_{pk} \geq c_0$ may not satisfy the quality condition $C_{pm} \geq c_0$. On the other hand, a process that satisfies the quality condition $C_{pm} \geq c_0$ may not satisfy the quality condition $C_{pk} \geq c_0$ either. But, a process does satisfy both quality conditions $C_{pk} \geq c_0$ and $C_{pm} \geq c_0$ if the process satisfies the quality condition $C_{pmk} \geq c_0$ since $C_{pmk} \leq c_{pk}$ and $C_{pmk} \leq c_{pm}$. Thus, the index C_{pmk} indeed provides more quality assurance with respect to process yield and process loss to the customers than the other two indices C_{pk} and C_{pm}. This is a desired property according to today's modern quality improvement theory, as reduction of process loss (variation from the target) is just as important as increasing the process yield (meeting the specifications). While the C_{pk} remains the more popular and widely used index, the index C_{pmk} is considered to be the most useful index to date for processes with two-sided specification limits. In practice, a manufacturing process is said to be inadequate if $C_{pmk} < 1.00$; it indicates that the process is not adequate with respect to the manufacturing tolerances, and/or the deviation of $|\mu - T|$, and/or the process variation σ^2 needs to be reduced (often using design of experiments). A manufacturing process is said to be marginally capable if $1.00 \leq C_{pmk} < 1.33$; it indicates that caution needs to be taken regarding the process consistency and some process control is required (usually using R or S control charts). The fraction of nonconformities for such process is within 66–2700 ppm (parts per million). A manufacturing process is said to be satisfactory if $1.33 \leq C_{pmk} < 1.67$; it indicates that process consistency is satisfactory, material substitution may be allowed, and no stringent precision control is required. The fraction of nonconformities for such process is within 0.54–66 ppm. A manufacturing process is said to be excellent if $1.67 \leq C_{pmk} < 2.00$; it indicates that process precision exceeds satisfactory. The fraction of nonconformities for such process

Table 1. Some commonly used capability requirement and the corresponding precision conditions.

Process types	C_{pmk}	**Process yield $\geq 2\Phi(3C_{pmk}) - 1$
Existing processes	1.33	≥99.9933896%
New processes	1.50	≥99.9993198%
Existing processes on safety, strength or critical parameters	1.50	≥99.9993198%
New processes on safety, strength or critical parameters	1.67	≥99.9999455%

$\Phi(\cdot)$ denoted the standard normal cumulative distribution function.

is within 0.002–0.54 ppm. Finally, a manufacturing process is said to be super if $C_{pmk} \geq 2.00$. The fraction of nonconformities for such process is less than 0.002 ppm.

Table 1 summarises the above five capability requirements and the corresponding C_{pmk} values. Some minimum capability requirements have been recommended in the manufacturing industry (Montgomery 2001), for specific process types, which must run under some more designated stringent quality conditions. For existing manufacturing processes, the capability must be no less than 1.33, and for new manufacturing processes, the capability must be no less than 1.50. For existing manufacturing processes on safety, strength, or critical parameters (such as manufacturing soft drinks or chemical solution bottled with glass containers), the capability must be no less than 1.50, and for new manufacturing processes on safety, strength, or critical parameters, the capability must be no less than 1.67.

Furthermore, Pearn *et al.* (1998) indicated the index $C_a = 1 - (|\mu - T|/d)$ for monitoring the accuracy of the manufacturing process, where $d = (USL - LSL)/2$ is half of the length of the specification interval. It is obvious that $C_a = 1$ when $\mu = T$ and $0 < C_a < 1$ when μ move away from T. In fact, C_a can be rewritten as the function of

$$C_1 = \frac{(USL - \mu)}{\left(3\sqrt{\sigma^2 + (\mu - T)^2}\right)}$$

and

$$C_2 = \frac{(\mu - LSL)}{\left(3\sqrt{\sigma^2 + (\mu - T)^2}\right)}$$

which is $C_a = 1 - |C_1 - C_2|/(C_1 + C_2)$. Huang *et al.* (2002) pointed out C_a cannot be too small since a smaller value of C_a implies the process is inaccurate in the sense that that the mean deviates from the target value too much and creates significant loss. In practice, the process accuracy is considered capable with $C_a \geq 0.750$.

Singhal (1990, 1991) proposed the multi-process performance analysis chart (MPPAC) based on process capability indices for controlling and monitoring multiple processes, which sets the priorities among multiple processes for capability improvement and indicates if reducing the variability or the departure of the process mean should be the focus of improvement. The MPPAC provides an easy way to process improvement by comparing the locations on the chart of the processes before and after the improvement effort. Pearn and Chen (1997–1998) proposed a modification to MPPAC combining the more-advanced process capability index, C_{pm} and C_{pmk}, to identify the problems causing the processes failing to centre around the target. Deleryd and Vännman (1999) provided a general framework using C_{dp} (measuring relative process variation) and C_{dr} (measuring relative process departure), which can be applied with appropriate modifications to develop various types of MPPAC charts using different indices, such as C_{pk}, C_{pm}, S_{pk}, and C_{pmk}. Chen *et al*. (2001) extended the MPPAC for controlling product reliability with multiple characteristics where the manufacturing tolerances could be symmetric or asymmetric. Pearn *et al*. (2002) introduced the MPPAC based on the incapability index, which is a simple transformation of C_{pm}.

All existing MPPAC researches develop the MPPAC by simply calculating and plotting the sample estimates of the process indices on the chart, then making conclusions on whether processes meet the capability requirement and directions need to be taken for further quality improvement. Such an approach is highly unreliable (it provides no confidence) since the sample point estimate is a random variable with no assessment to the sampling errors. Further, existing MPPAC researches only considered one single sample. Current quality control practice, however, is to estimate process capability using multiple groups of $\bar{X}$ and S control chart samples rather than one single sample. Therefore, in our work, the C_{pmk} MPPAC is developed to analytically and accurately calculate the lower confidence bounds based on multiple groups of control chart samples (rather than one single sample), which reliably (with designated level of confidence) estimates process capability. This method combines the accuracy index C_a with C_{pmk} to monitor/control and provides correct groupings for multiple processes. A real-world example taken from microelectronics device manufacturing processes is investigated to illustrate the applicability of the proposed MPPAC. As a conclusion, our MPPAC based on analytical approach (not seen in other MPPAC papers) is an advancement of existing technology.

