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Risk assessment for build-operate-transfer projects: a dynamic multi-objective programming approach

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Abstract

This study uses a dynamic multi-objective programming approach to establish a risk assessment model, and develops an iterative algorithm for the model solution. The results obtained show that the sum of the interactive utility value could determine whether or not the interactive relationship is characterized by independence among negotiators. In addition, our numerical example shows that the risk measurement model developed can reflect risk assessment made by the negotiation group for certain events, and can analyze interaction characteristics among negotiators.

Keywords: BOT; Dynamic programming; Interactive utility; Multi-objective programming; Risk assessment

1. Introduction

This paper develops a risk assessment model for analyzing large-scale infrastructure projects Build-Operate-Transfer (BOT) approach and with interactive utility among negotiators. The BOT approach is a process where the private sector is granted a concession to plan, design, construct, operate and maintain a project in a certain period of time and then transfer it to the government [1]. Private enterprises are invited by the government to participate in a BOT project in order to share the potential risks that occur in the project development process. There are many risks associated with BOT projects such as high financial risk, market risk, cost overrun risk, political risk, and operation

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risk [2]. The high-risk exposure associated with BOT projects implies that the decision-makers or negotiators of both the BOT concession company and the government must pay special attention to analyzing and managing risks [3].

Before negotiation, the negotiation groups from both the concession company and the government department will conduct internal risk assessment to examine uncertain factors in the contract. After risk evaluation, the negotiation will meet to discuss the risk events [4,5]. Previous studies have shown that methods such as utility theory, statistics, team theory, and mathematical programming can be employed to measure the risks of BOT projects. The utility function has been widely used in risk analysis. Among recent contributions, Zayed and Chang [6] used the concept of utility theory to derive the weighted expected value as a risk index of BOT projects, while ignoring the decision-makers' preference. David [7] developed the expected production cost for investor and the host utility using the statistics approach to obtain risk value for BOT projects. Bell [8] showed that the maximum value of expected utility can reveal the characteristics of high return and high risk. The Multi-attribute Utility (MAU) originally proposed by Keeney and Raiffa [9] has been adopted to study decision-making behavior [10], and to analyze risks of engineering projects [11]. Feng and Kang [4,5], Feng et al. [12] employed the MAU theory to evaluate risk for BOT concession contracts. Their studies examined risk preferences of the negotiators and determined the primary and secondary risks associated with BOT projects. However, no effect has yet been made to assess risks with interactive utility among negotiators.

Although the MAU model has additive utility and multiplicative utility, it assumes that the decision-maker's preference map [13] is independent and cannot be used to explain interactive behavior during the negotiation process [9]. It is worthy to mention that Debreu [14] proposed a theorem giving the exact conditions under which the preference map can be represented by a real-valued utility function. In the past, studies regarding utility interaction have included analysis of utility dependence (i.e., the Monte Carlo method), team theory [15–17] and mathematical programming [18,19]. In addition, Carbone [20] adopted the Monte Carlo method to simulate rank-dependent expected utility, and to distinguish the decision-making behavior for various models, such as pair-wise choice utility, expected utility, and weighted utility. One difficulty here is that when utility is simulated by the Monte Carlo method, all the utility values, simulated parameters, and probabilities must be pre-set. However, when utility values of any one of the decision-makers interact with those of other decision-makers, it is not easy to determine a priori the values for parameters and probability distributions.

The rank-dependent utility theory proposed by Belichrosdt and Quiggin [21] assumes that probability is one of the endogenous variables of utility function. The result of their study showed the relationships between utility and joint or marginal probability. In addition, Quiggin [22] relaxed the condition of independence for unrelated outcomes because a specific utility may have certain relationships with outcomes that are not completely independent. Quiggin [23] also presented a generalized expected utility theory that could be used in the analysis of the interaction between choices under uncertainty. The team theory proposed by Kim and Roush [17] is another approach to analyzing group decision-making behavior. It focuses on interaction between decision-makers. As to risk assessment via mathematical programming, Haimes [18] has recently proposed the concept of multi-objective programming and dynamic programming for risk assessment in decision-making.

Reviewing the above-mentioned studies, we can see that it is important to determine primary and secondary risks involved in BOT projects for both the concessionaire and the government.

The mathematical programming approach can be employed to analyze negotiation problems and to evaluate risks. This purpose of this study is to develop a risk assessment model for determining the primary and secondary risks. The rest of the paper is organized as follows. Section 2 describes the problem. Section 3 presents the model assumptions and develops a dependent utility model for the negotiation group. Section 4 develops an algorithm for the model solution. In Section 5, we use a numerical example to assess risk and to determine the primary and secondary risks for a BOT project. Finally, we draw some conclusions and present some thoughts for future research.

2. Problem description

In Taiwan, projects such as the High Speed Rail (HSR), Taipei Port, and Mass Rail Transit (MRT) have been carried out by the BOT approach. The concession company and the government will each carry out the contract negotiation process. The government negotiation team includes members of the transportation and environmental agencies, and local officials, while the concession company's negotiation team comprises lawyers, financial consultants, and engineering experts. The principal negotiator from each team is in charge of the negotiation process. Naturally, if the negotiation fails, the concession contract will not be signed. The negotiation process aims to discuss possible uncertainties in the contract, define the rights and obligations of each party and, lay down all agreements in a concessional format. Fig. 1 presents a conceptual diagram of this process.

As shown in Fig. 1, before signing the concession contract, the government and the BOT concession company will discuss which risk events are to be included in the contract. In the negotiation process, if both parties cannot accept a specific risk event, it will be the topic of negotiation in the next meeting; and each negotiation group will conduct internal discussion to re-assess the risk items. According to the concept of Fig. 1, during the discussion of a specific group other participants' decision variables may affect the utility results of a specific negotiator, while the utility of the negotiator may also affect the utility results of other participants. Whenever there is discussion

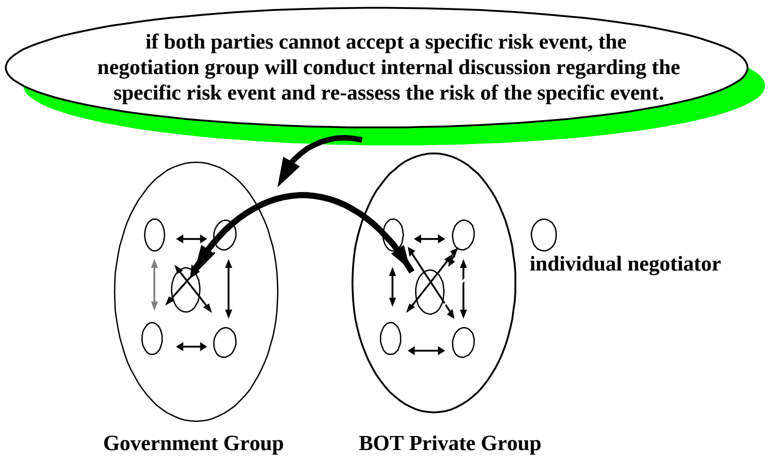

Fig. 1. Conceptual map of interactive utility among negotiators.

Table 1
Relationship between event state, attribute outcome, and assessment value

Event f	State (S)
	$s_1, s_2, \ldots, s_j, \ldots, s_n$
Outcome of attribute x	$x_1, x_2, \ldots, x_j, \ldots, x_n$
Probability p	$p_1, p_2, \ldots, p_j, \ldots, p_n$
Assessment value v	$v_1(x_1), v_2(x_2), \ldots, v_j(x_j), \ldots, v_n(x_n)$

among negotiators, there will be a utility interaction. The status of utility interaction may become stable after several rounds of discussion, and an internal consensus for a specific event can thus be obtained. If there is no utility interaction among negotiators, then the utility is independent.

3. The dependent utility model

A dependent utility model is developed in this section under the following assumptions.

(1) The agent's costs are independent of the negotiators' utility.
(2) The utility function of the negotiator is a continuous function.
(3) The negotiator makes decisions rationally, i.e., she/he optimizes utility in a risky environment.

Assumption (1) indicates that the negotiator is authorized by a specific organization; and if the agency cost is not zero, adverse-selection might occur in the negotiation process. Assumption (2) implies that the utility function satisfies the Von Neumann–Morgenstern (V–M) axioms. Assumption (3) satisfies the principle of maximizing utility while minimizing risk.

3.1. Definition of risk-state

Although some studies proposed using a risk evaluation model or a risk index to assess the risk level for BOT projects, most of them focused only on financial risk of BOT projects rather than on evaluating the primary and secondary risks of BOT projects among negotiators. To explore the issue, we develop a risk assessment model as follows.

Assume that an event f has n uncertain states, $s_1, s_2, \ldots, s_j, \ldots, s_n$, and has an outcome of attribute x. Every state corresponds to $x, x_1, x_2, \ldots, x_j, \ldots, x_n$, and every x_j corresponds to $v_j(x_j)$, where $v_j(x_j)$ is the value assessed by negotiators corresponding to the outcome of an attribute. In addition, p_j indicates the occurrence probability of state j. Let $p_j \times v_j(x_j)$ be the utility value for s_j and x_j, $j = 1, 2, \ldots, n$.

