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MATHEMATICAL MODEL AND BEARING CONTACTS OF THE ZK-TYPE WORM GEAR SET CUT BY OVERSIZE HOB CUTTERS

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Abstract—An oversize worm-type hob cutter is used to cut worm gears in this study. A mathematical model of the ZK-type worm gear set is developed based on the cutting mechanism and its tool parameters. Variations in bearing contact due to pitch diameter variations of oversize hob cutters and center distance variations of worm gear sets are also investigated. A numerical example is given to demonstrate the paths and dimensions of the bearing contacts of worm gear sets.

INTRODUCTION

The worm gear set is one of the most important devices for transmitting torque between spatially crossed axes. Due to their high transmission ratios, low noise and compact structure, worm gear sets are widely used in gear-reduction mechanisms. According to cutting methods and DIN standards, worm gear sets can be mainly categorized into four types—ZA, ZN, ZE (or ZI) and ZK types. The Klingelnberg worm gear set, or ZK-type worm gear set, is one of the most popular worm gear sets in industry, because its precision worm can be ground by a straight-edged grinding wheel.

Worm gear surface characteristics and contact nature have been studied by many researchers. Their researches improved the efficiency and working life time of worm gear sets. Winter and Wilkesmann [1], Simon [2], Bosch [3] and Bär [4] proposed different methods of obtaining more precise worm surfaces. Pencil-type milling cutters and disk-type grinding wheels for producing worm surfaces were studied by Litvin [5]. Kin [6] investigated the limitation of worms to avoid undercutting, and also studied the envelope existence of contact lines. Colbourne [7] investigated the undercutting, interference and non-conjugate contact of the ZK-type worm gear set. In his study, conjugate worm gear surfaces are obtained by considering the mating work surface as a series of rack cutters in its axial section. Wildhaber [8] was the first researcher to adopt surface curvatures to obtain an approximate tooth contact bearing diagram when worm gears are cut by oversize hob cutters. Design criteria, classification and regrind limitation of oversize hob cutters were also investigated by Wildhaber. Janninck [9] proposed a method for predicting the initial contact pattern and showed their results by applying a contact surface topology diagram over the entire worm gear surface. Colbourne [10] proposed a method for designing oversize hob cutters to cut worm gears. The contact surface separation topology concept was also adopted to show the results of his design. Litvin and Kin [11] also proposed a generalized tooth contact analysis (TCA) algorithm to determine the position of transfer points where an ideal contact line will turn into a real contact point. The influences of rotation axes misalignment and center distance offset on the conjugate worm gear set were also investigated in this study. Kin [12] studied the surface deviations of the produced worm gear tooth surfaces due to cutting edge deviations of hob cutters. Tolerances for worm gear tooth profiles can also be obtained by applying Kin's method.

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Theoretically, the ZK-type worm is cut by a straight-edged disk-type grinding wheel, and the worm gear is produced by a worm-type hob cutter which is identical with the worm. However, oversize worm-type hob cutters (i.e. oversize hob cutters) are usually used in industry for worm gear manufacturing. The bearing contacts of the worm gear set are point contacts rather than line contacts. Under loads, gear tooth surfaces contact will be spread over an area, usually elliptical, in the neighborhood of contact points, due to tooth surfaces elasticity. The bearing contact of tooth surfaces is formed by a set of contact ellipses moved over the tooth surfaces in the process of gear meshing. Nevertheless, if the pitch diameter of the oversize hob cutter is equal to that of the mating worm, the bearing contacts become line contacts again.

In this paper, a mathematical model of a grinding wheel is developed and worm surfaces are cut by this grinding wheel. Equations of the worm surfaces and the oversize hob cutter surfaces can be developed by considering the relationship of the grinding wheel to its cutting mechanism. Worm gear tooth profiles can then be obtained by simultaneously considering the locus equations of the oversize hob cutter together with the equation of meshing between oversize hob cutters and worm gears. Contact points of worm gear sets are determined by applying the developed mathematical model and tooth contact analysis (TCA) technique. Surface separation topology is adopted to determine the shapes and locus of bearing contacts. Variations on paths and dimensions of the bearing contact due to the change of pitch diameter of oversize hob cutters and the change of center distance of worm gear sets are also investigated in this paper. Finally, a numerical example is given to demonstrate the path and dimensions of bearing contacts of the ZK-type worm gear set.