3. The Li-BPIC manufacturing process

The following case is taken from a manufacturing factory located in a science-based industrial park at Hsinchu, Taiwan, making various types of Lithium-ion (Li-Ion) battery protection IC (Li-BPICs). Lithium-ion rechargeable batteries are quickly gaining popularity over nickel-based rechargeable batteries due to their superior light weight, energy density, higher cell voltage, and low self-discharge rate. These characteristics make them perfectly suited to today's high performance portable products. Lithium packs are constantly charged and discharged over their life cycle. An overcharge or over-discharge results in the temperature of the battery increasing. As the electrolyte solution heats up, it may decompose. This will result in a gas being produced or metal lithium being precipitated. These events could cause either a fire or an explosion. Therefore, safety concerns related to overcharging and short circuit protection have driven the industry to include battery protection circuits within the Li-Ion battery pack. Their purposes are to constantly monitor the cell voltage(s) and prevent over-charge or over-discharge by opening the current path if a cell is out of the normal operating voltage range. A typical protection circuit contains a protection IC that monitors the cell voltage. Two field effect transistors

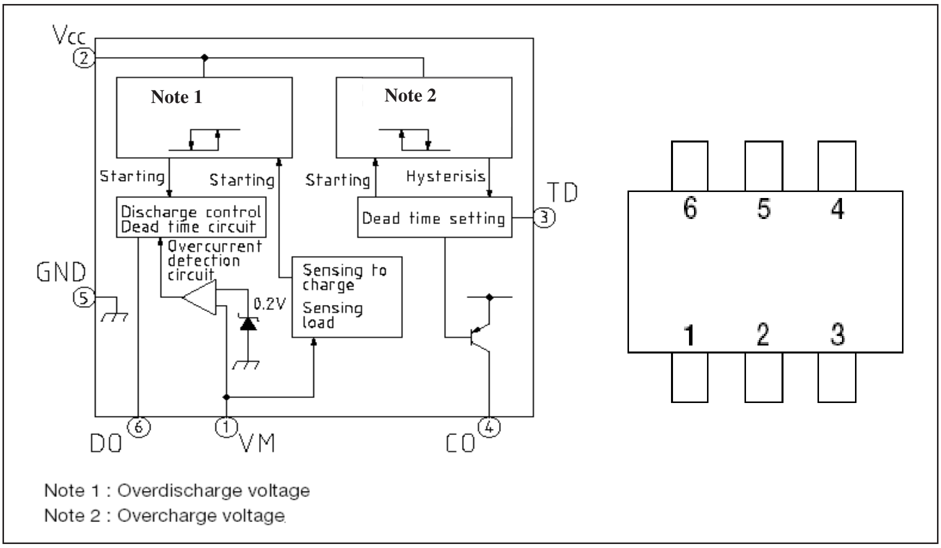
(FET) are used, one to limit the charge current and one to monitor the discharge current. This IC monitors the voltage of the battery connected to V_{CC} and GND pins and the differences in voltages between VM and GND pins to control charging and discharging. There are four conditions during operation as follows:

1. *Normal condition:* If the battery voltage (BV) is in the range from the over-discharge detection voltage (ODDV) to the overcharge detection voltage (OCDV), and the VM pin voltage is in the range from the charger detection voltage to the overcurrent detection voltage (OCUDV) the current flowing through the battery is equal to or lower than a specified value, both the charging and discharging control FETs turn on. In this condition, charging and discharging can be carried out freely.
2. *Overcharge condition:* If the BV becomes higher than the OCDV during charging under normal conditions and it continues for the overcharge detection delay time or longer, the charging control FET turns off to stop charging.
3. *Overdischarge condition:* If the BV falls below the ODDV during discharging under normal condition and it continues for the overdischarge detection delay time or longer, the discharging control FET turns off and discharging stops.
4. *Overcurrent condition (load short-circuiting):* If the discharging current becomes equal to or higher than a specified value (the VM pin voltage is equal to or higher than OCUDV) during discharging under normal conditions and it continues for the overcurrent detection delay time or longer, the discharging control FET turns off to stop discharging.

Therefore, OCDV, ODDV, and OCUDV of the Li-BPIC process are three essential quality characteristics for product reliability performance, which have significant impact on the Li-BPICs quality/reliability. The manufacturing Li-BPICs factory has first implemented a routine-basis production control plan using $\bar{X}$ and S control charts for monitoring/controlling quality characteristics stability of OCDV, ODDV, and OCUDV. Four manufacturing lines need to be controlled and monitored simultaneously in the factory making different types of Li-BPICs. Since OCDV, ODDV, and OCUDV characteristics are of bilateral specifications, using C_{pmk} MPPAC for this typical multiple processes environment is appropriate for product reliability control and improvement. The functional block diagram and the pin descriptions of a Li-BPIC and manufacturing OCDV, ODDV, and OCUDV specifications of the four Li-BPIC products (A, B, D, E) are depicted in figure 2 and table 2, respectively.

4. Development of the C_{pmk} MPPAC

The advantage of using MPPAC compared to using the capability index alone is that MPPACs instantly obtain visual information, simultaneously about the location and spread, as well as information about the capability. When the process is non-capable, the MPPAC are helpful when trying to understand if it is the variability, the deviation from target, or both that need to be reduced to improve the capability. In this way MPPACs provide an obvious guideline for quality improvement. The C_{pmk} MPPAC is shown in figure 3. Four contours for $C_{pmk} = 1.00$, 1.33, 1.67, and 2.00



Pin number	Pin name	Description
1	VM	Overcurrent detection input pin
2	V_{CC}	Positive power supply pin
3	TD	Overcharge detection dead time setting pin
4	CO	FET gate connection pin for charge control
5	GND	Negative power input pin
6	DO	FET gate connection pin for discharge control

Figure 2. The functional block diagram and the pin descriptions of a Li-BPIC.

Table 2. Manufacturing specifications of the four Li-BPIC products.

Product code	OCDV (V) 1			ODDV(V) 2			OCUDV(mV) 3		
	LSL	*T*	*USL*	*LSL*	*T*	*USL*	*LSL*	*T*	*USL*
A	3.175	3.2	3.225	1.20	1.3	1.40	124	150	176
B	4.150	4.2	4.250	2.18	2.3	2.42	172	200	228
D	4.425	4.5	4.575	2.28	2.4	2.52	200	230	260
E	5.400	5.5	5.600	3.40	3.6	3.80	218	250	282

represent different categories of characteristic conditions and the bold supplementary (narrow) lines evaluate characteristic accuracy using the $C_a = 0.750$ measures. Process conditions and the corresponding C_{pmk} and C_a values of the process capability zones are summarised in table 3. On the C_{pmk} MPPAC, we note that:

i. The 45° target line represents the points where the process mean equal to the target ($\mu = T$) and the values of C_1 and C_2 are equal.

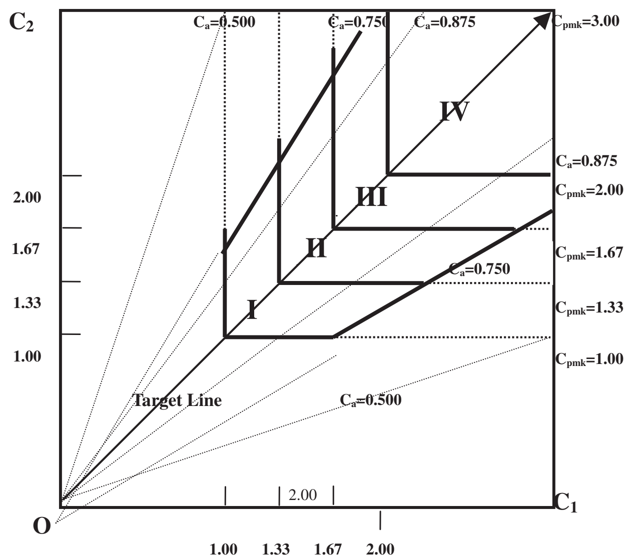


Figure 3. The C_{pmk} MPPAC.