Table 1 shows the relationship between event state, attribute outcome, and assessment value. Before exploring the risk, this study first defines the risk-state following the concept of Table 1. According to the risk defined by Buhalmann [24], the risk preference defined by Keeney and Raiffa [9], and the risk assessment approach proposed by Zayed and Chang [6], we define the risk-state as

Table 2
Data for the example of land acquisition event of the HSR BOT project

Land acquisition event 1st negotiator	State (delay in number of year)							
	0	1	2	3	4	5	6	7
Outcome of attribute: increased construction cost (NT$: 100 Million dollars)	0	2.98	4.45	6.89	9.68	11.98	15.45	20.36
Assessment value $v(x_j)$	0.95	0.82	0.64	0.55	0.32	0.11	0.06	0.005
Probability p	0.2115	0.0294	0.0602	0.0762	0.0995	0.097	0.09448	0.0806
$p \times v(x_j)$	0.2009	0.0241	0.0385	0.0419	0.0318	0.0107	0.0057	0.0004
$u_1^f(x_j)$	1.0000	0.1182	0.1898	0.2066	0.1565	0.0511	0.0262	0.0000

shown in Eq. (1).

$$u^*(x_j) \leqslant \bar{u}(x), \ \forall j, \tag{1}$$

where $u^*(x_j)$ is the utility value of the negotiator regarding state j for a specific event, $u^*(x_j) = p_j v_j(x_j)$. Since $v_j(x_j) \in [0,1]$ and $p_j \in [0,1]$, hence $u^*(x_j) \in [0,1]$. In addition, $\bar{u}(x)$ represents the mean utility value of $u^*(x_j)$, where $\bar{u}(x) = (1/n)\sum_j^n u(x_j)$.

Eq. (1) shows a risk-state under state s_j through a negotiator evaluation of a specific event if $u^*(x_j) < \bar{u}(x)$. According to Eq. (1), although $0 \leqslant v_j(x_j) \leqslant 1$, $0 \leqslant p_j \leqslant 1$, the value of $\bar{u}(x)$ may be greater than 1. To simplify comparison, we utilize the concept of transformation utility proposed by Keeney and Raiffa to modify Eq. (1) as follows:

$$u_q^f(x_j) = \frac{p_j \times v_j(x_j) - \min_j\{p_j \times v_j(x_j)\}}{\max_j\{p_j \times v_j(x_j)\} - \min_j\{p_j \times v_j(x_j)\}},$$
$$\max_j\{p_j \times v(x_j)\} \neq \min_j\{p_j \times v(x_j)\}, \ \forall j, \tag{2}$$

where $u_q^f(x_j)$ is the utility value of the qth negotiator for attribute-outcome x_j of event f and is the normalized utility value; f is defined as the land acquisition event for BOT projects; and q is the number of negotiators, $q = 1, 2, \ldots, Q$. If $\max_j\{p_j \times v(x_j)\} = \min_j\{p_j \times v(x_j)\}$, then let $u_q^f(x_j) = 0$ which is the non-risk state for attribute-outcome x_j. Eq. (2) states that the outcome of attribute x_j for the specific event under state s_j is a risk-state as $u_q^f(x_j) < \bar{u}(x)$ by according to the evaluation of the qth negotiator.

Take the land acquisition event of the HSR BOT project as an example. If the government cannot acquire the land for the route and stations in time, the company cannot start construction on schedule, and then delays on both construction and operation will occur. Let the delay time be $0, 1, 2, \ldots, 7$ years, and the increased construction cost, x, be the outcome of the attribute due to a year-long delay. Given the assessment value $v(x_j)$ of each negotiator, and the probability value p of event occurrence, the data are shown in Table 2.

We can obtain the utility values using $p \times v(x_j)$ as 0.2009, 0.0241, …, and 0.0004, respectively. As can be seen, the maximum and minimum value of $p \times v(x_j)$ are 0.2009 and 0.0004, respectively.

Substituting the data of Table 2 into Eq. (2), we can get

$$u_1^f(x_1)=\frac{0.2009-0.0004}{0.2009-0.0004}=1,\ u_1^f(x_2)=\frac{0.0241-0.0004}{0.2009-0.0004}=0.1182,\ldots,$$

$$u_1^f(x_6)=0.0262,\ u_1^f(x_7)=0$$

and the mean utility value $\bar{u}(x)=0.21855$. This shows that state 0, that is no delay, is a non-risk state because $u_1^f(x_1)>\bar{u}(x)$, while the other states are risk-states.

3.2. Dependent utility model for negotiation group

To construct a dependent utility model among negotiators, we use the concept of utility linear transformation. We first construct a dependent utility model for three negotiators, and then extend it to a generalized dependent utility model.

Assume that there are three negotiators for a negotiation group, and define $\alpha_{2,1}^f(x_j,t)$ as the interactive utility value (IUV); where $\alpha_{2,1}^f(x_j,t)$ means that the utility value of negotiator #2 affects the utility of negotiator #1 for the outcome of attribute x_j of event f at discussion #t. Similarly, $\alpha_{1,3}^f(x_j,t)$, $\alpha_{3,1}^f(x_j,t)$, $\alpha_{2,3}^f(x_j,t)$, $\alpha_{3,2}^f(x_j,t)$, and $\alpha_{1,2}^f(x_j,t)$ are constants, and the IUV mentioned above is located between 0 and 1. Assume that the utility function satisfies the N-M axiom. According to the utility linear transformation concept of Fishburn [25], we can state the following:

If the utility function satisfies the continuity, transitivity and weak independent axioms, then the utility function can be a linear transformation; i.e., $\exists\delta\in[0,1]$ and $u_1^f(t),u_2^f(t)\in U^f$, such that $U^f(t)=\delta u_1^f(t)+(1-\delta)u_2^f(t)$, the $U^f(t)$ still satisfies the binary preference relation.

Using the utility linear transformation and IUV, we can thus derive the utility value of the three negotiators in our example. The transformed utility model is shown in Eqs. (3)–(5).

$$u_1^f(x_j,t+1)=(1-\alpha_{2,1}^f(x_j,t)-\alpha_{3,1}^f(x_j,t))u_1^f(x_j,t)+\alpha_{2,1}^f(x_j,t)u_2^f(x_j,t)+\alpha_{3,1}^f(x_j,t)u_3^f(x_j,t), \quad (3)$$

$$u_2^f(x_j,t+1)=(1-\alpha_{1,2}^f(x_j,t)-\alpha_{3,2}^f(x_j,t))u_2^f(x_j,t)+\alpha_{1,2}^f(x_j,t)u_1^f(x_j,t)+\alpha_{3,2}^f(x_j,t)u_3^f(x_j,t), \quad (4)$$

$$u_3^f(x_j,t+1)=(1-\alpha_{1,3}^f(x_j,t)-\alpha_{2,3}^f(x_j,t))u_3^f(x_j,t)+\alpha_{1,3}^f(x_j,t)u_1^f(x_j,t)+\alpha_{2,3}^f(x_j,t)u_2^f(x_j,t), \quad (5)$$

where $u_1^f(x_j,t)$ is the utility value of negotiator #1 for attribute-outcome x_j of event f at discussion #t; $u_2^f(x_j,t)$ is the utility value of negotiator #2 for attribute-outcome x_j of event f at discussion #t; $u_3^f(x_j,t)$ is the utility value of negotiator #3 for attribute-outcome x_j of event f at discussion #t; and

$$u_q^f(x_j,t)=\frac{p_j\times v_j(x_j,t)-\min_j\{p_j\times v_j(x_j,t)\}}{\max_j\{p_j\times v_j(x_j,t)\}-\min_j\{p_j\times v_j(x_j,t)\}},$$
$$\max_j\{p_j\times u_j(x_j,t)\}\neq\min_j\{p_j\times u_j(x_j,t)\}, \quad (6)$$

where t is the discussion variable, with $t=0,1,2,\ldots,T$; and $q=1,2,3$. If $\max_j\{p_j\times u_j(x_j,t)\}$ equals $\min_j\{p_j\times u_j(x_j,t)\}$, then $u_q^f(x_j,t)=0$. When $t=0$ Eq. (6) will reduce to Eq. (2), which is the initial utility value for a negotiator. Thus, Eq. (2) is a special case of Eq. (6). As shown in Eqs. (3)–(5), previous discussion of a specific negotiator may affect the utility of other negotiators through the

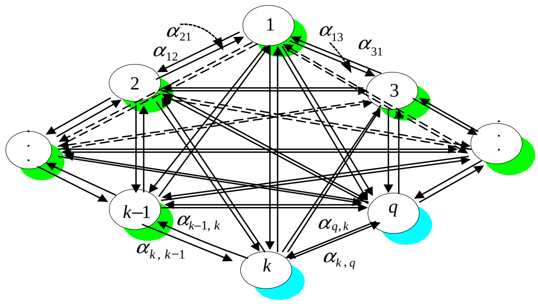

Fig. 2. Diagram of the interactive utility among Q negotiators.