MATHEMATICAL MODEL OF THE GRINDING WHEEL

The ZK-type worm is cut by a cone-shaped disk-type grinding wheel with two straight-edged revolving surfaces in its axial section, as shown in Fig. 1. The surface equations for the grinding wheel, represented in coordinate system $S_c(X_c, Y_c, Z_c)$, are expressed by:

$$x_c = u_1 \cos\alpha_1 \cos\theta_1,$$

$$y_c = u_1 \cos\alpha_1 \sin\theta_1$$

and

$$z_c = \pm(b_1 - u_1 \sin\alpha_1) \tag{1}$$

where θ_1 and u_1 are surface parameters of the disk-type grinding wheel, $0 \leqslant \theta_1 < 2\pi$ and $u_{min} \leqslant u_1 \leqslant u_{max}$. Design parameter α_1 is the apex angle of grinding wheels formed by the straight edge and the Z_c-axis, as shown in Fig. 1(a). The plus sign of z_c-expressed in equation (1) is associated with the right-hand grinding wheel surface and the minus sign of z_c is associated with the left-hand grinding wheel surface. Parameter b_1 defined in Fig. 1(a) can be expressed as follows:

$$b_1 = \frac{s_{n1}}{2} + r_{c1} \tan\alpha_1 \tag{2}$$

where parameter s_{n1} is the normal groove width of worms measured along the worm pitch cylinder, as shown in Fig. 1(a). Parameter r_{c1} is the mean grinding wheel radius, where the width of grinding wheel is equal to the normal width of worm groove s_{n1}. The surface unit normal vector of grinding wheels can be expressed as follows:

$$n_{xc} = \pm\sin\alpha_1 \cos\theta_1,$$

$$n_{yc} = \pm\sin\alpha_1 \sin\theta_1$$

and

$$n_{zc} = \cos\alpha_1 \tag{3}$$

where the plus and minus signs shown in equation (3) are associated with the right-hand and left-hand sides of the grinding wheel surfaces, respectively.

MATHEMATICAL MODELS OF THE ZK-TYPE WORM AND OVERSIZE HOB CUTTER

The ZK-type worm is produced by a straight-edged disk-type grinding wheel with its rotation axis inclined to a lead angle β_1 relative to the worm rotation axis, as shown in Fig. 1(c). Therefore, the worm surface geometry depends on the design parameters of the grinding wheel and on machine-tool settings. The ZK-type worm surface equations can be obtained by simultaneously considering the locus of grinding wheels, represented in worm coordinate system $S_1(X_1, Y_1, Z_1)$ (as shown in Fig. 2), and the equation of meshing between the worm and grinding wheel.

The ZK-type worm cutting mechanism can be simplified by considering the relative motion between coordinate systems $S_1(X_1, Y_1, Z_1)$ and $S_c(X_c, Y_c, Z_c)$, as shown in Fig. 2. Coordinate system $S_1(X_1, Y_1, Z_1)$ is associated with the worm surface, and coordinate system $S_f(X_f, Y_f, Z_f)$ is the reference coordinate system; Z_1-axis and Z_c-axis are the rotation axes of the worm and grinding wheel, respectively. These two axes have the shortest distance S_d, and are inclined to a lead angle β_1, as shown in Fig. 2(a). The locus of the generating tool surface (i.e. the grinding wheel surface) can be represented in coordinate system S_1 by applying the following homogeneous position vector transformation matrix equation:

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\psi_1 & \sin\psi_1\cos\beta_1 & -\sin\psi_1\sin\beta_1 & S_d\cos\psi_1 \\ -\sin\psi_1 & \cos\psi_1\cos\beta_1 & -\cos\psi_1\sin\beta_1 & -S_d\sin\psi_1 \\ 0 & \sin\beta_1 & \cos\beta_1 & -P_1\psi_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix} \quad (4)$$