Table 3. Process conditions and the corresponding C_{pmk} and C_a values of the process capability zones.

Process conditions	Capability zones	Bilateral specifications
Marginally capable	I	$1.00 \leq C_{pmk} < 1.33$ and $C_a \geq 0.75$
Satisfactory	II	$1.33 \leq C_{pmk} < 1.67$ and $C_a \geq 0.75$
Excellent	III	$1.67 \leq C_{pmk} < 2.00$ and $C_a \geq 0.75$
Super	IV	$2.00 \leq C_{pmk}$ and $C_a \geq 0.75$

ii. For the points inside the area to the right of the 45° target line, represents processes where the process mean is towards the lower specification limit (process mean is lower than target value). On the other hand, for the points inside the area to the left of the 45° target line represents processes where the process mean is towards the upper specification limit (process mean is higher than target value).

iii. The origin point represents a process with $C_1 = C_2 = 0$ which means that the standard deviation of the process is infinite. As the distance from origin of the projection of the plotted point on the target line increases, the variability of the corresponding process decreases.

iv. Manufacturing processes in the process capability zone I are judged to be marginally capable, those in zone II are judged to be satisfactory, in zone III are judged to be excellent, in zone IV are judged to be super, and those outside the process capability zones are judged to be incapable.

In general, we never know the true values of the process parameters μ and σ^2, C_{pmk} and C_a, either. In purchasing agreements, many customers ask their suppliers to record process capability indices for the critical product characteristics on a regular basis. Kirmani *et al.* (1991) indicated that a common practice of the process capability estimation in the manufacturing industry is to first implement a routine-basis production control plan for monitoring/controlling the process stability. A routine-basis data collection procedure is executed to run $\bar{X}$ and S control charts (for moderate sample sizes), then to analyse the past in-control data. Hence, these parameters need to be estimated based on rational multiple samples without further sampling and calculation and sampling error of the index C_{pmk} and C_a needs to be considered for product reliability purpose. In the next section, sampling distributions of C_{pmk} and C_a based on multiple samples are obtained to compute the lower confidence bound on C_{pmk} and C_a for evaluating processes performance.

5. Measuring process capability C_{pmk}

The past in-control data consisting of m_s multiple samples, with variable sample sizes n_i, $(x_{i1}, x_{i2}, \ldots, x_{in_i})$, are chosen randomly from a stable process which follows a normal distribution $N(\mu, \sigma^2)$, is then analysed to compute the manufacturing capability.

Let

$$\overline{X}_i = \frac{\sum_{j=1}^{n_i} x_{ij}}{n_i} \quad \text{and} \quad S_i = \sqrt{\frac{\sum_{j=1}^{n_i} \left(x_{ij} - \overline{X}_i\right)^2}{n_i}},$$

be the ith sample mean and the sample standard deviation, respectively and total number of observations $N = \sum_{i=1}^{m_s} n_i$. We consider the following natural estimator of C_{pmk} based on multiple samples:

$$\hat{C}_{pmk} = \min\left\{\frac{USL - \overline{\overline{X}}}{3\sqrt{S_p^2 + \left(\overline{\overline{X}} - T\right)^2}}, \frac{\overline{\overline{X}} - LSL}{3\sqrt{S_p^2 + \left(\overline{\overline{X}} - T\right)^2}}\right\},$$

where the overall sample mean is $\overline{\overline{X}} = \sum_{i=1}^{m_s} \overline{X}_i / m_s$ and $S_p^2 = \sum_{i=1}^{m_s} (n_i) S_i^2 / N$ is the pooled variance. Then, the estimator $\hat{C}_{pmk}$ can be rewritten as follows:

$$\hat{C}_{pmk} = \frac{d - \left|\overline{\overline{X}} - T\right|}{3\sqrt{S_P^2 + \left(\overline{\overline{X}} - T\right)^2}}.$$

From (A2) and (A3) in the Appendix, changing the variable with $y = t^2$, the CDF and PDF of $\hat{C}_{pmk}$ can be expressed in terms of a mixture of the Chi-square distribution and the normal distribution:

$$F_{\hat{C}_{pmk}}(x) = 1 - \int_0^{b\sqrt{N}/(1+3x)} F_K\left(\frac{\left(b\sqrt{N} - t\right)^2}{9x^2} - t^2\right)\left[\phi\left(t + \xi\sqrt{N}\right) + \phi\left(t - \xi\sqrt{N}\right)\right]dt,$$

$$f_{\hat{C}_{pmk}}(x) = \int_0^{b\sqrt{N}/(1+3x)} f_K\left(\frac{\left(b\sqrt{N} - t\right)^2}{9x^2} - t^2\right)\frac{2\left(b\sqrt{N} - t\right)^2}{9x^3}\left[\phi\left(t + \xi\sqrt{N}\right) + \phi\left(t - \xi\sqrt{N}\right)\right]dt,$$

for $x > 0$, where $b = d/\sigma$ and $\xi = (\mu - T)/\sigma$, $F_K(\cdot)$ and $f_K(\cdot)$ are the CDF and PDF of the ordinary central Chi-square distribution $\chi^2_{N-m_s}$, and $\phi(\cdot)$ is the PDF of the standard normal distribution $N(0,1)$. Figure 4(a)–(d) display the PDF plots of $\hat{C}_{pmk}$

