

# The role of optoelectronic feedback on Franz-Keldysh voltage modulation of transistor lasers

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## ABSTRACT

Possessing both the high-speed characteristics of heterojunction bipolar transistors (HBTs) and enhanced radiative recombination of quantum wells (QWs), the light-emitting transistor (LET) which operates in the regime of spontaneous emissions has achieved up to 4.3 GHz modulation bandwidth. A 40 Gbit/s transmission rate can be even achieved using transistor laser (TL). The transistor laser provides not only the current modulation but also direct voltage-controlled modulation scheme of optical signals via Franz-Keldysh (FK) photon-assisted tunneling effect. In this work, the effect of FK absorption on the voltage modulation of TLs is investigated. In order to analyze the dynamics and optical responses of voltage modulation in TLs, the conventional rate equations relevant to diode lasers (DLs) are first modified to include the FK effect intuitively. The theoretical results of direct-current (DC) and small-signal alternating-current (AC) characteristics of optical responses are both investigated. While the DC characteristics look physical, the intrinsic optical response of TLs under the FK voltage modulation shows an AC enhancement with a 20 dB peak, which however is not observed in experiment. A complete model composed of the intrinsic optical transfer function and an electrical transfer function fed back by optical responses is proposed to explain the behaviors of voltage modulation in TLs. The abnormal AC peak disappears through this optoelectronic feedback. With the electrical response along with FK-included photon-carrier rate equations taken into account, the complete voltage-controlled optical modulation response of TLs is demonstrated.

**Keywords:** Transistor laser, modulator, light-emitting transistor, Franz-Keldysh effect.

## 1. INTRODUCTION

As the technology development and information circulation grow rapidly, the amount of data explodes dramatically. The need for high-speed and low-power interconnects becomes an important issue for next-generation communication systems. Integrating photonics and electronics together is one of the popular trends in the future circuits, and therefore photonically-embedded electronic devices begin to attract the more attention. The light-emitting transistor (LET) based on the InGaP/GaAs structure was first proposed by Feng and Holonyak in 2004 [1, 2]. The device is unique in that electrical and optical outputs are present simultaneously. The high-speed optical modulation can be achieved in LETs because of the heavily doped and tilted-charge distribution in the active region [3]. Furthermore, LETs can be substantially modified and turned into a transistor laser (TL) in the presence of a cavity with high quality ( $Q$ ) factors [4]. According to previous experiments, the TL exhibited a high modulation bandwidth and 40 Gb/s transmission capability [5]. It has a great potential for optical interconnects and optoelectronic integrated circuit (OEIC) in the future.

As the TL is operated in the forward-active mode, the junction between the emitter and base (EB junction) is forward biased, but that between the base and collector (BC junction) is reverse biased. Because of the large electric field in the BC junction, the photons generated from the active region may be absorbed even though the energy of photons is less than the bandgap energy there. This phenomenon is known as the photon-assisted tunneling or Franz-Keldysh (FK) absorption. Due to FK effect in the BC junction [6], TLs provide not only current-controlled but also voltage-controlled modulations. In this work, we propose an analytical model composed of carrier-photon interaction, optical and electrical feedbacks, and laser modulation as the TL is biased in the forward-active and common emitter operation with the significant FK absorption and possible breakdown mechanism. A complete circuit model is further constructed from this analytical one to explain the behaviors of voltage modulations in TLs.

This paper is organized as follows. The theoretic model of the TL is presented in section 2. The DC characteristics, small signal analysis, and results are demonstrated in section 3. In section 4, we present the electrical circuit model which contains the optoelectronic feedback and is capable of explaining the behaviors of TLs under the high-frequency modulation. The conclusion is given in section 5.