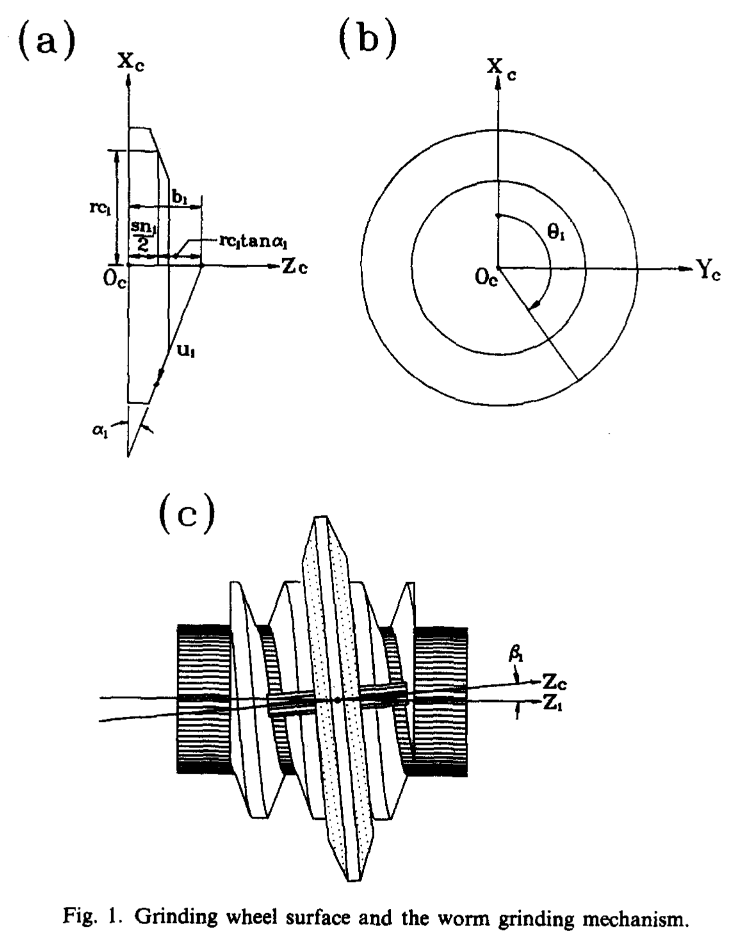

Fig. 1. Grinding wheel surface and the worm grinding mechanism.

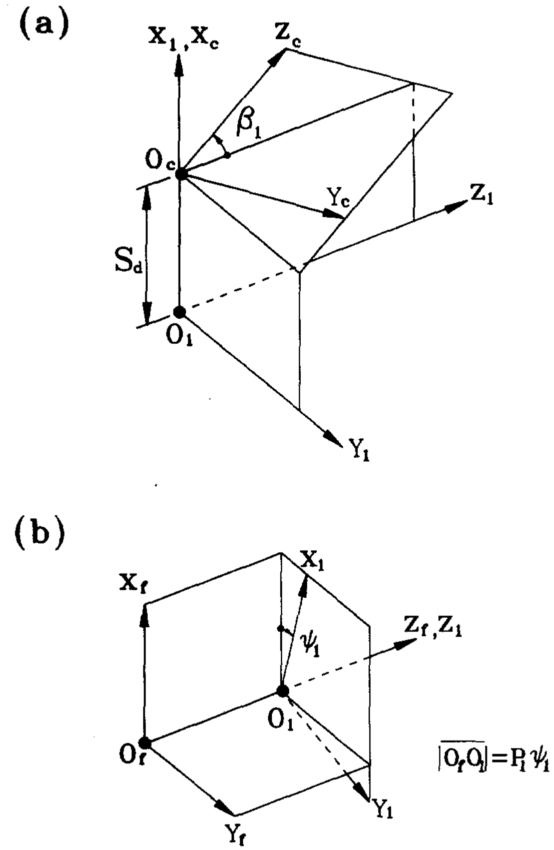

Fig. 2. Relationship between the worm and grinding wheel coordinate systems.

where parameter P_1 is the lead of the worm surface per radian revolution, and ψ_1 is the rotation angle in screw motion. Substituting equation (1) into equation (4), the locus equation becomes:

$$x_1 = A_1 \sin \psi_1 + B_1 \cos \psi_1,$$

$$y_1 = A_1 \cos \psi_1 - B_1 \sin \psi_1$$

and

$$z_1 = C_1 - P_1 \psi_1 \tag{5}$$

where

$$A_1 = u_1 \cos \alpha_1 \cos \beta_1 \sin \theta_1 \pm (u_1 \sin \alpha_1 - b_1) \sin \beta_1,$$

$$B_1 = u_1 \cos \alpha_1 \cos \theta_1 + S_d$$

and

$$C_1 = u_1 \cos \alpha_1 \sin \beta_1 \sin \theta_1 \mp (u_1 \sin \alpha_1 - b_1) \cos \beta_1 .$$