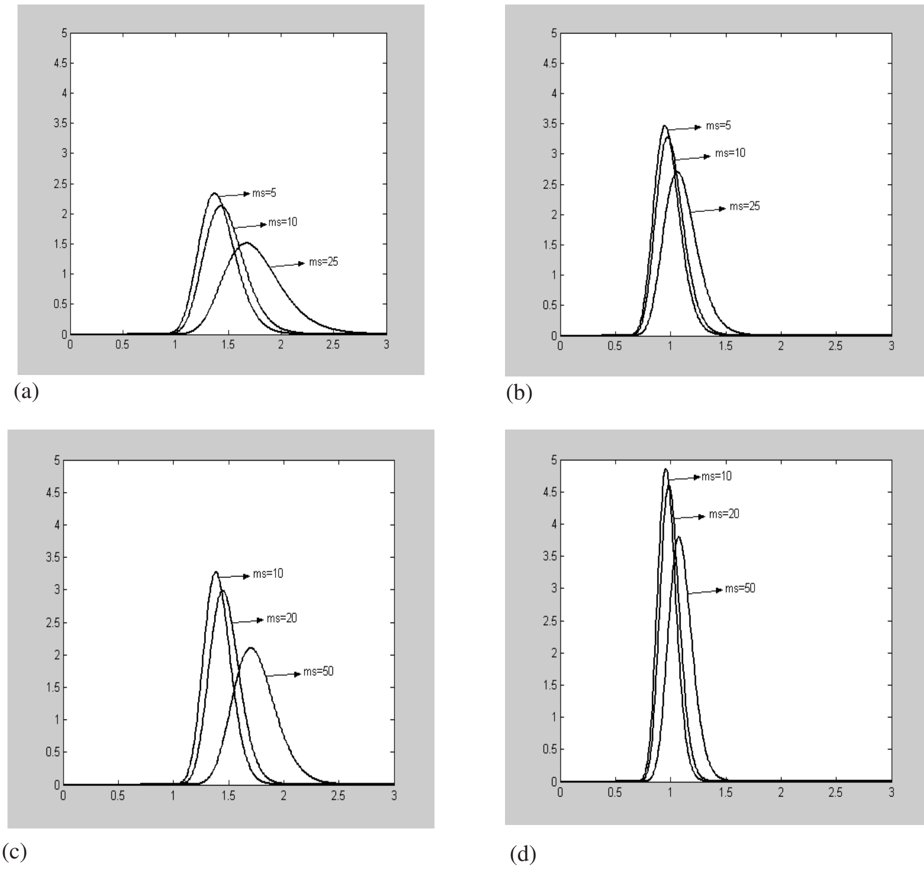


Figure 4. (a) PDF plots of $\hat{C}_{\rm pmk}$ with $\xi = 0.5$, $b = 5$, $d = 3$, and $N = 50$, $m_s = 5$, 10, 25. (b) PDF plots of $\hat{C}_{\rm pmk}$ with $\xi = 1.0$, $b = 5$, $d = 3$, and $N = 50$, $m_s = 5$, 10, 25. (c) PDF plots of $\hat{C}_{\rm pmk}$ with $\xi = 0.5$, $b = 5$, $d = 3$, and $N = 100$, $m_s = 10$, 20, 50. (d) PDF plots of $\hat{C}_{\rm pmk}$ with $\xi = 1.0$, $b = 5$, $d = 3$, and $N = 100$, $m_s = 10$, 20, 50.

for $\xi = 0.5$ and 1, $b = 5$, $d = 3$, with $N = 50$ and 100 and various m_s. From these figures, we observe as m_s increases, the bias increases and the distributions are skew and have large bias and spread for fixed N with increasing m_s.

5.1 *Lower confidence bounds on C_{pmk}*

The index C_{pmk} may be rewritten as the following:

$$C_{pmk} = \frac{d - |\mu - T|}{3\sqrt{\sigma^2 + (\mu - T)^2}} = \frac{d/\sigma - |\xi|}{3\sqrt{1 + \xi^2}}$$

When $C_{pmk} = \mathrm{C}$, $b = d/\sigma$ can be expressed as $b = 3C\sqrt{1 + \xi^2} + |\xi|$. Hence, given the total sample size N with m_s, the confidence level γ, the estimated value $\hat{C}_{pmk}$, and the parameter ξ the lower confidence bounds C_{pmk}^L can be obtained using numerical integration technique with iterations, to solve the following equation (1) (Matlab program is available upon request). In practice, the parameter $\xi = (\mu - T)/\sigma$

is unknown, but it can be calculated from the sample data as $\hat{\xi} = (\overline{\overline{X}} - T)/S_p$. It should be noted that the equation is an even function of $\tilde{\xi}$. Thus, for both $\xi = \xi_0$ and $\xi = -\xi_0$ we have the same lower confidence bounds.

$$\int_0^{b\sqrt{N}/(1+3\hat{C}_{pmk})} F_K\left(\frac{(b\sqrt{N} - t^2)}{9\hat{C}_{pmk}^2} - t^2\right)\left[\phi\left(t + \xi\sqrt{N}\right) + \phi\left(t - \xi\sqrt{N}\right)\right]dt = 1 - \gamma. \quad (1)$$

To eliminate the need for further estimating the distribution characteristic parameter ξ, we examine the behaviour of the lower confidence bound values C_{pmk}^L against the parameter ξ Pearn and Shu (2004) investigated the behaviour of the lower confidence bound versus the process characteristic parameter ξ based on one single sample, which resulted that the lower confidence bound attains its minimal value at $\xi = 0.5$. For multiple samples, we perform extensive calculations to obtain the C_{pmk}^L for $\xi = 0(0.05)3.00$, $\hat{C}_{pmk} = 0.7(0.1)3.0$, the total number of observations $N = 50(50)200$ with various m_s for confidence level $\gamma = 0.90$, 0.95, 0.975, and 0.99. The results indicate that the lower confidence bound first decreases as ξ increases, obtains its minimum value at $\xi = 0.45$ or 0.5, then increases again within the range of $\xi \in [0.5, 3.0]$ in all cases. Hence, for practical purpose we may solve equation (1) with $\xi = 0.5$ to obtain the required sample accuracy for given N, m_s and γ, without having to further estimate the parameter ξ. The proposed decision-making procedure ensures that the risk of making a wrong decision will be no greater than the preset type I error $1 - \gamma$.

6. Measuring process departure C_a

We consider the natural estimator $\hat{C}_a$ to estimate the accuracy index C_a, which can be expressed as the following:

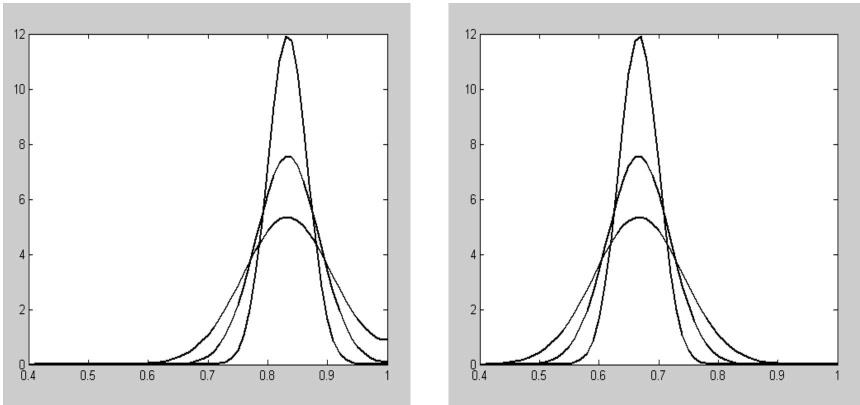
$$\hat{C}_a = 1 - \frac{\left|\overline{\overline{X}} - T\right|}{d} = 1 - \frac{\sigma}{d\sqrt{N}} \times \frac{\left|\overline{\overline{X}} - T\right|}{\sigma/\sqrt{N}},$$

where $\sqrt{N}|\overline{\overline{X}} - T|/\sigma$ is distributed as the folded normal distribution with parameter $\sqrt{N}|\mu - T|/\sigma$ as defined by Leone *et al.* (1961). From (A4) and (A5) in the Appendix, changing variable with $y = t^2$, the CDF and PDF of $\hat{C}_a$ can be expressed in terms of a mixture of the normal distribution:

$$F_{\hat{C}_a}(x) = 1 - \int_0^{b\sqrt{N}(1-x)} \left[\phi\left(t + \xi\sqrt{N}\right) + \phi\left(t - \xi\sqrt{N}\right)\right]dt, \quad \text{for} \quad x < 1$$

$$f_{\hat{C}_a}(x) = b\sqrt{N}\left[\phi\left(b\sqrt{N}(1-x) + \xi\sqrt{N}\right) + \phi\left(b\sqrt{N}(1-x) - \xi\sqrt{N}\right)\right], \quad \text{for} \quad x < 1$$

Figure 5(a)–(b) display the PDF plots of $\hat{C}_a$ for $\xi = 0.5$ and 1, $b = 3$, $d = 3$, with various sample sizes $N = 20, 40$, and 100. From both figures, we observe that the estimate $\hat{C}_a$ is approximately unbiased and the spread decreases with increasing N.