IUV, while the other negotiators will also affect that specific negotiator during current discussion through the IUV, so there is indeed an interactive relation with feedback features.

We use the concept of Eqs. (3)–(5) to construct the generalized dependent utility model. Assume that there are Q negotiators in the negotiation group, let $q = 1, 2, \ldots, Q$. In addition, $u_q^f(x_j, t)$ and $u_q^f(x_j, t+1)$ represent the utility of negotiator $\#q$ for outcome of attribute x_j for event f at discussions $\#t$ and $\#t+1$; respectively, where $u_q^f(x_j, t+1) \in [0, 1]$ and $u_q^f(x_j, t) \in [0, 1]$. After linear transformation of utility functions of negotiator #1 and other negotiators, the resulting utility function is shown in Eq. (7). The conceptual diagram for dependent utility among Q negotiators in the negotiation group is shown in Fig. 2.

$$
\begin{aligned}
u_1^f(x_j, t+1) = {} & (1 - \alpha_{2,1}^f(x_j, t) - \cdots - \alpha_{k,1}^f(x_j, t) - \cdots - \alpha_{Q,1}^f(x_j, t)) u_1^f(x_j, t) \\
& + \alpha_{2,1}^f(x_j, t) u_2^f(x_j, t) + \Lambda + \alpha_{k,1}^f(x_j, t) u_k^f(x_j, t) + \cdots + \alpha_{Q,1}^f(x_j, t) u_Q^f(x_j, t) \\
= {} & \left(1 - \sum_{q=2}^{Q} \alpha_{q,1}^f(x_j, t) \right) u_1^f(x_j, t) + \sum_{q=2}^{Q} \alpha_{q,1}^f(x_j, t) u_q^f(x_j, t),
\end{aligned} \tag{7}
$$

where $\alpha_{q,1}^f(x_j, t)$ and $\alpha_{1,q}^f(x_j, t)$ are the IUV, $q = 1, 2, \ldots, Q$.

Similarly, the utility functions of negotiators #2, $\#k$, and $\#Q$ can undergo linear transformation, with results shown in Eqs. (8)–(10).

$$
\begin{aligned}
u_2^f(x_j, t+1) = {} & \left(1 - \sum_{q=1, q \neq 2}^{Q} \alpha_{q,2}^f(x_j, t) \right) u_2^f(x_j, t) + \sum_{q=1, q \neq 2}^{Q} \alpha_{q,2}^f(x_j, t) u_q^f(x_j, t) \\
= {} & \left(1 - \sum_{q=1, q \neq 2}^{Q} \alpha_{q,2}^f(x_j, t) \right) u_2^f(x_j, t) + \sum_{q=1, q \neq 2}^{Q} \alpha_{q,2}^f(x_j, t) u_q^f(x_j, t),
\end{aligned} \tag{8}
$$

$$u_k^f(x_j,t+1)=\left(1-\sum_{q=1,q\neq k}^{Q}\alpha_{q,k}^f(x_j,t)\right)u_k^f(x_j,t)+\sum_{q=1,q\neq k}^{Q}\alpha_{q,k}^f(x_j,t)u_q^f(x_j,t), \tag{9}$$

$$u_Q^f(x_j,t+1)=\left(1-\sum_{q=1,q\neq Q}^{Q}\alpha_{q,Q}^f(x_j,t)\right)u_Q^f(x_j,t)+\sum_{q=1,q\neq Q}^{Q}\alpha_{q,Q}^f(x_j,t)u_q^f(x_j,t). \tag{10}$$

Eq. (7) shows that $u_1^f(x_j,t+1)$ changes as both $\sum_{q=2}^{Q}\alpha_{q,1}^f(x_j,t)$, $u_1^f(x_j,t)$ and $\sum_{q=2}^{Q}\alpha_{q,1}^f(x_j,t)u_q^f(x_j,t)$ change meaning that other negotiators and IUV among negotiators will also affect the utility of negotiator #1 at discussion #t. Since $u_1^f(x_j,t)\in[0,1]$ and $u_q^f(x_j,t)\in[0,1]$, therefore, $0\leqslant u_1^f(x_j,t+1)\leqslant 1$ and the relation $0\leqslant\sum_{q=2}^{Q}\alpha_{q,1}^f(x_j,t)\leqslant 1$ will also hold. Letting $\sum_{q=2}^{Q}\alpha_{q,1}^f(x_j,t)=0$, then $u_1^f(x_j,t+1)=u_1^f(x_j,t)$, which indicates that negotiator #1 is not affected by other negotiators, i.e., he/she is utility-independent. In other words, the utility assessed by negotiator #1 remains unchanged for attribute x_j of event f at discussions #$t+1$ and #t. However, when $\sum_{q=2}^{Q}\alpha_{q,1}^f(x_j,t)\neq 0$, it represents that other negotiators are related to negotiator #1 via interdependent utilities. When $\sum_{q=2}^{Q}\alpha_{q,1}^f(x_j,t)=1$, it represents that negotiator #1 was affected completely by other negotiators, and gave up his/her original assessed value. We can see that when $0<\sum_{q=2}^{Q}\alpha_{q,1}^f(x_j,t)<1$, then Eq. (7) is the dependent utility model for negotiator #1 and other negotiators. When $\sum_{q=2}^{Q}\alpha_{q,1}^f(x_j,t)\to 1$, the degree of utility-dependence is higher; contrarily, when $\sum_{q=2}^{Q}\alpha_{q,1}^f(x_j,t)\to 0$, the degree of utility-dependence is lower. In Eqs. (8)–(10), whether the negotiators are independent will depend on whether variables $\sum_{q=1,q\neq 2}^{Q}\alpha_{q,2}^f(x_j,t)$, $\sum_{q=1,q\neq k}^{Q}\alpha_{q,k}^f(x_j,t)$ and $\sum_{q=1,q\neq Q}^{Q}\alpha_{q,Q}^f(x_j,t)$ are zero.

In Eqs. (7)–(10), there is a linear relationship between the utility of negotiators q at discussion #$t+1$ and the utility of other negotiators at discussion #t. And since q negotiators affect each other's utility through the IUV, so IUV is the endogenous variable of the utility function of both the individual negotiator and the negotiation group. This is $GU^f(x_j,t+1)=U(u_1^f(\sum\alpha_{q,1}^f(x_j,t)), u_2^f(\sum\alpha_{q,2}^f(x_j,t)),\ldots,u_Q^f(\sum\alpha_{q,Q}^f(x_j,t)))$; where $GU^f(x_j,t+1)$ is the utility value of the negotiation group when attribute-outcome x_j of event f at discussion #$t+1$. Since $u_q^f(x_j,t)$ and $u_k^f(x_j,t)$ satisfy the N-M axiom, therefore, $GU^f(x_j,t+1)$ still satisfies the binary preference relation through linear transformation in utility [25]. Thus, the utility of the negotiation group can be represented by the expected utility value, and $GU^f(x_j,t+1)$ can be obtained by the concept of the preference decomposition theory [21]. Therefore, summing up the utility function of individual negotiators, and obtaining the negotiation group's utility value, we get $GU^f(x_j,t+1)$, as shown in Eq. (11).

$$GU^f(x_j,t+1)=U(u_1^f(x_j,t),u_2^f(x_j,t),\ldots,u_Q^f(x_j,t))=\sum_{q=1}^{Q}u_q^f(x_j,t)$$

$$=\left[\left(1-\sum_{q=2}^{Q}\alpha_{q,1}^f(x_j,t)\right)u_1^f(x_j,t)+\sum_{q=2}^{Q}\alpha_{q,1}^f(x_j,t)u_q^f(x_j,t)\right]$$

$$+\left[\left(1-\sum_{q=1,q\neq 2}^{Q}\alpha_{q,2}^{f}(x_j,t)\right)u_2^f(x_j,t)+\sum_{q=1,q\neq 2}^{Q}\alpha_{q,2}^{f}(x_j,t)u_q^f(x_j,t)\right]+\cdots$$

$$+\left[\left(1-\sum_{q=1,q\neq k}^{Q}\alpha_{q,k}^{f}(x_j,t)\right)u_k^f(x_j,t)+\sum_{q=1,q\neq k}^{Q}\alpha_{q,k}^{f}(x_j,t)u_k^f(x_j,t)\right]+\cdots$$

$$+\left[\left(1-\sum_{q=1,q\neq Q}^{Q}\alpha_{q,Q}^{f}(x_j,t)\right)u_Q^f(x_j,t)+\sum_{q=1,q\neq Q}^{Q}\alpha_{q,Q}^{f}(x_j,t)u_q^f(x_j,t)\right]$$

$$=\sum_{q=1}^{Q}\left[\left(1-\sum_{q=1;k\neq q}^{Q}\alpha_{q,k}^{f}(x_j,t)\right)u_k^f(x_j,t)\right]$$

$$+\sum_{q=1}^{Q}\left(\sum_{q=1;k\neq q}^{Q}\alpha_{q,k}^{f}(x_j,t)\right)u_q^f(x_j,t),\forall q=1,2,\ldots,Q. \tag{11}$$