The surface unit normal vector of the ZK-type worm thread can also be obtained by applying the vector transformation matrix equation:

$$\begin{bmatrix} n_{x1} \\ n_{y1} \\ n_{z1} \end{bmatrix} = \begin{bmatrix} \cos \psi_1 & \sin \psi_1 \cos \beta_1 & -\sin \psi_1 \sin \beta_1 \\ -\sin \psi_1 & \cos \psi_1 \cos \beta_1 & -\cos \psi_1 \sin \beta_1 \\ 0 & \sin \beta_1 & \cos \beta_1 \end{bmatrix} \begin{bmatrix} n_{xc} \\ n_{yc} \\ n_{zc} \end{bmatrix} . \tag{6}$$

Substituting equation (3) into equation (6), the surface unit normal vector of the worm thread, represented in coordinate system S_1, can be expressed as follows:

$$n_{x1} = D_1 \sin\psi_1 + E_1 \cos\psi_1,$$

$$n_{y1} = D_1 \cos\psi_1 - E_1 \sin\psi_1$$

and

$$n_{z1} = \pm\sin\alpha_1 \sin\beta_1 \sin\theta_1 + \cos\alpha_1 \cos\beta_1 \tag{7}$$

where

$$D_1 = \pm\sin\alpha_1 \sin\beta_1 \sin\theta_1 - \cos\alpha_1 \sin\beta_1$$

and

$$E_1 = \pm\sin\alpha_1 \cos\theta_1.$$

For the conjugate action between grinding wheels and produced worm surfaces, both surfaces are in continuous tangency at every moment during the cutting process. Therefore, the relative velocity between these two surfaces must lie on the common tangent plane. Hence, the following equation must be observed:

$$\mathbf{n}_c \cdot \mathbf{V}_c^{1c} = 0 \tag{8}$$

where $\mathbf{n}_c$ is the surface common unit normal vector of mating surfaces, and $\mathbf{V}_c^{1c}$ is the relative velocity of mating surfaces represented in coordinate system S_c. Equation (8) is known in the theory of gearing as the equation of meshing. The relative velocity between grinding wheel and worm surfaces can be obtained by applying the equation

$$\mathbf{V}_c^{1c} = (\boldsymbol{\omega}_1 - \boldsymbol{\omega}_c) \times \mathbf{R}_c + \boldsymbol{\omega}_1 \times \mathbf{R} - \frac{d\mathbf{R}}{dt} \tag{9}$$

where $\mathbf{R}_c$ is the position vector of any common contact point, represented in coordinate system S_c, as expressed in equation (1). $\boldsymbol{\omega}_1$ and $\boldsymbol{\omega}_c$ are the angular velocities of worm and grinding wheel surfaces, respectively. According to the cutting mechanism shown in Fig. 2, the grinding wheel is rotated about its rotation axis Z_c while the worm surface performs a screw motion. Position vector $\mathbf{R}$ is measured from the rotation axis of the grinding wheel to the rotation axis of the worm, represented in coordinate system S_c.

Substituting equations (1), (3) and (9) into equation (8), the equation of meshing between grinding wheel and worm surfaces can be obtained as follows:

$$u_1 = \pm(S_d \cot\beta_1 + P_1)\sin\alpha_1 \tan\theta_1 - r_{c1}\frac{\cos\alpha_1}{\cos\theta_1} + b_1 \sin\alpha_1 \tag{10}$$

The mathematical model of the ZK-type worm surface is obtained by simultaneously considering equations (5) and (10). The oversize worm-type hob cutter can also be produced on the same machine by increasing the shortest center distance S_d, as shown in Fig. 2(a). Therefore, the pitch diameter of the hob cutter is larger than that of the worm, and the hob cutter is called an oversize hob cutter. Hence, the mathematical model of oversize hob cutters is similar to that of worms, and can be obtained from equations (5) and (10) by replacing the subscript "$_1$" with "$_o$".