(a) (b)

Figure 5. (a) PDF plots of $\hat{C}_a$ with $\xi = 0.5$, $b = 3$, $d = 3$, and $N = 20$, 40, 100 (bottom to top). (b) PDF plots of $\hat{C}_a$ with $\xi = 1.0$, $b = 3$, $d = 3$, and $N = 20$, 40, 100 (bottom to top).

6.1 *Lower confidence bounds on C_a*

Given $C_{pmk} \geq C_{pmk}^L$, we can obtain a lower bound on C_a as $3C_{pmk}^L/(3C_{pmk}^L + 1)$. Furthermore, the index C_a may be rewritten as the following:

$$C_a = 1 - \frac{|\mu - T|}{d} = 1 - \frac{|\xi|}{d/\sigma}.$$

When $C_a = C^*$, $b = d/\sigma$ can be expressed as $b = |\xi|/(1 - C^*)$. Hence, given a sample of size N, the confidence level γ, the estimated value $\hat{C}_a$, and the parameter ξ, the lower confidence bounds C_a^L can be obtained using numerical integration technique with iterations, to solve the following equation:

$$\int_0^{b\sqrt{N}\left(1-\hat{C}_a\right)} \left[\phi\left(t + \xi\sqrt{N}\right) + \phi\left(t - \xi\sqrt{N}\right)\right] dt = 1 - \gamma \tag{2}$$

In order to obtain a meaningful and reliable lower confidence bound on C_a, $\max(3C_{pmk}^L/(3C_{pmk}^L + 1),\ C_a^L)$ is applied to the real application. In the next section, the lower confidence bounds on C_{pmk} and C_a will be used in the MPPAC to provide more reliable capability assurance for the Li-BPIC products.

7. Manufacturing quality computation

We collected sample data for the investigated characteristics from some OCDV, ODDV, and OCUDV processes, justified by stable $\overline{X}$ and S control charts of four manufacturing processes making different kinds of Li-BPIC devices. Data of the sample mean and the sample standard deviation for 24 multiple samples each with sample size 5 are taken and simply calculated the overall mean, the pooled sample variance, and the estimate $\hat{C}_{pmk}$ and $\hat{C}_a$. The product codes, the estimated index values, the lower confidence bounds (LCBs), and the corresponding maximum fractions of nonconformities (in ppm) for the four processes are tabulated in table 4. Figure 6 plots the C_{pmk} MPPAC for the four processes based on the minimum true values tabulated in table 4. We analyse these process points in figure 6 and obtain the following critical summary information of the capability condition for all characteristics.

i. The plotted points A2, B1, B3, D3, and E1 are not located within capability zones. It indicates that the process has a very low capability both on accuracy and precision. Actions must be taken to improve the process quality, either to shift the process mean close to the process target or to reduce the process variation.

ii. The plotted points E2 are not located within capability zones. It indicates that the process has a very low capability. Since the points E2, $\max(3C^L_{pmk}/(3C^L_{pmk}+1), C^L_a) = 0.98$, this process presents that the process mean is close to the target value, and the poor capability is mainly contributed by the significant process variation. Thus, immediate quality improvement actions must be taken for reducing the process variance for this process.

iii. The plotted points A1, B2, D1, and E3 lie within the capability zone I. It indicates that these processes are marginal capable. The points D1 and E3 lies inside the area to the right of the 45° target line represents processes where the process mean is towards the lower specification limit (process mean is lower than target value). On the other hand, the point A1 and B2 lies inside the area, which is to the left of the 45° target line represents processes where the process mean is towards the upper specification limit (process mean is higher than target value). Thus, quality improvement effort for these processes should be first focused on reducing their process departure from the target value T, then the reduction of the process variance.

iv. Process D2 and A3 lie inside the capability zone II and III. Both processes are considered performing satisfactory and excellent, respectively, and no immediate improvement activities needed to be taken. Both processes have the lower priority in allocating quality improvement efforts than other processes.

Table 5 displays the manufacturing quality and capability groupings for the 12 processes using the estimated C_{pmk} and C_a values (uncorrected) and the lower confidence bounds (corrected) (with asterisks * indicating incorrect groupings). The C_{pmk} MPPAC for the twelve processes based on the estimated C_{pmk} and C_a index values (an approach widely used in current industrial applications) rather than using the lower confidence bounds, is displayed in figure 7. We note that such MPPAC obviously conveys unreliable information and is misleading, which should be avoided in real applications.

8. Conclusions

In this paper, we developed the C_{pmk} MPPAC, which incorporates with the accuracy index C_a to analyse the performance of a group of manufacturing processes. The proposed C_{pmk} MPPAC prioritises the order of those processes for which the quality improvement effort should focus on, either to move the process mean closer to the target value or reduce the process variation on one single chart. We obtained analytically the sampling distributions and the corresponding probability density

Table 4. Calculated statistics and LCBs of the estimated C_{pmk} and C_a, and the maximum fractions of nonconformities (in ppm) of the four Li-BPIC products.

ProductCode	Calculated statistics, LCBs, and maximum fractions of nonconformities (in ppm)									
	$\overline{\overline{X}}$	S_p	$\hat{C}_1$	$\hat{C}_2$	$\hat{C}_{pmk}$	$\hat{C}_a$	C^L_{pmk}	C^L_a	$\max((3C^L_{pmk}/3C^L_{pmk}+1),C^L_a)$	ppm
A1	3.204	0.000176	1.626	2.292	1.626	0.83	1.292	0.76	0.80	106.23
A2	1.321	0.009487	1.143	1.750	1.143	0.79	0.898	0.70	0.73	7060.11
A3	151.04	3.290897	2.411	2.612	2.411	0.96	1.931	0.94	0.94	0.006935
B1	4.18	0.000105	1.167	0.50	0.500	0.60	0.372	0.43	0.53	264422.2
B2	2.310	0.020901	1.600	1.878	1.600	0.92	1.271	0.89	0.89	137.34
B3	205.32	3.959222	1.140	1.675	1.140	0.81	0.896	0.73	0.73	7188.23
D1	4.486	0.001095	2.082	1.417	1.417	0.81	1.122	0.73	0.77	762.78
D2	2.389	0.014144	2.450	2.045	2.045	0.91	1.643	0.87	0.87	0.8277
D3	243.5	0.298329	0.407	1.074	0.407	0.55	0.296	0.36	0.47	374540.7
E1	5.518	0.023637	0.920	1.324	0.920	0.82	0.716	0.74	0.74	31713.6
E2	3.604	0.069388	0.940	0.978	0.940	0.99	0.732	0.98	0.98	28091.8
E3	246	5.5000	1.765	1.372	1.372	0.875	1.109	0.82	0.82	877.98