However, there is an iterative and recursive relation between $GU^f(x_j,t+1), u_k^f(x_j,t)$ and $u_q^f(x_j,t)$. This problem can be formulated as a dynamic programming problem with other constraints and its optimal solution can be obtained by the Lingo package. To explore the character of "discussion" among negotiators, we substitute Eqs. (7)–(10) into Eq. (11), rearrange it, and obtain Eq. (12).

$$GU^f(x_j,t+1)=\sum_{q=1}^{Q}\left[\left(\left(1-\sum_{k=1,q\neq k}^{Q}\alpha_{q,k}^{f}(x_j,t)\right)^{t}u_k^f(x_j,1)\right)\right.$$

$$+\left(\left(1-\sum_{k=1,q\neq k}^{Q}\alpha_{q,k}^{f}(x_j,t)\right)^{t-1}\sum_{k=1,q\neq k}^{Q}\alpha_{q,k}^{f}(x_j,t)u_q^f(x_j,1)\right)$$

$$+\left(\left(1-\sum_{k=1,k\neq q}^{Q}\alpha_{q,k}^{f}(x_j,t)\right)^{t-1}\sum_{k=1,k\neq q}^{Q}\alpha_{q,k}^{f}(x_j,t)u_k^f(x_j,1)\right)$$

$$+\left(\left(1-\sum_{k=1,k\neq q}^{Q}\alpha_{q,k}^{f}(x_j,t)\right)^{t-3}\sum_{k=1,k\neq q}^{Q}\alpha_{q,k}^{f}(x_j,t)u_q^f(x_j,1)\right)$$

$$+\sum_{k=1,k\neq q}^{Q}\alpha_{q,k}^{f}(x_j,t)\left(\left(1-\sum_{k=1,k\neq q}^{Q}\alpha_{q,k}^{f}(x_j,t)\right)^{t-3}u_k^f(x_j,1)\right)$$

$$+\left(\left(1-\sum_{k=1,k\neq q}^{Q}\alpha_{q,k}^{f}(x_j,t)\right)^{t-3}\left(\sum_{k=1,k\neq q}^{Q}\alpha_{q,k}^{f}(x_j,t)\right)^{t-2}u_q^{f}(x_j,1)\right)$$

$$+\left(\left(1-\sum_{k=1,k\neq q}^{Q}\alpha_{q,k}^{f}(x_j,t)\right)^{t-3}\left(\sum_{k=1,k\neq q}^{Q}\alpha_{q,k}^{f}(x_j,t)\right)^{t-2}u_k^{f}(x_j,1)\right)$$

$$+\left.\left(\sum_{k=1,k\neq q}^{Q}\alpha_{q,k}^{f}(x_j,t)\right)^{t}u_q^{f}(x_j,1)\right],\quad q=1,2,\ldots,k,\ldots,Q. \tag{12}$$

Eq. (12) is the utility value of the negotiation group when attribute-outcome x_j of event f at discussion #$t+1$ is realized. This value can be obtained through weighting the utility of the individual negotiator at discussion #1. If it converges after the first discussion among negotiators, then there is no dispersion or iteration through the IUV during the discussion process. We can substitute $t=1$ into the first and second items of the right side of Eq. (12) and obtain $GU^f(x_j,2)=\sum_{k=1,q\neq k}^{Q}(u_k^f(x_j,1)+\sum_{q=1,q\neq k}^{Q}\alpha_{q,k}^f(x_j,1)u_q^f(x_j,1))$. With $t=1$ and $\sum_{q=1,q\neq k}^{Q}\alpha_{q,k}^f(x_j,1)=0$, Eq. (12) becomes $GU^f(x_j,2)=\sum_{q=1}^{Q}u_q^f(x_j,1)$, and the utility value of the negotiation group is the sum of all individual negotiators' utility. This implies that when there is utility independence among negotiators, the utility value of the negotiation group can be obtained through adding of the original utility of all individual negotiators. The result turns out to be the same as the additive-utility of Belichrosdt and Quiggin [21], and the additive independent-utility of Luce and Fishburn [26].

When $t=1$ and $\sum_{q=1,q\neq k}^{Q}\alpha_{q,k}^f(x_j,1)=1$, Eq. (12) becomes $GU^f(x_j,2)=\sum_{q=1}^{Q}\sum_{q=1,q\neq k}^{Q}\alpha_{q,k}^f(x_j,1)\,u_q^f(x_j,1)$, which shows that the utility value of the negotiation group from a realization of the outcome of attribute x_j of event f is the sum of the IUV values multiplied by the utility of the individual negotiators. This shows that although there is no independence phenomenon among negotiators, if the negotiators reach consensus during the first discussion, then there is no iteration relation among negotiators. At this point, $GU^f(x_j,2)$ is the weight of the individual negotiator's utility and the IUV value. When $t\geqslant 3$, and $0<\sum_{k=1,q\neq k}^{Q}\alpha_{q,k}^f(x_j,1)<1$, this means that when discussion among negotiators is completed after three or more times, the utility value of the negotiation group toward the outcome of attribute x_j of event f will be affected and changed by the utility of the individual negotiator, the IUV values, and variable t. Since $0<\sum_{q=1,q\neq k}^{Q}\alpha_{q,k}^f(x_j,t)<1$, $0<u_k^f(x_j,t)<1$ and $0<u_q^f(x_j,t)<1$, then $GU^f(x_j,t+1)$ will decrease with increasing number of discussions and the IUV values will become stable. This completes our demonstration. We now turn to some technical issues in the context of the multi-objective programming paradigm.

4. The dynamic multi-objective programming model

Because there are many events or uncertainties associated with BOT projects [1] and some events are dependent on other events, to provide a simple illustration, we assume that event f is independent of other events. To assess the risk of event f under the condition of discussion among negotiators, we construct a dynamic multi-objective programming model.

Assume that each individual negotiator pursues maximization of the utility, so it implies that the negotiation group will also pursues utility maximization. The discussion behavior of a negotiation group regarding event f can be formulated as Eqs. (13)–(22).

$$\text{Max} \quad GU^f(t+1) \tag{13}$$

$$\begin{aligned}
\text{Max} \quad GU^f(x_j, t+1) = \sum_{q=1}^{Q} \Bigg[& \left(\left(1 - \sum_{q=1, q \neq k}^{Q} \alpha_{q,k}^f(x_j, t) \right)^t u_q^f(x_j, 1) \right) \\
& + \left(\left(1 - \sum_{k=1, q \neq k}^{Q} \alpha_{q,k}^f(x_j, t) \right)^{t-1} \sum_{k=1, q \neq k}^{Q} \alpha_{q,k}^f(x_j, t) u_q^f(x_j, 1) \right) \\
& + \left(\left(1 - \sum_{k=1, q \neq k}^{Q} \alpha_{q,k}^f(x_j, t) \right)^{t-1} \sum_{k=1, q \neq k}^{Q} \alpha_{q,k}^f(x_j, t) u_k^f(x_j, 1) \right) \\
& + \left(\left(1 - \sum_{k=1, q \neq k}^{Q} \alpha_{q,k}^f(x_j, t) \right)^{t-3} \sum_{k=1, q \neq k}^{Q} \alpha_{q,k}^f(x_j, t) u_q^f(x_j, 1) \right) \\
& + \sum_{k=1, q \neq k}^{Q} \alpha_{q,k}^f(x_j, t) \left(\left(1 - \sum_{k=1, q \neq k}^{Q} \alpha_{q,k}^f(x_j, t) \right)^{t-3} u_k^f(x_j, 1) \right) \\
& + \left(\left(1 - \sum_{k=1, q \neq k}^{Q} \alpha_{q,k}^f(x_j, t) \right)^{t-3} \left(\sum_{k=1, q \neq k}^{Q} \alpha_{q,k}^f(x_j, t) \right)^{t-2} u_q^f(x_j, 1) \right) \\
& + \left(\left(1 - \sum_{k=1, q \neq k}^{Q} \alpha_{q,k}^f(x_j, t) \right)^{t-3} \left(\sum_{k=1, q \neq k}^{Q} \alpha_{q,k}^f(x_j, t) \right)^{t-2} u_k^f(x_j, 1) \right) \\
& + \left(\sum_{k=1, q \neq k}^{Q} \alpha_{q,k}^f(x_j, t) \right)^t u_q^f(x_j, 1) \Bigg]
\end{aligned} \tag{14}$$

$$\text{s.t.} \quad u_q^f(x_j, t+1) = \left(1 - \sum_{q=1, q \neq k}^{Q} \alpha_{q,k}^f(x_j, t) \right) u_k^f(x_j, t) + \sum_{q=1, q \neq k}^{Q} \alpha_{q,k}^f(x_j, t) u_q^f(x_j, t) \tag{15}$$

$$u_k^f(x_j,t+1)=\left(1-\sum_{q=1,q\neq k}^{Q}\alpha_{q,k}^f(x_j,t)\right)u_k^f(x_j,t)+\sum_{q=1,q\neq k}^{Q}\alpha_{q,k}^f(x_j,t)u_q^f(x_j,t), \tag{16}$$