## 2. THEORETICAL MODEL

In our device structure, an intrinsic-GaAs absorption layer is present between the base and collector. The reverse bias there results in the FK photon absorption, which converts the photons generated by the spontaneous or stimulated emissions from the active quantum well (QW) in the base back into electron-hole pairs in the BC junction. The reverse bias will also force electrons and holes generated there to flow oppositely into the collector and base (active) region, respectively. Due to the requirement of charge neutrality (conservation of the current flux), additional electrons need to be injected from the emitter into active region. These additional carriers in the active region would participate in the carrier-photon interaction again. The layer structure of the TL was shown in more detail is in reference [6]. In order to include the FK effect and breakdown mechanism, we modify the conventional rate equations which are often utilized to describe the modulation characteristics of diode lasers (DLs). A new term related to the photon-assisted tunneling and breakdown current due to the impact ionization are added into rate equations of the carrier and photon densities. The modified rate equations are expressed as

$$\frac{dN}{dt} = \frac{\eta_i i_B}{eV} - R - v_g g N_p + \eta_{BC} \left[ \frac{\Gamma_{J_{BC}}}{\Gamma_a} v_g \alpha N_p M + \frac{1}{eV} I_{CBO} M + \frac{I_C}{eV} (M - 1) \right], \quad (1)$$

$$\frac{dN_p}{dt} = \left( \Gamma_a v_g g - \frac{1}{\tau_p} \right) N_p + \Gamma_a \beta_{sp} R_{sp} - \Gamma_{J_{BC}} v_g \alpha N_p, \quad (2)$$

where  $N$  is the carrier density;  $N_p$  is the photon density;  $\eta_i$  is the injection efficiency;  $i_B$  is the base current;  $e$  is the electron charge;  $V$  is the volume of active region;  $R$  is the recombination rate, in which only the radiative recombination characterized by a spontaneous emission lifetime  $\tau_{sp}$  is considered ( $R = N / \tau_{sp} = R_{sp}$ );  $v_g$  is the group velocity;  $g = a_0(N - N_{tr}) / (1 + \varepsilon N_p)$  is the optical gain based on the linear gain model with nonlinear gain saturation from the photon density ( $a_0$  and  $N_{tr}$  are the differential gain and transparency carrier density, respectively; and  $\varepsilon$  is the nonlinear gain saturation coefficient);  $\eta_{BC}$  is the injection efficiency of the FK current from the collector to base region;  $\Gamma_{J_{BC}} = V_{BC} / V_p$  is the confinement factor of the BC junction and is the ratio between the volume  $V_{BC}$  of the BC junction and effective volume  $V_p$  of the photonic mode; and  $\Gamma_a = V / V_p$  is the confinement factor of the active region;  $\alpha$  is the FK absorption coefficient;  $M$  is the current multiplication factor due to breakdown, which is a function of the collector-to-base voltage;  $I_{CBO}$  is the collector-base current under the reverse-bias condition;  $I_C$  is the collector current;  $\tau_p$  is the photon lifetime;  $\beta_{sp}$  is the spontaneous-emission coupling factor defined as the percentage of the total spontaneous emissions coupled into the lasing mode; and  $R_{sp}$  is the spontaneous emission rate.

Different from DLs, a few additional terms appear in the rate equations of TLs. The last term  $\Gamma_{J_{BC}} v_g \alpha N_p$  in Eq. (2) is resulted from the FK absorption in the BC junction. A similar term  $\eta_{BC} (\Gamma_{J_{BC}} / \Gamma_a) v_g \alpha N_p M$  in Eq. (1) corresponds to the extra electron-hole pairs due to the holes which are generated by the the FK absorption and flow into the active region. The introduction of the multiplication factor  $M$  is a result of the impact ionization at a large reverse bias. The last term  $\eta_{BC} [I_{CBO} M / e / V + I_C (M - 1) / e / V]$  in Eq. (1) is related to the reverse-biased current and diffusion current in the BC junction of the transistor. The values of various parameters are listed in Table 1. Because heavily doped and tilted-charge distribution profile in the base region, the spontaneous lifetime  $\tau_{sp}$  of the TL could be shorter than those of DLs. We set it to 100 ps in these simulations.