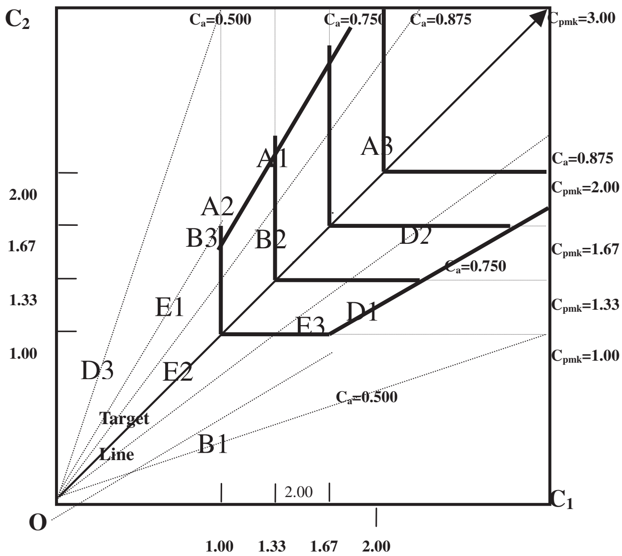


Figure 6. The C_{pmk} MPPAC groups for the Li-BPIC processes.

and cumulative distribution functions of the estimated C_{pmk} and C_a based on multiple samples, to compute the lower confidence bounds on C_{pmk} and C_a. The lower confidence bounds of the estimated C_{pmk} and C_a are then employed to the MPPAC, to provide reliable capability grouping for those processes. Our implementation of the complicated statistical theory for manufacturing quality assessment bridges the gap between the theoretical development and the factory applications.

Appendix

Sampling distributions of C_{pmk} and C_a based on multiple samples

Given a set of m_s multiple samples, with variable sample sizes $n_i, (x_{i1}, x_{i2}, \ldots, x_{in_i})$, taken randomly from a stable normally distributed process. Let

$$\overline{X}_i = \sum\nolimits_{j=1}^{n_i} x_{ij}/n_i \quad \text{and} \quad S_i = \left[(n_i)^{-1}\sum\nolimits_{j=1}^{n_i}\left(x_{ij} - \overline{X}_i\right)^2\right]^{1/2}$$

be the ith sample mean and the sample standard deviation, with total number of observations $N = \sum_{i=1}^{m_s} n_i$. We consider the following natural estimator of C_{pmk} and C_a based on multiple samples as

$$C_{pmk} = \min\left\{\frac{USL - \overline{\overline{X}}}{3\sqrt{S_p^2 + \left(\overline{\overline{X}} - T\right)^2}}, \frac{\overline{\overline{X}} - LSL}{3\sqrt{S_p^2 + \left(\overline{\overline{X}} - T\right)^2}}\right\} \quad \text{and} \quad \hat{C}_a = 1 - \frac{\left|\overline{\overline{X}} - T\right|}{d},$$

where the overall sample mean is $\overline{\overline{X}} = \sum_{i=1}^{m_s} \overline{X}_i/m_s$ and $S_p^2 = \sum_{i=1}^{m_s} (n_i)S_i^2/N$ is the pooled variance. Then, the estimator $\hat{C}_{pmk}$ can be rewritten as follows:

$$\hat{C}_{pmk} = \frac{d - \left|\overline{\overline{X}} - T\right|}{3\sqrt{S_P^2 + \left(\overline{\overline{X}} - T\right)^2}},$$

Table 5. Estimated and corrected (LCB) capabilities and their groupings for the Li-BPIC processes.

Code	Estimated value$_{(\hat{C}_{pmk}, \hat{C}_a)}$	Grouping	LCB $[C^L_{pmk}, \max((3C^L_{pmk}/3C^L_{pmk}+1), C^L_a)]$	Grouping
A1	(1.626, 0.83)	Satisfactory*	(1.292, 0.80)	Marginally capable
A2	(1.143, 0.79)	Marginally capable*	(0.898, 0.73)	Incapable
A3	(2.411, 0.96)	Super*	(1.931, 0.94)	Excellent
B1	(0.50, 0.60)	Incapable	(0.372, 0.53)	Incapable
B2	(1.60, 0.92)	Satisfactory*	(1.271, 0.89)	Marginally capable
B3	(1.14, 0.81)	Marginally capable*	(0.896, 0.73)	Incapable
D1	(1.417, 0.81)	Satisfactory*	(1.122, 0.77)	Marginally capable
D2	(2.045, 0.91)	Super*	(1.643, 0.87)	Satisfactory
D3	(0.407, 0.55)	Incapable	(0.296, 0.47)	Incapable
E1	(0.92, 0.82)	Incapable	(0.716, 0.74)	Incapable
E2	(0.94, 0.99)	Incapable	(0.732, 0.98)	Incapable
E3	(1.372, 0.875)	Satisfactory*	(1.109, 0.82)	Marginally capable

*Indicating incorrect groupings.

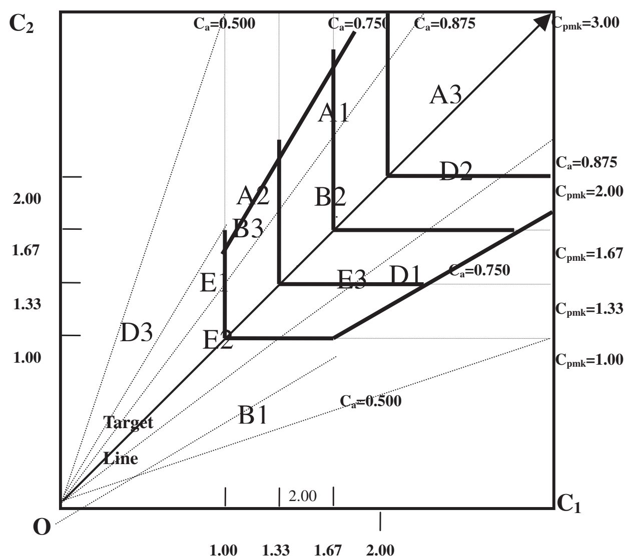


Figure 7. The C_{pmk} MPPAC based on $\hat{C}_{\text{pmk}}$ and $\hat{C}_a$.