$$0\leqslant u_q^f(x_j,t),u_q^f(x_j,t+1)\leqslant 1,\ \ t=0,1,2,\ldots,T, \tag{17}$$

$$0\leqslant \sum_{q=1,q\neq k}^{Q}\alpha_{q,k}^f(x_j,t),\quad \sum_{k=1,q\neq k}^{Q}\alpha_{q,k}^f(x_j,t)\leqslant 1, \tag{18}$$

$$0\leqslant \sum_{q=1}^{Q}\sum_{k=1,q\neq k}^{Q}\alpha_{q,k}^f(x_j,t)\leqslant 1, \tag{19}$$

$$0\leqslant GU^f(x_j,t+1),GU_{mau}^f(x_j,t+1)\leqslant 1, \tag{20}$$

$$GU^f(t+1)=\begin{cases} GU^f(x_j,t+1))\times\psi(x_j),\ if\ \sum_{q,k\neq q}\alpha_{q,k}(x_j,t+1)=\sum_{k,k\neq q}\alpha_{k,q}(x_j,t+1), \\ GU_{mau}^f(x_j,t+1))\times\psi(x_j),\ if\ \sum_{q,k\neq q}\alpha_{q,k}(x_j,t+1)\neq\sum_{k,k\neq q}\alpha_{k,q}(x_j,t+1), \end{cases} \tag{21}$$

$$0\leqslant \alpha_{q,k}^f(x_j,t)\leqslant 1,\forall q\neq k;\ \text{decision variable},$$

$$q=1,2,\ldots,Q;\ \ j=1,2,\ldots,n, \tag{22}$$

where $GU^f(t+1)$ is the utility value of a negotiation group for event f at discussion $\#t+1$; $GU^f(x_j,t+1)$ the utility value of a negotiation group for the outcome of attribute x_j for event f at discussion $\#t+1$; $GU_{mau}^f(x_j,t+1)$ the utility value of a negotiation group for the outcome of attribute x_j for event f at discussion $\#t+1$; if consensus cannot be reached through discussion among negotiators, the utility value will be calculated as an independent state; $u_q^f(x_j,t)$, $u_q^f(x_j,t+1)$ the utility value of individual negotiator $\#q$ for the outcome of attribute x_j toward event f at discussions $\#t$ and $\#t+1$, respectively; $\sum_{q=2}^{Q}\alpha_{q,1}^f(x_j,t)$ the sum value of IUVs where the utility value of negotiator $\#q$ affects negotiator $\#1$ at discussion $\#t$; $\sum_{q=1,q\neq k}^{Q}\alpha_{q,k}(x_j,t)$ the sum value of IUVs where the utility value of negotiator $\#q$ affects negotiator $\#k$ at discussion t; $\alpha_{q,k}^f(x_j,t)$ the utility value of negotiator $\#q$, which affects the utility of negotiator $\#k$ for the outcome of attribute x_j of event f at discussion $\#t$, $k\neq q;\psi(x_j)$ the maximum probability value for all states of event f by q negotiators; and t the index of the number of discussions, $t\in\{0,1,2,\ldots,T\}$.

Eq. (13) is an objective function of the multi-objective programming model. It represents the utility measured by the negotiation group regarding event f through discussion $\#t+1$, and $GU^f(t+1)\in[0,1]$. Another objective of this model is Eq. (14), which is derived from the utility value of the negotiation group regarding the outcome of attribute x_j for each state of event f at discussion $\#t+1$. It can be easily seen that constraints (15)–(22) can affect Eqs. (13) and (14). Constraints (15) and (16) represent the utility value of negotiators q and k at discussion $\#t+1$; respectively. The utility function related to the utility of other negotiators at discussion $\#t$, and the utility of negotiators q and k at discussion $\#t+1$ will affect the utility of other negotiators in the subsequent discussion. In addition, constraint (17) is to ensure that the utility of all the negotiators can satisfy the condition of being in the closed interval [0,1]. Eqs. (18) and (19) are to ensure that the sum of the IUV values among negotiators satisfies the condition of being in the same closed interval. Eq. (20) is also to ensure that the utility value of the negotiation group is not negative. Meanwhile, the utility value of the negotiation group in Eq. (20) meets the constraint of being limited between 0 and 1. Eq. (21) shows that if consensus cannot be reached through discussion among negotiators, the utility value will be calculated as if at an independent state. On the other hand, if consensus is reached during discussion, then the utility value after the discussion can be substituted. Eq. (22) shows that the IUV value among negotiators must be between 0 and 1, and that $\alpha^f_{q,k}(x_j,t)$ are the decision variables.

5. Algorithm

From the dynamic multi-objective programming model, the decision variables are $\alpha^f_{q,k}(x_j,t)$, while the objective functions are $GU^f(x_j,t+1)$ and $GU^f(t+1)$. We develop an iterative algorithm for solving the model. The algorithm steps are described as below.

Step 0: Set the number of discussions, probability values $\psi(x_j), \varepsilon$, and δ value, where ε is a very small value, and δ is an allowable tolerance-error value.

Step 1: *Set the initial utility value*. To find $u^f_q(x_j,0)$, which represents the utility before discussion for negotiator $\# q$, add ε value to represent the initial utility value during discussion. $u^f_q(x_j,1)$ represents the utility values for discussion #1 for negotiators $\# q, q=1,2,\ldots,Q$.

Step 2: *Obtain the initial interactive utility value (IUV)*. During discussion of the negotiation group, assume that the main negotiator (such as the chairman or key negotiator) speaks first; thus, we can obtain the initial IUV for the key negotiator. Then, calculate the initial IUVs of the other negotiators. The calculation procedure is as follows. Apply values for $u^f_q(x_j,0)$ and $u^f_k(x_j,0)$ together with $u^f_q(x_j,1)$ and $u^f_k(x_j,1)$ to Eqs. (A.4) and (A.5) in Appendix A. The initial IUV among negotiators is thus obtained as shown in Eq. (23). Take the absolute value of Eq. (23) to make both $\alpha^f_{q,k}(x_j,t+1)$ and $\alpha^f_{k,q}(x_j,t+1)$ satisfy the non-negativity condition.

$$\alpha^f_{q,k}(x_j,t+1) = \left| \frac{u^f_k(x_j,t+1) - u^f_k(x_j,t)}{(u^f_q(x_j,t+1) - u^f_k(x_j,t))} \right|,$$

$$\alpha^f_{k,q}(x_j,t+1) = (1-\alpha^f_{q,k}(x_j,t+1)) \left| \frac{u^f_q(x_j,t) - u^f_q(x_j,t+1)}{u^f_k(x_j,t+1) - u^f_q(x_j,t)} \right|. \tag{23}$$

When $u^f_q(x_j,t+1) = u^f_q(x_j,t)$, let $u^f_q(x_j,t+1) - u^f_q(x_j,t) = \varepsilon$; or as $u^f_q(x_j,t) = u^f_k(x_j,t+1)$, then let $u^f_q(x_j,t) - u^f_k(x_j,t+1) = \varepsilon$.

Step 3: *Normalize the interactive utility value*. Taking into consideration the different preferences among negotiators, Keeney and Raiffa [9] proposed the concept of the normalization of utility. This step is to simulate conditions with respect to those negotiators who affect others by their own views (strong-minded negotiators) as well as those who are easily affected by others (obedient negotiators). In either case, it will be difficult for the model to converge because the utility values of the strong minded negotiators or the obedient negotiators would not easily change their measured utility. Therefore, we normalize Step 2, as shown in Eq. (24).

$$\alpha_{k,q}^{adj}(x_j,t+1)=\frac{\alpha_{k,q}^{f}(x_j,t+1)-\alpha_{\min}(x_j,t+1)}{\alpha_{\max}(x_j,t+1)-\alpha_{\min}(x_j,t+1)},$$
$$\alpha_{q,k}^{adj}(x_j,t+1)=\frac{\alpha_{q,k}^{f}(x_j,t+1)-\alpha_{\min}(x_j,t+1)}{\alpha_{\max}(x_j,t+1)-\alpha_{\min}(x_j,t+1)}, \tag{24}$$

where $\alpha_{k,q}^{adj}(x_j,t+1)$ and $\alpha_{q,k}^{adj}(x_j,t+1)$ are normalized IUV; $\alpha_{\max}=\max\{\alpha_{k,q}^{f}(x_j,t+1), q=1,2,\ldots,Q, q\neq k\}$; and $\alpha_{\min}=\min\{\alpha_{k,q}^{f}(x_j,t+1), q=1,2,\ldots,Q, q\neq k\}$.