MATHEMATICAL MODEL OF THE WORM GEAR

As mentioned above, in order to produce a point contact worm gear set, the worm gear is usually cut with a worm-type oversize hob cutter. The mathematical model of the worm gear can be obtained by considering the locus of the oversize hob cutter together with the equation of meshing of oversize hob cutters and worm gears. The cutting/mating mechanism of a worm gear is shown in Fig. 3. The coordinates system, $S_1(X_1, Y_1, Z_1)$ is associated with the worm, $S_o(X_o, Y_o, Z_o)$ is associated with the oversize hob cutter and $S_2(X_2, Y_2, Z_2)$ is associated with the worm gear; and the fixed systems $S_w(X_w, Y_w, Z_w)$, $S_h(X_h, Y_h, Z_h)$ and $S_g(X_g, Y_g, Z_g)$ are the reference coordinate systems. Axes Z_1, Z_o and Z_2 are the rotation axes of the worm, oversize hob cutter and worm gear, respectively. Parameters γ_h and γ_w are setting angles measured from Z_2-axis to Z_h-axis and Z_w-axis, respectively. Parameters S_h and S_w are the shortest center distance measured from the rotation axis

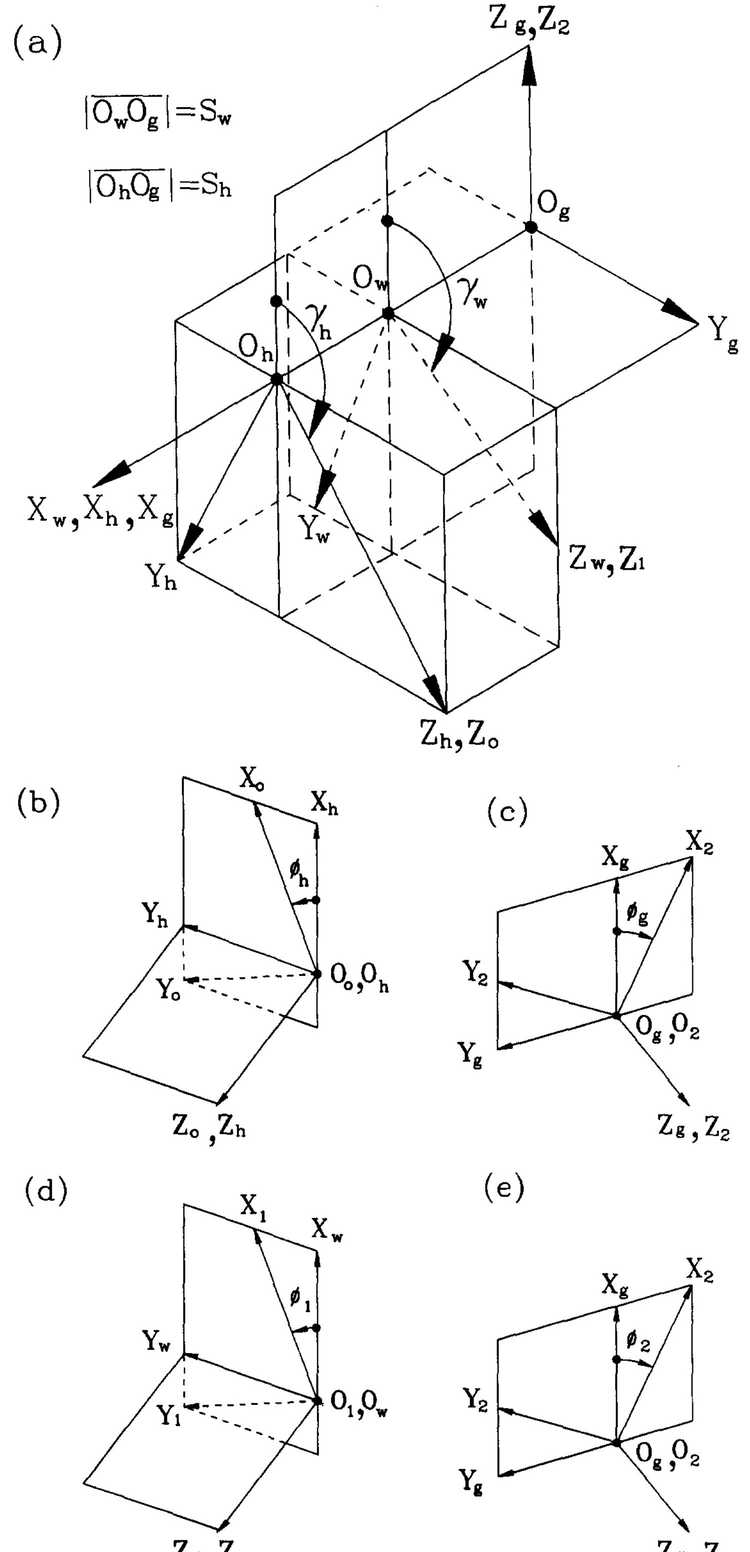

Fig. 3. Coordinate systems among the worm, worm gear and oversize hob cutter.