To derive the probability density function (PDF) and the cumulative distribution function (CDF) of $\hat{C}_{pmk}$ and $\hat{C}_a$, we define:

i. $D = \sqrt{N}d/\sigma$,
ii. $K = NS_P^2/\sigma^2$, which is distributed as $\chi^2_{N-m_s}$,
iii. $Z = \sqrt{N}(\overline{\overline{X}} - T)/\sigma$, which is distributed as $N(\delta, 1)$, where $\delta = \sqrt{N}(\mu - T)/\sigma$,
iv. $Y = Z^2$, then, the PDF of Y can be expressed as:

$$f_Y(y) = \frac{1}{2\sqrt{y}}\left(f_Z\left(-\sqrt{y}\right) + f_Z\left(\sqrt{y}\right)\right), \quad \text{for } y > 0, \tag{A1}$$

We note that the estimator $\hat{C}_{pmk}$ can be rewritten as:

$$\hat{C}_{pmk} = \frac{D - \sqrt{Y}}{3\sqrt{K + Y}}.$$

$$F_{\hat{C}_{pmk}}(x) = 1 - P\left\{\frac{D - \sqrt{Y}}{3\sqrt{K + Y}} > x\right\} = 1 - \int_0^\infty P\left\{\sqrt{K + Y} < \frac{D - \sqrt{Y}}{3x} \,\middle|\, Y = y\right\} f_Y(y)dy,$$

$$P\left\{\sqrt{K + y} < \frac{D - \sqrt{y}}{3x}\right\} = 0, \quad \text{for } x > 0 \quad \text{and} \quad y > D^2,$$

$$F_{\hat{C}_{pmk}}(x) = 1 - \int_0^{D^2} P\left\{\sqrt{K + y} < \frac{D - \sqrt{y}}{3x}\right\} f_Y(y)dy.$$

Since,

$$P\left\{K < \frac{\left(D - \sqrt{y}\right)^2}{9x^2} - y\right\} = 0, \quad \text{for } [D/(1 + 3x)]^2 < y \le D^2,$$

then

$$F_{\hat{C}_{pmk}}(x) = 1 - \int_0^{D^2/(1+3x)^2} P\left\{K < \frac{\left(D - \sqrt{y}\right)^2}{9x^2} - y\right\} f_Y(y)dy, \quad x > 0.$$

Using the expression in (A1), we obtain the CDF and PDF of $\hat{C}_{pmk}$ as:

$$F_{\hat{C}_{pmk}}(x) = 1 - \int_0^{D^2/(1+3x)^2} F_K\left(\frac{\left(D-\sqrt{y}\right)^2}{9x^2} - y\right)\frac{1}{2\sqrt{y}}\left(f_Z\left(-\sqrt{y}\right) + f_Z\left(\sqrt{y}\right)\right)dy, \quad \text{(A2)}$$

$$f_{\hat{C}_{pmk}}(x) = \int_0^{D^2/(1+3x)^2} f_K\left(\frac{\left(D-\sqrt{y}\right)^2}{9x^2} - y\right)\frac{\left(D-\sqrt{y}\right)^2}{9x^3\sqrt{y}}\left(f_Z\left(-\sqrt{y}\right) + f_Z\left(\sqrt{y}\right)\right)dy, \quad \text{(A3)}$$

where $F_K(\cdot)$ and $f_K(\cdot)$ are the CDF and PDF of the ordinary central Chi-square distribution $\chi^2_{N-m_s}$. Similarly, we note that the estimator $\hat{C}_a$ can be rewritten as:

$$\hat{C}_a = 1 - \frac{\sqrt{Y}}{D}$$

$$F_{\hat{C}_a}(x) = P\left\{1 - \frac{\sqrt{Y}}{D} \leq x\right\} = 1 - P\left(Y \leq [D(1-x)]^2\right) = 1 - \int_0^{[D(1-x)]^2} f_Y(y)dy.$$

Using the expression in (A1), we obtain the CDF and PDF of $\hat{C}_a$ as:

$$F_{\hat{C}_a}(x) = 1 - \int_0^{[D(1-x)]^2} \frac{1}{2\sqrt{y}}\left[f_Z\left(-\sqrt{y}\right) + f_Z\left(\sqrt{y}\right)\right]dy \quad \text{(A4)}$$

$$f_{\hat{C}_a}(x) = D[f_Z(-D(1-x)) + f_Z(D(1-x))], \quad \text{for} \quad x > 1. \quad \text{(A5)}$$

Moments *of* $\hat{C}_{pmk}$ *and* $\hat{C}_a$

To obtain the rth moment of $\hat{C}_{pmk}$ and $\hat{C}_a$, we apply the method similar to that used in Pearn *et al.* (1992, 1998), and Vännman (1995). The derivation is shown below. To derive the expected value and variance of $\hat{C}_{pmk}$, we first calculate rth moment as follows:

$$\hat{C}_{pmk} = \frac{D - \sqrt{Y}}{3\sqrt{K+Y}} = 3^{-1}\left(D - \sqrt{Y}\right)(K+Y)^{-1/2}.$$

Thus,

$$E\left[\hat{C}^r_{pmk}\right] = 3^{-r}\sum_{i=0}^{r}\binom{r}{i}(-1)^i D^{r-i} E\left[Y^{i/2}(K+Y)^{-r/2}\right],$$

$$E\left[Y^{i/2}(K+Y)^{-r/2}\right] = \frac{e^{-\lambda/2}}{2\sqrt{\pi}}\sum_{j=0}^{\infty}\frac{2^{j/2}\lambda^{j/2}}{j!}\Gamma\left(\frac{1+j}{2}\right)E\left[Y_j^{i/2}\left(K+Y_j\right)^{-r/2}\right]$$
$$+ \frac{e^{-\lambda/2}}{2\sqrt{\pi}}\sum_{j=0}^{\infty}\frac{2^{j/2}(-\lambda)^{j/2}}{j!}\Gamma\left(\frac{1+j}{2}\right)E\left[Y_j^{i/2}\left(K+Y_j\right)^{-r/2}\right],$$

where $Y_j \sim \chi^2_{1+j}$. Let $e_j = Y_j/(K+Y_j)$ and $W_j = K + Y_j$. Under the assumption of normality e_j and W_j are independent (Johnson *et al.* (1994) or Vännman (1995)), and e_j is distributed as Beta(α, β) distribution with $\alpha = (1+j)/2$, and $\beta = (N-m_s)/2$.