Step 4: *Solve the IUV and utility value that occurs after discussion*. After normalizing the $\alpha_{k,q}^{adj}(x_j,t+1)$ and $\alpha_{q,k}^{adj}(x_j,t+1)$, since $\alpha_{k,q}^{adj}(x_j,t+1)$ and $\alpha_{q,k}^{adj}(x_j,t+1)$ are pre-discussion IUV values, let $\alpha_{k,q}^{adj}(x_j,t+1)$ and $\alpha_{q,k}^{adj}(x_j,t+1)$ be the new IUV, and substitute the values of Step 3 and utility value of Step 1 into the dynamic multi-objective programming model. Thus, we can obtain the solution value of $\alpha_{k,q}^{f*}(x_j,t+1)$ and $\alpha_{q,k}^{f*}(x_j,t+1)$ to represent the utility values after discussion. We can also obtain the solution values of $u_q^{f*}(x_j,t+1)$ and $u_k^{f*}(x_j,t+1)$ after discussion.

Step 5: *Determine whether or not the IUV will converge*. We first check if $u_q^{f*}(x_j,t+1), \alpha_{k,q}^{f*}(x_j,t+1)$ and $\alpha_{q,k}^{f*}(x_j,t+1)$, as obtained from Steps 1–4, converge or not. The convergence condition can be verified as in Appendix B. As shown in Appendix B, when $\alpha_{k,q}^{f*}(x_j,t+1)=\alpha_{q,k}^{f*}(x_j,t+1)$ or $\sum_{q,k\neq q}\alpha_{q,k}^{f*}(x_j,t+1)=\sum_{k,k\neq q}\alpha_{k,q}^{f*}(x_j,t+1)$ satisfies the convergence condition, this indicates that the views among the negotiators are very close. Therefore, $\alpha_{k,q}^{f*}(x_j,t+1)=\alpha_{q,k}^{f*}(x_j,t+1)=0$ or $\alpha_{k,q}^{f*}(x_j,t+1)=\alpha_{q,k}^{f*}(x_j,t+1)=1$ is one of the convergence conditions. With the convergence condition satisfied, we go to Step 9 below to determine the utility value for the negotiation group. If there is no convergence, then proceed to Step 6, modify the IUV and start the next discussion.

Step 6: *Modify the IUV*. When the model cannot converge, we modify the IUV among negotiators, as below. When $|\alpha_{k,q}^{f*}(x_j,t+1)-\alpha_{q,k}^{f*}(x_j,t+1)|>\delta$, then let $\alpha_{k,q}^{f*}(x_j,t+1)=\alpha_{q,k}^{f*}(x_j,t)-\delta$, where $\alpha_{k,q}^{f*}(x_j,t+1)$ and $\alpha_{q,k}^{f*}(x_j,t+1)$ are solution values of the model at $t+1$ discussion.

Step 7: *Modify the individual's utility value*. Let $\alpha_{k,q}^{*}(x_j,t+1)$ and $\alpha_{q,k}^{*}(x_j,t+1)$ of the modified IUV (obtained in Step 6) be the new IUV. Then substitute the new IUV (i.e., $\alpha_{k,q}^{*}(x_j,t+1)$ and $\alpha_{q,k}^{*}(x_j,t+1)$) into the dynamic multi-objective programming model.

Step 8: Repeat Steps 4–7 until the model converges or the end of discussion.

Step 9: Calculate the utility values for individual negotiators and negotiation groups.

When the obtained solution satisfies the convergence condition, we can obtain the solution value for $\alpha_{q,k}^{f*}(x_j,t)$ and $\alpha_{k,q}^{f*}(x_j,t)$, t, $u_q^{f*}(x_j,t), GU^f(x_j,t)$, and $GU^f(t)$.

6. A numerical example

For illustration, we provide a numerical example to describe the model algorithm developed in this study, using the data from Kang [27].

Table 3
Utility value obtained by negotiators regarding the bank loan credit ratio

Negotiator	States						
	6.5%	7.0%	7.5%	8%	8.5%	9%	10%
Negotiator # 1	0.1990	0.4701	0.8737	0.4020	0.0613	0.0121	0.0089
Negotiator # 2	0.9589	0.8693	0.4330	0.2432	0.0992	0.0449	0.0020
Negotiator # 3	0.4680	0.3306	0.2906	0.0546	0.0660	0.0171	0.0090
Negotiator # 4	0.1157	0.5478	0.6089	0.4990	0.0024	0.0002	0.0090
Negotiator # 5	0.8404	0.6843	0.4235	0.2908	0.0261	0.0018	0.0011
Negotiator # 6	0.2016	0.7225	0.7280	0.3570	0.0990	0.0121	0.0001

Source: Kang [27].

6.1. Description of the loan credit ratio event

The concession company must pay loan interest to the bankers within the BOT concession period. So the loan credit is an event for the BOT concession company's negotiation group. If the credit ratio increases, the interest cost will also increase, meaning increased risk. Let the credit ratio associated with this event be 6.5%, 7%, 7.5%, 8%, 8.5%, 9%, and 10%. A total of seven states exist for the level of the loan credit ratio. The attribute outcome for this event is interest cost. Assume that there are six negotiators in the negotiation group of the BOT concession company, and they discuss the bank loan credit ratio. The outcomes of attribute, the utility value of each negotiator and the probability of a specific state negotiator for each event are given. Before discussion, each individual negotiator measures the utility of each state of the event. We use Eq. (2) with the data from Kang [27] to find the utility value for each negotiator. The measurement results are shown in Table 3.

We used Turbo Pascal 7.0 to write the simulation program and to calculate the post-discussion values to obtain the IUV, the utility individual negotiators, the number of discussions after convergence, and the utility value of the negotiation group. The steps are detailed as follows.

Step 0: Set the number of discussions, probability values $\psi(x_j), \varepsilon$ and δ value.

Consider that the discussion behavior has a limitation for a group, so the number of discussions cannot be infinite in the real world, and to simplify the analysis, we set the maximum number of discussions to be 50, which is $t \leqslant 50$. In addition, the probability value, $\psi(x_j)$, are 0.9589, 0.9150, 0.9540, 0.6112, 0.45, 0.2211, and 0.101, respectively as obtained from Kang [27]. Let δ be 0.0001 and $\varepsilon = 0.00001$.

Step 1: Consider the state 9% in Table 3, where the utility value of each negotiator is 0.0121, 0.0449, 0.0171, 0.0002, 0.0018, and 0.0121, respectively. Add the ε value, $\varepsilon = 0.00001$, to the utility of each negotiator and obtain the negotiators' utilities as 0.01211, 0.04491, 0.01711, 0.00021, 0.00181, and 0.01211, respectively, which are the initial utility values during discussion.

Step 2: Substitute the initial utility value of Step 1 into Eq. (23), and obtain the initial IUV, $\alpha^f_{q,k}(9\%, 1)$, as in Table 4. For example, $\alpha^f_{2,1}(9\%, 1) = 0.00030$ indicates that at discussion # 1, the utility value of negotiator # 2 affecting negotiator # 1 is 0.0003, and the rest of the IUVs can be

Table 4
Initial IUVs of the bank loan credit ratio at 9%

Interactive utility value	Negotiator # 1	Negotiator # 2	Negotiator # 3	Negotiator # 4	Negotiator # 5	Negotiator # 6
Negotiator # 1	NA	0.00030	0.00200	0.00084	0.00097	0.00000
Negotiator # 2	0.00030	NA	0.00032	0.09998	0.00566	0.00087
Negotiator # 3	0.00200	0.10000	NA	0.00032	0.00059	0.00180
Negotiator # 4	0.00084	0.00022	0.00059	NA	0.00000	0.00000
Negotiator # 5	0.00097	0.10000	0.10000	0.00625	NA	0.00087
Negotiator # 6	1.00000	0.10000	0.10000	0.00084	0.10000	NA

Table 5
IUVs among negotiators after discussion # 1

Interactive utility value	Negotiator # 1	Negotiator # 2	Negotiator # 3	Negotiator # 4	Negotiator # 5	Negotiator # 6	Total
Negotiator # 1	NA	0.00000	0.00000	0.00000	0.00000	0.00425	0.00425
Negotiator # 2	0.29185	NA	0.34434	0.21415	0.2221	0.29185	1.36429
Negotiator # 3	0.26944	0.04846	NA	0.07972	0.08805	0.26944	0.75511
Negotiator # 4	1.00000	0.32844	0.86871	NA	1.00000	1.00000	4.19715
Negotiator # 5	1.00000	0.29859	0.84112	1	NA	1.00000	4.13971
Negotiator # 6	0.00000	0.07850	0.51493	0.21636	0.24997	NA	1.05976
Total	2.56129	0.75399	2.56910	1.51023	1.56012	2.56554	NA

deduced accordingly. According to Table 4, the IUV among some negotiators is symmetric, while others are not equal.

Step 3: As shown in Table 4, the differences between the negotiators are great. This reflects that there are many points of view among negotiators; therefore, we apply Eq. (24) to normalize the IUVs in Table 4.