## 3. DC CHARACTERISTICS AND SMALL SIGNAL ANALYSIS

Figure 1 shows the DC light-versus-current-voltage (LIV) surface calculated from the steady-state rate equations. Both

Table 1. List of common parameters for various TL models.

	$\eta_i$	$V$	$\tau_{sp}$	$a_0$
Value	0.95	$0.3216 (\mu m^{-3})$	$100 (ps)$	$1.265 \times 10^{-19} (m^2)$
	$\eta_{BC}$	$\Gamma_{J_{BC}}$	$\Gamma_a$	$v_g$
Value	0.95	0.3659	0.074481	$7.5 \times 10^7 (m/s)$
	$N_{tr}$	$\varepsilon$	$\tau_p$	$\beta_{sp}$
Value	$4.5 \times 10^{23} (m^{-3})$	$1.5 \times 10^{-23} (m^{-3})$	$3.98 (ps)$	$8.69 \times 10^{-5}$

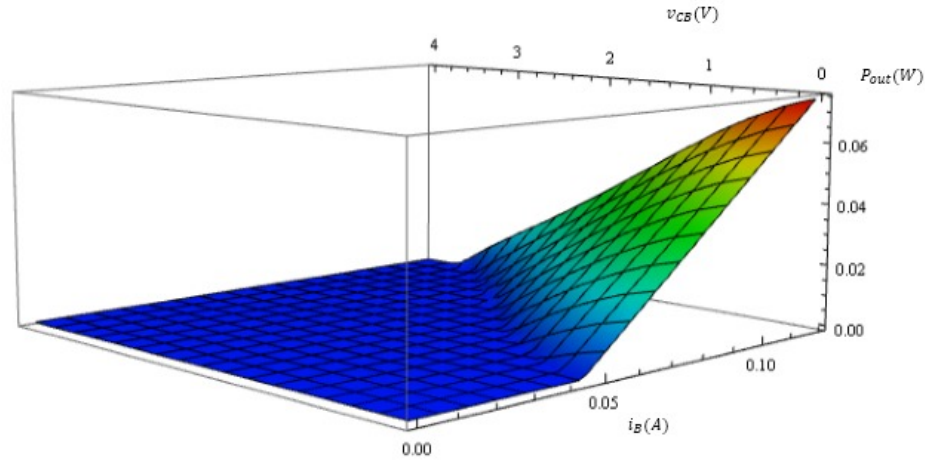


Figure 1. The LIV curve of the TL. Due to the FK absorption in the BC junction, as  $V_{CB}$  becomes larger, the light output turns smaller.

time derivatives in the two rate equations are set to zero when solving the carrier and photon densities as a function of the base current and collector-to-base voltage. The turning point of the base current as power suddenly pops up is the threshold current, which is approximately 45 mA here. The threshold current increases with the collector-to-base voltage because the larger reverse bias voltage results in the more significant FK absorption and hence the fewer photons in the lasing mode. In short, as  $V_{CB}$  becomes larger, the light output turns smaller.

The bias-dependent FK absorption in the BC junction can be utilized as a mechanism for the voltage-controlled modulation. In order to understand the dynamical responses of TLs, we analyze Eqs. (1) and (2) with their small-signal models. Taking  $N$ ,  $N_p$ ,  $g$ ,  $\alpha$ ,  $M$ , and  $I_{CBO}$  as dynamic variables, we expand Eqs. (1) and (2) as

$$\begin{aligned} \frac{d\Delta N}{dt} = & -\Delta R_{sp} - v_g \Delta g N_p + v_g g \Delta N_p + \eta_{BC} \left[ \frac{\Gamma_{J_{BC}}}{\Gamma_a} (v_g \Delta \alpha N_p M + v_g \alpha \Delta N_p M + v_g \alpha N_p \Delta M) \right. \\ & \left. + \frac{1}{eV} \Delta I_{CBO} M + \frac{1}{eV} I_{CBO} \Delta M + \frac{I_C}{eV} \Delta M \right], \end{aligned} \quad (3)$$