Further, W_j is distributed as $\chi^2_{N-m_s+1+j}$, a chi-square distribution with $(N-m_s+1+j)$ degrees of freedom. Therefore,

$$E\left[Y_j^{i/2}\left(K+Y_j\right)^{-r/2}\right]=E\left[W_j^{-(r-i)/2}\right]E\left[e_j^{i/2}\right],$$

$$E\left[W_j^{-(r-i)/2}\right]=2^{-(r-i)/2}\frac{\Gamma(c-a)}{\Gamma(c-i/2)},\quad E\left[e_j^{i/2}\right]=\frac{\Gamma(c-i/2)\Gamma(b)}{\Gamma(b-i/2)\Gamma(c)},$$

where $a=r/2$, b $(1+I+j)/2$, and $c=(N-m_s+1+I+j)/2$. Combining these results, we can obtain the rth moment of $\hat{C}_{pmk}$ as expressed in the following:

$$E\left[\hat{C}_{pmk}^r\right]=3^{-r}\sum_{i=0}^{r}\binom{r}{i}(-1)^iD^{r-i}\left\{\frac{e^{-\lambda/2}}{2^{(r-i+2)/2}\sqrt{\pi}}\sum_{j=0}^{\infty}\frac{2^{j/2}\lambda^{j/2}}{j!}\Gamma\left(\frac{1+j}{2}\right)\frac{\Gamma(c-a)\Gamma(b)}{\Gamma(b-i/2)\Gamma(c)}\right.$$
$$\left.+\frac{e^{-\lambda/2}}{2^{(r-i+2)/2}\sqrt{\pi}}\sum_{j=0}^{\infty}\frac{2^{j/2}(-1)^j\lambda^{j/2}}{j!}\Gamma\left(\frac{1+j}{2}\right)\frac{\Gamma(c-a)\Gamma(b)}{\Gamma(b-i/2)\Gamma(c)}\right\}.$$

Let $j=2l$

$$E\left[\hat{C}_{pmk}^r\right]=3^{-r}\sum_{i=0}^{r}(-1)^i\binom{r}{i}\left(\frac{D}{\sqrt{2}}\right)^{r-i}\sum_{\ell=0}^{\infty}\frac{\lambda^\ell e^{-\lambda/2}}{2^\ell \ell!}\times\frac{\Gamma(c-a)\Gamma(b)}{\Gamma(b-i/2)\Gamma(c)},$$

$$E\left[\hat{C}_{pmk}\right]=\frac{D}{3\sqrt{2}}\sum_{\ell=0}^{\infty}\left[\frac{\lambda^\ell e^{-\lambda/2}}{2^\ell \ell!}\times\frac{\Gamma((N-m_s)/2+\ell)}{\Gamma((N-m_s+1)/2+\ell)}\right]$$
$$-\frac{1}{3}\sum_{\ell=0}^{\infty}\left[\frac{\lambda^\ell e^{-\lambda/2}}{2^\ell \ell!}\times\frac{\Gamma(1+\ell)\Gamma((N-m_s+1)/2+\ell)}{\Gamma(1/2+\ell)\Gamma((N-m_s+2)/2+\ell)}\right],$$

$$E\left(\hat{C}_{pmk}^2\right)=\left(\frac{D}{3}\right)^2\sum_{\ell=0}^{\infty}\frac{\lambda^\ell e^{-\lambda/2}}{2^\ell \ell!(N-m_s-1+2\ell)}-\frac{2\sqrt{2}D}{9}\sum_{\ell=0}^{\infty}\frac{\lambda^\ell e^{-\lambda/2}}{2^\ell(N-m_s+2\ell)\Gamma(1/2+\ell)},$$
$$+\frac{1}{9}\sum_{\ell=0}^{\infty}\frac{\lambda^\ell e^{-\lambda/2}(1+2\ell)}{2^\ell \ell!(N-m_s+1+2\ell)},\quad \text{thus}\quad \mathrm{Var}\left(\hat{C}_{pmk}\right)=E\left(\hat{C}_{pmk}^2\right)-E^2\left(\hat{C}_{pmk}\right).$$

For $\hat{C}_a$ the rth moment of $\hat{C}_a$ can be obtained as:

$$E\left(\hat{C}_a^r\right)=E\left[\left(1-\frac{\sqrt{Y}}{D}\right)^r\right]=\sum_{i=0}^{r}\binom{r}{i}(-1)^iD^{-i}E\left(Y^{i/2}\right).$$

Hence,

$$E\left(\hat{C}_a\right)=C_a-\frac{\sqrt{2}e^{-\lambda/2}}{D\sqrt{\pi}}+2(1-C_a)\Phi(-|\delta|),$$

$$E\left(\hat{C}_a^2\right)=C_a^2+\frac{1}{D^2}-\frac{2\sqrt{2}e^{-\lambda/2}}{3D\sqrt{\pi}}+4(1-C_a)\Phi(-|\delta|),\quad \mathrm{Var}\left(\hat{C}_a\right)=E\left(\hat{C}_a^2\right)-E^2\left(\hat{C}_a\right).$$

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References

Chan, L.K., Cheng, S.W. and Spiring, F.A., A new measure of process capability: C_{pm}. *J. Quality Tech.*, 1988, **20**(3), 162–175.

Chen, K.S., Huang, M.L. and Li, R.K., Process capability analysis for an entire product. *Int. J. Prod. Res.*, 2001, **39**(17), 4077–4087.

Deleryd, D. and Vännman, K., Process capability plots—a quality improvement tool. *Qual. Rel. Eng. Int.*, 1999, **15**, 213–227.

Huang, M.L., Chen, K.S. and Hung, Y.H., Integrated process capability analysis with an application in backlight module. *Microelect. Rel.*, 2002, **42**(12), 2009–2014.

Johnson, N.L., Kotz, S. and Pearn, W.L., Flexible process capability indices. *Pakistan J. Stat.*, 1994, **10A**(1), 23–31.

Kane, V.E., Process capability indices. *J. Qual. Technol.*, 1986, **18**(1), 41–52.

Kirmani, S.N.U.A., Kocherlakota, K. and Kocherlakota, S., Estimation of σ and the process capability index based on sub-samples. *Communications in Statistics: Theory and Methods*, 1991, **20**(1), 275–291.

Leone, F.C., Nelson, L.S. and Nattingham, R.B., The folded normal distribution. *Technometrics*, 1961, **3**(4), 543–550.

Montgomery, D.C., *Introduction to Statistical Quality Control*, 4th Edn, 2001 (John Wiley & Sons, Inc.: New York, NY).

Pearn, W.L., Kotz, S. and Johnson, N.L., Distributional and inferential properties of process capability indices. *J. Qual. Technol.*, 1992, **24**, 216–231.

Pearn, W.L. and Chen, K.S., Multiprocess performance analysis: a case study. *Qual. Eng.*, 1997–98, **10**(1), 1–8.

Pearn, W.L., Lin, G.H. and Chen, K.S., Distributional and inferential properties of the process accuracy and process precision indices. *Communications in Statistics: Theory and Methods*, 1998, **27**(4), 985–1000.

Pearn, W.L., Ko, C.H. and Wang, K.H., A multiprocess performance analysis chart based on the incapability index C_{pp}: an application to the chip resistors. *Microelect. Rel.*, 2002, **42**(7), 1121–1125.

Pearn, W.L. and Shu, M.-H., Measuring manufacturing capability based on lower confidence bounds of C_{pmk} applied to current transmitter process. *Int. J. Adv. Manuf. Tech.*, 2004, **23**, 116–125.

Singhal, S.C., A new chart for analyzing multiprocess performance. Qual. Eng., 1990, **2**(4), 379–390.

Singhal, S.C., Multiprocess performance analysis chart (MPPAC) with capability zones. *Quality Engineering*, 1991, **4**(1), 75–81.

Vannman, K., A unified approach to capability indices. *Statistica Sinica*, 1995, **5**, 805–820.