Step 4: Substitute the initial value of steps 0 and 1 and the normalized IUV value of Step 2 into the basic model to obtain the negotiator's utility values after discussion # 1, which are $u_1^f(9\%, 1) = 0.0376$, $u_2^f(9\%, 1) = 0.0196$, $u_3^f(9\%, 1) = 0.05068$, $u_4^f(9\%, 1) = 0.11378$, $u_5^f(9\%, 1) = 0.22152$, and $u_6^f(9\%, 1) = 0.01045$, respectively. These are the utility values after discussion # 1, and are different from the minor adjusted utility of Step 1. Compared with the result obtained in Step 1, the negotiator's utility show obvious changes after discussion # 1. Then substitute the utility value of each of the six negotiators into Eq. (23), and obtain the IUV, $\alpha_{q,k}^f(9\%, 1)$, after discussion # 1, as shown in Table 5.

Step 5: To determine whether the IUVs converge or not, we use the result of Step 4 to sum the column values and row values, respectively. The summation of columns is 0.00425, 1.34629, 0.75511, 4.19715, 4.13971 and the summation of rows are 1.05976, 2.56129, 0.75399, 2.56910, 1.56012, and 2.56554, respectively. This indicates that a specific negotiator's utility that is affected by other negotiators is not equal to the utility of other negotiators that are affected by the specific

Table 6
Results of IUV among negotiators under the convergent condition

	Bank	Load	Credit	Ratio:	9%		
Interactive utility value	Negotiator # 1	Negotiator # 2	Negotiator # 3	Negotiator # 4	Negotiator # 5	Negotiator # 6	Total
Negotiator # 1	NA	0.00001	0.00000	0.00000	0.00000	0.00000	0.00001
Negotiator # 2	0.00001	NA	0.00000	0.00000	0.00000	0.00000	0.00001
Negotiator # 3	0.00000	0.00000	NA	0.00000	0.00000	0.00425	0.00425
Negotiator # 4	0.00000	0.00000	0.00000	NA	0.00000	0.00425	0.00425
Negotiator # 5	0.00000	0.00000	0.00000	0.00000	NA	0.00000	0.00000
Negotiator # 6	0.00000	0.00000	0.00425	0.00425	0.00000	NA	0.00850
Total	0.00001	0.00001	0.00425	0.00425	0.00000	0.00850	0.01702

negotiator; and it does not satisfy the convergence condition of the model, requiring further revision as shown in Step 6.

Step 6: From Table 5, $\alpha^{f*}_{2,1}(9\%,1)-\alpha^{f*}_{1,2}(9\%,1) > 0.0001$, we let $\alpha^{f}_{2,1}(9\%,2)=\alpha^{f*}_{1,2}(9\%,1)-0.0001$. This is the input value for the revised IUV of discussion # 2. The revision of other IUVs are similar.

Step 7: The multi-objective programming model performs the second simulation, using the IUV value of Step 6 and the utility value of Step 4.

Step 8: Repeat the calculation from Steps 4–7 using the model.

Through the repeated simulation of the algorithm, the model reaches the convergence condition after discussion # 5, where $\alpha^{f*}_{q,k}$ (9%,5) is as shown in Table 6. The IUV shows symmetry in this case, and the utility impact of negotiator # 4 affecting other negotiators is 0, 0, 0, 0, 0, and 0.00425, which is the same as the utility impact of the other negotiators affecting negotiator # 4. The sum of IUVs for each negotiator is 0.00001, 0.00001, 0.00425, 0.00425, 0.00000, and 0.00850, which satisfies the condition for the model, so the discussion can end at this state. In other words, these six negotiators reach an "agreement" at discussion # 5. Then, we can obtain $u^{f*}_1(9\%,5)=0.0376, u^{f*}_2(9\%,5)=0.0449, u^{f*}_3(9\%,5)=0.00002, u^{f*}_4(9\%,5)=0.00002, u^{f*}_5(9\%,5)=0.00002$, and $u^{*}_6(9\%,5)=0.00001$ for the negotiation group regarding bank credit ratio at 9%. In addition, $GU^f(9\%,5)=0.05068$, where $GU^f(9\%,5)$ is the risk assessment for post-discussion utility of the six negotiators toward the bank loan credit ratio (i.e., event f) at 9%.

We assume that the state of the event is independent, and we simulate the discussion among negotiators by applying data in Table 3 with the algorithm and Pascal program. We obtain the number of discussion, utility change and convergence of each state of the event; and the results are summarized in Table 7. Simulating all the states of the event for 50 times yields the number of discussions for each state as 28, 50, 50, 50, 4, 5, and 6. The utility values of the negotiation group for each state are 0.59044, 3.21145, 2.95137, 1.59945, 0.05949, 0.05068, and 0.53806, respectively. As for credit ratios of 7.0%, 7.5%, and 8.0%, they cannot reach convergence even over 50 simulations, so no convergent solution is obtained for the IUV. The utility value for other states are all less than one, meeting the convergence condition.

For utility changing of all individual negotiators regarding the credit ratio of 6.5%, the post-discussion utility value of negotiators # 2, # 3, # 5, and # 6 is greatly decreased compared with the

Table 7
Pre- and post-discussion measured utility value regarding the bank loan credit ratio event

State								
Utility	Discussion	6.5%	7.0%	7.5%	8.0%	8.5%	9.0%	10.0%
Nego.# 1	Pre	0.19900	0.47010	0.87370	0.40200	0.06130	0.01210	0.00890
	Post	0.17350	0.44460	0.84820	0.37650	0.03580	0.03760	0.03440
	Variation	−0.02550	−0.02550	−0.02550	−0.02550	−0.02550	0.02550	0.02550
Nego.# 2	Pre	0.95890	0.86930	0.43300	0.24320	0.09920	0.04490	0.00200
	Post	0.11534	0.79091	0.43905	0.25192	0.02369	0.01301	0.05261
	Variation	−0.84356	−0.07839	0.00605	0.00872	−0.07551	−0.03189	0.05061
Nego. # 3	Pre	0.46800	0.33060	0.29060	0.05460	0.06600	0.01710	0.00900
	Post	0.06166	0.44195	0.41883	0.19242	0.00000	0.00002	0.17901
	Variation	−0.40634	0.11135	0.12823	0.13782	−0.06600	−0.01708	0.17001
Nego. # 4	Pre	0.11570	0.54780	0.60890	0.49900	0.00240	0.00020	0.00900
	Post	0.11981	0.50176	0.41507	0.26065	0.00000	0.00002	0.17901
	Variation	0.00411	−0.04604	−0.19383	−0.23835	−0.00240	−0.00018	0.17001
Nego. # 5	Pre	0.84040	0.68430	0.42350	0.29080	0.02610	0.00180	0.00110
	Post	0.07877	0.51609	0.41062	0.26004	0.00000	0.00002	0.46513
	Variation	−0.76164	−0.16821	−0.01288	−0.03076	−0.02610	−0.00178	0.46403
Nego. # 6	Pre	0.20160	0.72250	0.72800	0.35700	0.09900	0.01210	0.00010
	Post	0.04136	0.51614	0.41960	0.25792	0.00000	0.00001	0.46513
	Variation	−0.16024	−0.20636	−0.30840	−0.09908	−0.09900	−0.01209	0.46503
Converge/diverge		√	×	×	×	√	√	√
Number of discussions		28	50	50	50	4	5	5
Pre-Dis. $GU_{mau}(x_j)$		0.42820	0.56250	0.53640	0.28870	0.05700	0.01360	0.00560
Post-Dis. $GU(x_j)$		0.59044	3.21145	2.95137	1.59945	0.05949	0.05068	0.53806

Note:"×" indicates divergence; "√" indicates convergence.

value before discussion. The variation in credit ratio at 10.0% is also great, except for negotiator # 1, the utility value after discussion of all other negotiators is clearly increased. This reveals that all six negotiators change their original assessed utility after discussion, as shown in Table 7. In addition, for utility changing of the negotiation group, the pre-discussion $GU^f_{mau}(x_j)$ is obtained by using the additive MAU model from Kang [27]. As for the post-discussion, $GU^f(x_j, t)$ is obtained by applying the dynamic multi-objective programming model. Comparing $GU^f_{mau}(x_j, t)$ and $GU^f(x_j, t)$, respectively, shows that during discussion, one negotiator was significantly affected by other negotiators and changed the original risk assessment for the utility, while the others showed no significant change.

For risk assessment of this event, if the negotiator's utility is independent, the utility measured by the negotiation group toward the loan credit ratio event is 0.4001 which is greater than the mean

Table 8
Comparison between pre- and post-discussion utility measured

Event of bank loan credit ratio	Utility value of group negotiator	Mean utility value	Risk/non-risk
Pre-discussion	0.4001	0.2877	Non-risk
Post-discussion	0.56617	0.25689	Non-risk

utility value of 0.2877. This implies that the pre-discussion utility measured by the negotiation group toward the bank loan credit ratio appears to be a non-risk event. After 50 simulations, the utility value of event post-discussion as obtained by multi-objective programming is 0.56617, which is greater the mean utility value of 0.25689. Therefore, comparing the pre- and post-discussion utility value measured by the negotiation group that the bank loan credit ratio is a non-risk event. The results are shown in Table 8.

From this numerical example, we can summarize the following key points of this study. (1) If the utility differences between negotiators become smaller, the model converges more easily; otherwise, it is difficult for the model to converge and the number of discussions will increase. (2) With more discussions, the IUVs among negotiators will decrease.