$$\frac{d\Delta N_p}{dt} = \Gamma_a v_g \Delta g N_p + \Gamma_a v_g g \Delta N_p - \frac{\Delta N_p}{\tau_p} - \Gamma_{J_{BC}} v_g \Delta \alpha N_p - \Gamma_{J_{BC}} v_g \alpha \Delta N_p + \Gamma_a \Delta (\beta_{sp} R_{sp}). \quad (4)$$

In Eqs. (3) and (4), the quantities preceded by capital delta ( $\Delta$ ) mean the corresponding small variations. Also, the variation  $\Delta i_B$  of the base current is set to zero in absence of the current-controlled modulation. The gain variation  $\Delta g$  can be expanded in terms of the carrier and photon density variations:

$$\Delta g = a\Delta N - a_p\Delta N_p, \quad (5)$$

$$a = \frac{\partial g}{\partial N} = \frac{a_0}{(1 + \varepsilon N_p)}, \quad (6)$$

$$a_p = -\frac{\partial g}{\partial N_p} = \frac{\varepsilon g}{(1 + \varepsilon N_p)}. \quad (7)$$

For convenient, we use matrix form to rewrite Eqs. (3) and (4) as follows:

$$\frac{d}{dt} \begin{pmatrix} \Delta N \\ \Delta N_p \end{pmatrix} = \begin{pmatrix} -\gamma_{NN} & -\gamma_{NP} \\ \gamma_{PN} & -\gamma_{PP} \end{pmatrix} \begin{pmatrix} \Delta N \\ \Delta N_p \end{pmatrix} + \begin{pmatrix} \gamma_{N\alpha}\Delta\alpha + \gamma_{NM}\Delta M + \gamma_{NI_{CBO}}\Delta I_{CBO} \\ -\gamma_{N\alpha}\Delta\alpha \end{pmatrix}, \quad (8)$$

where various coupling coefficients are

$$\begin{aligned} \gamma_{PN} &\equiv v_g a \Gamma_a N_p, \quad \gamma_{PP} \equiv v_g a_p \Gamma_a N_p - \Gamma_a v_g g + \frac{1}{\tau_p} + \Gamma_{J_{BC}} v_g \alpha, \quad \gamma_{P\alpha} \equiv \Gamma_{J_{BC}} v_g N_p, \\ \gamma_{NN} &\equiv v_g a N_p + \frac{1}{\tau_{\Delta N}}, \quad \gamma_{NP} \equiv v_g g - v_g a_p N_p - \eta_{bc} \frac{\Gamma_{J_{BC}}}{\Gamma_a} v_g \alpha M, \quad \gamma_{N\alpha} \equiv \eta_{bc} \frac{\Gamma_{J_{BC}}}{\Gamma_a} v_g N_p M, \\ \gamma_{NM} &\equiv \eta_{bc} \frac{\Gamma_{J_{BC}}}{\Gamma_a} v_g \alpha N_p + \frac{\eta_{bc}}{eV} I_{CBO} + \frac{\eta_{bc} I_C}{eV}, \quad \gamma_{NI_{CBO}} \equiv \frac{\eta_{bc}}{eV} M. \end{aligned} \quad (9)$$

In the sinusoidal steady state, the modulation voltage can be expressed as  $\Delta V_{CB} = \text{Re}[V_1 \exp(j\omega t)]$ , where  $V_1$  is the phasor of  $\Delta V_{CB}$ ; and  $\omega$  is the modulation frequency. Similarly for other small variations, we will mark their phasors with a subscript 1. We then substitute the time derivative  $d/dt$  with  $j\omega$  and note that  $\Delta\alpha$ ,  $\Delta M$ , and  $\Delta I_{CBO}$  all depend on  $\Delta V_{CB}$  and therefore on  $V_1$ . In this way, Eq. (8) is converted into two algebraic equations, from which we can derive the ratio between the phasor  $N_{p1}$  of photon-density variation  $N_p$  and that  $V_1$  of  $\Delta V_{CB}$  as follows:

$$\begin{aligned} \frac{N_{p1}}{V_1} &= [v_g a \Gamma_a N_p (\eta_{bc} \frac{\Gamma_{J_{BC}}}{\Gamma_a} v_g N_p M K + \eta_{bc} \frac{\Gamma_{J_{BC}}}{\Gamma_a} v_g \alpha N_p P + \frac{\eta_{bc}}{eV} M S + \frac{\eta_{bc}}{eV} I_{CBO} P + \frac{\eta_{bc}}{eV} I_C P) \\ &\quad \times (j\omega + v_g a N_p + \frac{1}{\tau_{\Delta N}})^{-1} - \Gamma_{J_{BC}} v_g N_p K] / [j\omega - v_g a \Gamma_a N_p (v_g a_p N_p - v_g g + \eta_{bc} \frac{\Gamma_{J_{BC}}}{\Gamma_a} v_g \alpha M) \\ &\quad \times (j\omega + v_g a N_p + \frac{1}{\tau_{\Delta N}})^{-1} + v_g a_p \Gamma_a N_p - \Gamma_a v_g g + \frac{1}{\tau_p} + \Gamma_{J_{BC}} v_g \alpha]. \end{aligned} \quad (10)$$

Using Eq. (10), we can obtain the intrinsic optical response in the logarithmic scale ( $10 \log |N_{p1} / V_1|^2$ ). After the proper normalization with its zero-frequency response, we then present the related normalized optical response. The intrinsic optical responses at a fixed base current of 110 mA but different collector-to-base voltages from 1.0 to 2.5 V are shown in Figure 2. As the collector-to-base voltage increases, the FK absorption in the BC junction turns stronger. The optical bandwidth decreases as the collector-to-base voltage increases due to the fewer photons in the cavity. Surprisingly, the intrinsic optical response of TLs under the FK voltage modulation shows an AC peak with an enhancement of about 20 dB, which is not observed in experiment, however. In next section, an electrical circuit model which contains the optoelectronic feedback from the intrinsic optical response is proposed to demonstrate the correct behavior of voltage modulation in TLs.

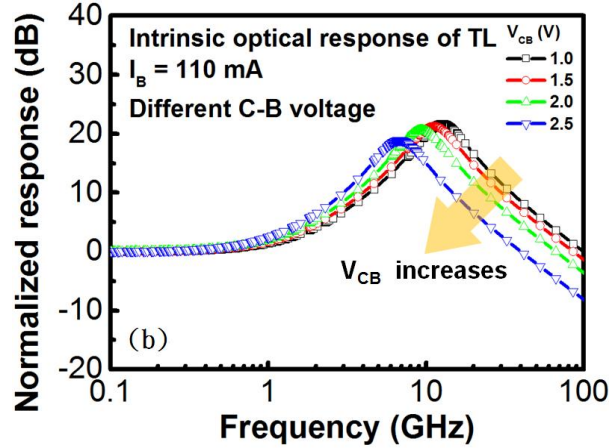


Figure 2. The intrinsic optical response at a fixed biased base current  $i_B$  of 110 mA but different collector-to-base voltages  $V_{CB}$  from 1.0 to 2.5 Volt.