7. Conclusions

In this paper, we have relaxed the independent utility condition for negotiators, and demonstrated the interactive behavior among negotiators and risk assessment for a specific event. Dynamic multi-objective programming is adopted to formulate and test a risk assessment model for interactive negotiators. As shown in the model and numerical example, the risk assessed by the negotiation group of a BOT concession company with respect to the event of bank loan credit ratio shows that it is a non-risk event both before and after discussion. It can be seen that the dynamic multi-objective programming model developed in this paper could be used as the basis for negotiation discussion, to explain the interactive utility among negotiators and risk assessment of the negotiation group with respect to some specific events.

The model developed in this paper treats IUVs as endogenous variables of the negotiator's utility function. Besides the utility interaction among negotiators, factors that may affect the utility function include the learning capability of negotiators as well as incomplete information. These factors can be incorporated into the model for further study. In the future, the assumption of event independence made in this paper can be relaxed and the model can be revised by developing a risk assessment utility model for multiple events discussed. Future research can also attempt to improve upon the model algorithm in this paper in order to make the algorithm computationally more efficient.

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Appendix A. Solving the equations to derive (dependent) utility values of negotiators

Assume that the linear dependent utility for negotiators k and q are as shown in Eqs. (A.1) and (A.2), and solve for $\alpha_{k,q}(x_j,t)$ and $\alpha_{q,k}(x_j,t)$ by the simultaneous equation system below:

$$u_k^f(x_j,t+1) = (1-\alpha_{q,k}^f(x_j,t))u_k^f(x_j,t) + \alpha_{q,k}^f(x_j,t)u_q^f(x_j,t), \tag{A.1}$$

$$u_q^f(x_j,t+1) = (1-\alpha_{k,q}^f(x_j,t))u_q^f(x_j,t) + \alpha_{k,q}^f(x_j,t)u_k^f(x_j,t), \tag{A.2}$$

$k,q = 1,2,\ldots,Q,\ k \neq q; t = 0,1,2,\ldots,T.$ t is the number of discussions. Substitute Eq. (A.1) into Eq. (A.2) to get Eq. (A.3).

$$u_k^f(x_j,t) = \frac{1}{1-\alpha_{q,k}^f(x_j,t)}[u_k^f(x_j,t+1) - \alpha_{q,k}^f(x_j,t)u_q^f(x_j,t)]. \tag{A.3}$$

Substituting Eq. (A.3) into the variable $u_k^f(x_j,t)$ of Eq. (A.2) yields $\alpha_{q,k}^f(x_j,t)$ value, shown as Eq. (A.4); therefore, substitute $\alpha_{q,k}^f(x_j,t)$ into Eq. (A.3) to obtain $\alpha_{k,q}^f(x_j,t)$, with the result shown as Eq. (A.5).

$$\alpha_{q,k}^f(x_j,t) = \left(\frac{u_k^f(x_j,t+1) - u_k^f(x_j,t)}{u_q^f(x_j,t) - u_k^f(x_j,t)}\right), \tag{A.4}$$

$$\alpha_{k,q}^f(x_j,t) = (1-\alpha_{q,k}^f(x_j,t))\left(\frac{u_q^f(x_j,t) - u_q^f(x_j,t+1)}{u_k^f(x_j,t+1) - u_q^f(x_j,t)}\right), \quad \forall k \neq q. \tag{A.5}$$

Appendix B. Derivation of convergence condition

To show the convergence condition of the iterative algorithm for dynamic multi-objective programming, we modify Eqs. (A.1)–(B.1) by applying Eqs. (A.1) and (A.2). The left-hand side of Eq. (B.1) represents the incremental utility difference between negotiators k and q. The right-hand side utility of Eq. (B.1) represents the utility of negotiator k as affected by negotiator q. The IUVs can become equal and Eq. (B.1) can therefore be represented by Eq. (B.2).

$$u_k^f(x_j,t+1) - u_k^f(x_j,t) = \alpha_{q,k}^f(x_j,t)(u_q^f(x_j,t) - u_k^f(x_j,t)). \tag{B.1}$$

Let $\Delta u_k^f(x_j,t+1) = u_k^f(x_j,t+1) - u_k^f(x_j,t)$, and $\Delta u_q^f(x_j,t) = (u_q^f(x_j,t) - u_k^f(x_j,t))$, Eq. (B.1) can thus be arranged as

$$\Delta u_k^f(x_j,t+1)/\Delta u_q^f(x_j,t) = \alpha_{q,k}^f(x_j,t). \tag{B.2}$$

Similarly, modify Eq. (A.2) into (B.3), which represents the difference in incremental utility value of negotiators q and k, which can also be shown as the utility incremental method, such as Eq. (B.4).

$$u_q^f(x_j,t+1) - u_q^f(x_j,t) = \alpha_{k,q}^f(x_j,t)(u_k^f(x_j,t) - u_q^f(x_j,t)), \tag{B.3}$$

$$\Delta u_q^f(x_j,t+1)/\Delta u_k^f(x_j,t) = \alpha_{k,q}^f(x_j,t). \tag{B.4}$$

Eq. (B.4) indicates the incremental ratio of the negotiator's utility. If the change in utility becomes stable, then the utility incremental ratios of Eqs. (B.2) and (B.4) tend to be equal, which is $\Delta u_q^f(x_j,t+1)/\Delta u_k^f(x_j,t) = \Delta u_k^f(x_j,t+1)/\Delta u_q^f(x_j,t)$. This implies that the IUV of negotiators k and q tend to be equal, which is $\alpha_{q,k}^f(x_j,t) = \alpha_{k,q}^f(x_j,t)$, and the development procedure is as below.

As the incremental utility becomes stable, indicating that the smaller the difference between the utility incremental ratio of negotiators k and q, the better the case is, which is

$$\min[(\Delta u_q^f(x_j,t+1)/\Delta u_k^f(x_j,t)) - (\Delta u_k^f(x_j,t+1)/\Delta u_q^f(x_j,t))].$$

$$\min[(\Delta u_q^f(x_j,t+1)/\Delta u_k^f(x_j,t)) - (\Delta u_k^f(x_j,t+1)/\Delta u_q^f(x_j,t))] = \min(\alpha_{q,k}^f(x_j,t) - \alpha_{k,q}^f(x_j,t)). \tag{B.5}$$

To keep $0 \leqslant \alpha_{q,k}^f(x_j,t) - \alpha_{k,q}^f(x_j,t) \leqslant 1$, let $\gamma_{q,k} = |\alpha_{q,k}^f(x_j,t) - \alpha_{k,q}^f(x_j,t)|$, modify Eq. (B.5) to Eq. (B.6).

$$\min\gamma_{q,k} = \min|\alpha_{q,k}(x_j,t) - \alpha_{k,q}(x_j,t)|. \tag{B.6}$$

We can modify Eq. (B.6) as

$$\min\gamma_{q,k}^2 = \min(\alpha_{q,k}^f(x_j,t) - \alpha_{k,q}^f(x_j,t))^2. \tag{B.7}$$

Eq. (B.7) indicates the minimum difference in IUV, so we differentiate $\alpha_{q,k}^f(x_j,t)$ to get Eq. (B.8).

$$\min\gamma_{q,k}^2 = \min(\alpha_{q,k}^f(x_j,t) - \alpha_{k,q}^f(x_j,t))^2,$$

$$\frac{\partial\gamma_{q,k}^2}{\partial\alpha_{q,k}^f(x_j,t)} = 2(\alpha_{q,k}^f(x_j,t) - \alpha_{k,q}^f(x_j,t)).$$

Let $\partial\gamma_{q,k}^2/\partial\alpha_{q,k}^f(x_j,t) = 0$, therefore $2(\alpha_{q,k}^f(x_j,t) - \alpha_{k,q}^f(x_j,t)) = 0$ and

$$\alpha_{q,k}^f(x_j,t) = \alpha_{k,q}^f(x_j,t). \tag{B.8}$$

Similarly, we differentiate $\alpha_{k,q}^f(x_j,t)$, getting the same result of as from Eq. (B.8). That is, $\alpha_{q,k}^f(x_j,t) = \alpha_{k,q}^f(x_j,t) \Rightarrow (\Delta u_q^f(x_j,t+1)/\Delta u_k^f(x_j,t)) = (\Delta u_k^f(x_j,t+1)/\Delta u_q^f(x_j,t))$.

As shown in Eq. (B.8), when the utilities of negotiators k and q become stable, their IUVs tend to be equal. In other words, when $\alpha_{q,k}^f(x_j,t) = \alpha_{k,q}^f(x_j,t)$, there is a convergent solution for the model. When $\sum_{q,q\neq k}\alpha_{q,k}^f(x_j,t) = \sum_{q,q\neq k}\alpha_{k,q}^f(x_j,t)$, this satisfies the convergence condition of the model. The meaning of the convergent solution is that there is no utility change among the negotiators due to discussion, and the negotiators have reached "consensus".

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