#### 4. COMPLETE CIRCUIT MODEL OF TL

The development of small-signal circuit for heterojunction bipolar transistor is mature [7]. In a TL, the additional current which is resulted from the FK absorption effect in the BC junction should be incorporated into the small-signal analysis. In our electrical circuit model, an AC voltage source with an amplitude of 0.01 V is connected to the external collector port ( $C$ ). Other parasitic elements are set to typical values of the small-signal model for LETs and TLs. Figure 3 shows small-signal circuit model of the TL. The parameters  $R_E$  and  $L_E$  stand for the parasitic resistance and inductance of the emitter port;  $R_C$ ,  $L_C$  are the counterparts of the collector port. Due to the common emitter configuration in this study, the emitter is connected to the ground. The additional current  $i_{add}$  which is resulted from FK absorption in the BC junction can be derived from the rate equation of carrier density:

$$i_{add} = \eta' \left[ \frac{eV\Gamma_{JBC}}{\Gamma_a} C\alpha N_p M + I_{CBO}M + I_C(M-1) \right], \quad (11)$$

where  $\eta'$  is efficiency due to the electron-hole recombination. The small-signal admittance  $Y$  corresponding to this additional current is then deduced from Eq. (11). Through the small-signal analysis, we have

$$\Delta i = \Delta i_{add} = Y \Delta V_{C'B}, \quad (12)$$

$$Y = eV_a \eta' \frac{\Gamma_{JBC}}{\Gamma_a} v_g \left( \frac{\partial \alpha}{\partial V_{C'B}} M N_p + \alpha N_p \frac{\partial M}{\partial V_{C'B}} + \alpha M \frac{\partial N_p}{\partial V_{C'B}} \right) + \eta' I_{CBO} \frac{\partial M}{\partial V_{C'B}} + \eta' (M-1) \frac{\partial I_{CBO}}{\partial V_{C'B}}. \quad (13)$$

The intrinsic optical response obtained in previous calculation [Eq. (10)] is in fact the modulation response related to the voltage  $V_{C'B}$  of the internal collector port ( $C'$ ). In practice, we can only monitor the modulation response corresponding to the external voltage  $V_{CB}$ . Therefore, the overall response which is experimentally observed should be represented as follows:

$$T_{overall} = \frac{\Delta N_p}{\Delta V_{CB}} = \frac{\Delta N_p}{\Delta V_{C'B}} \frac{\Delta V_{C'B}}{\Delta V_{CB}} = T_p \frac{\Delta V_{C'B}}{\Delta V_{CB}} = T_p \times T_e, \quad (14)$$

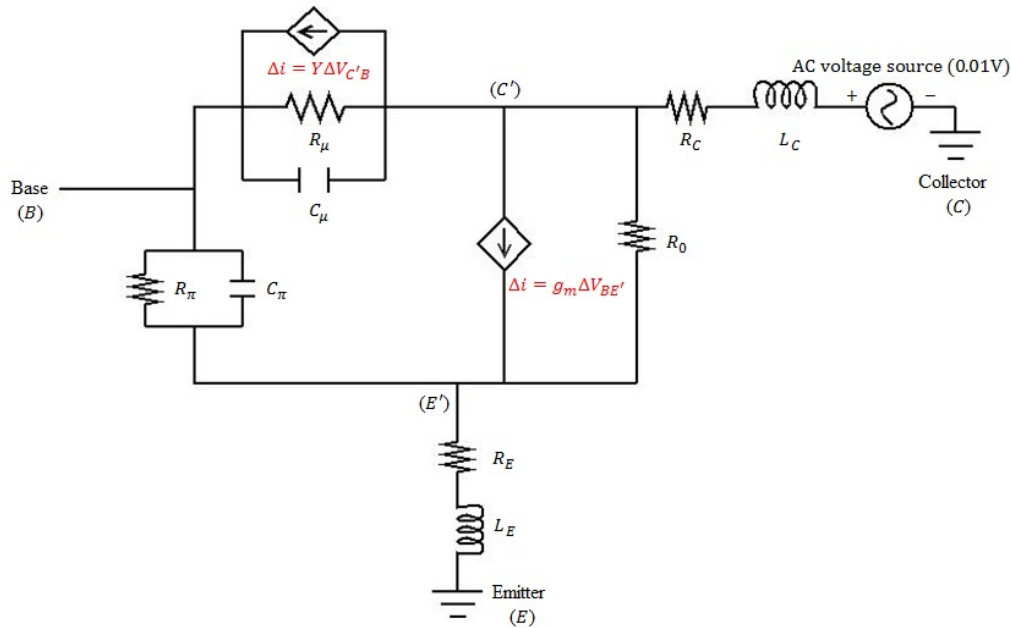


Figure 3. The electrical circuit model which includes the FK effect of the TL.

where  $T_p$  and  $T_e = \Delta V_{C'B} / \Delta V_{CB}$  are the intrinsic optical response and electrical transfer function, respectively. We note that the electrical transfer function  $T_e$  actually depends on  $T_p$  and, which marks the feedback by the intrinsic optical response of TLs.

The overall responses of voltage modulation for the TL at different bias voltages (2.0, 2.2 and 2.4 V) is shown as Figure 4. The electrical transfer function  $T_e$  which can be deduced from Eq. (13) is also compared with the intrinsic optical response  $T_p$  there. The magnitude of the admittance  $Y$  is maximal as the magnitude of the intrinsic optical response is at its peak. Therefore, the magnitude of the voltage drop across the admittance would decrease. That is, the magnitude of the electrical transfer function is at its minimum when the intrinsic optical response is at its maximal magnitude, which can be also observed in Figure 4. This circuit model can explain the behaviors of voltage modulation in TLs. The abnormal AC peak disappears through this optoelectronic feedback.

## 5. SUMMARY

The effect of FK absorption on the voltage modulation of transistor lasers is investigated. The small-signal model of TLs is first established through the incorporation of the FK absorption into the conventional photon-carrier rate equations for the modulation characteristics of DLs. The theoretical results of direct-current and alternating-current characteristics of optical responses for TLs are both investigated. A complete model including not only the intrinsic optical characteristics but also an electrical transfer function fed back by intrinsic optical responses is required to get rid of a giant but nonphysical AC enhancement of the FK voltage modulation, which is experimentally absent.

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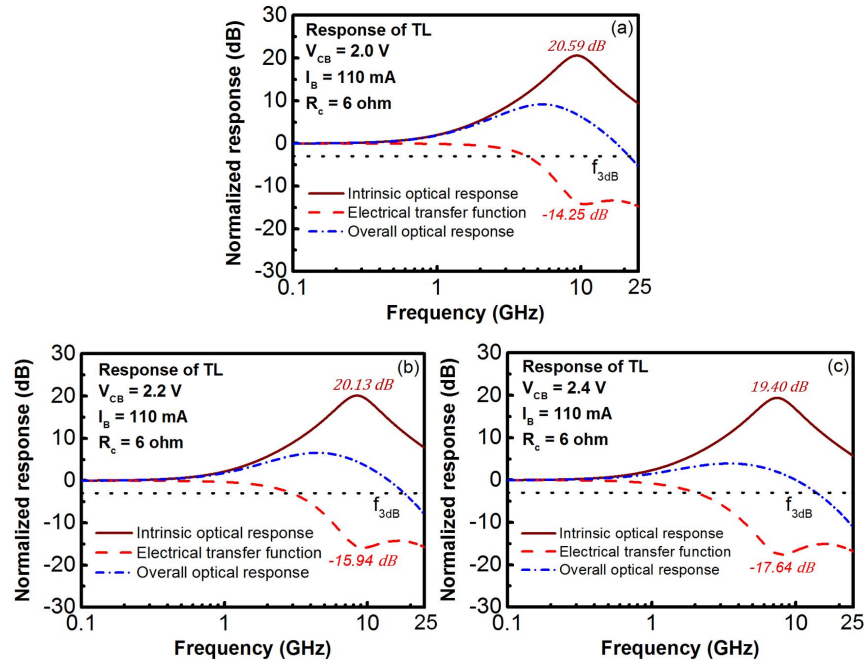


Figure 4. The comparison among various responses related to the voltage modulations for the TL.

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