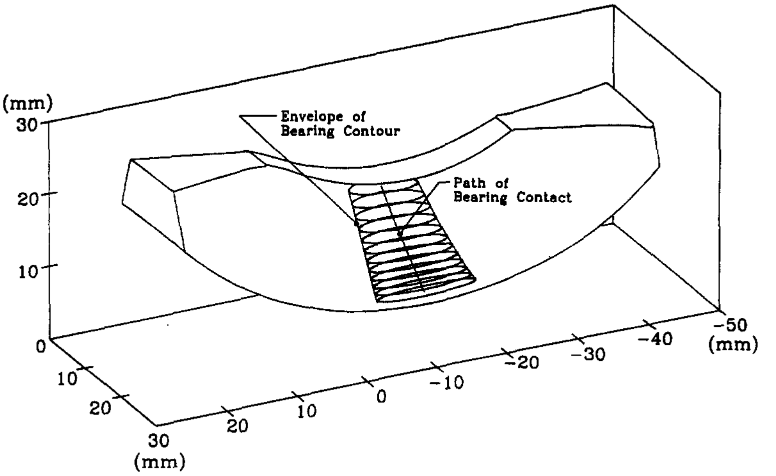

Fig. 4. Envelope and path of bearing contacts.

of the worm gear to the rotation axis of the oversize hob cutter and worm, respectively. The shortest center distances can be expressed by equations

$$S_h = r_o + r_2$$

and

$$S_w = r_1 + r_2 \tag{11}$$

where r_o, r_1 and r_2 are the pitch radii of oversize hob cutter, worm and worm gear, respectively. Rotation angles ϕ_h and ϕ_g, shown in Figs 3(b) and 3(c), are associated with the rotation angles of the oversize hob cutter and worm gear, respectively, during the cutting process; rotation angles ϕ_1 and ϕ_2, shown in Figs 3(d) and 3(e), are associated with the rotation angles of the worm and worm gear, respectively, during the mating process. Because the oversize hob cutter and the produced worm gear remain in conjugate action during the cutting process, rotation angles ϕ_h and ϕ_g can be related by the equation

$$\phi_g = \frac{z_h \cdot \phi_h}{z_g} \tag{12}$$

where z_g and z_h are the tooth numbers of the worm gear and oversize hob cutter, respectively. The locus of the oversize hob cutter represented in coordinate system S_2 can be obtained by applying the following coordinate transformation matrix equation:

$$\mathbf{R}_2 = [M_{2o}] \cdot \mathbf{R}_o \tag{13}$$

where $\mathbf{R}_2$ and $\mathbf{R}_o$ are the locus equations of the oversize hob cutters represented in coordinate systems S_2 and S_o, respectively. Because the geometry of the oversize hob cutter actually is a worm-type cutter, the surface equation $\mathbf{R}_o$ can be obtained from equations (5) and (10) by replacing subscript "1" with "o". Matrix $[M_{2o}]$ is the coordinate transformation matrix which transforms position vectors from coordinate system S_o to S_2, and is represented as follows:

$$[M_{2o}] = \begin{bmatrix} a_{11} & a_{12} & -\sin\phi_g \sin\gamma_h & S_h \cos\phi_g \\ a_{21} & a_{22} & \cos\phi_g \sin\gamma_h & S_h \sin\phi_g \\ -\sin\gamma_h \sin\phi_h & -\sin\gamma_h \cos\phi_h & \cos\gamma_h & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{14}$$

where

$$a_{11} = -\sin\phi_g \cos\gamma_h \sin\phi_h + \cos\phi_g \cos\phi_h,$$

$$a_{12} = -\sin\phi_g \cos\gamma_h \cos\phi_h - \cos\phi_g \sin\phi_h,$$

$$a_{21} = \cos\phi_g \cos\gamma_h \sin\phi_h + \sin\phi_g \cos\phi_h$$

and

$$a_{22} = \cos\phi_g \cos\gamma_h \cos\phi_h - \sin\phi_g \sin\phi_h.$$

Parameter γ_h represents the setting angle of the oversize hob cutter. Substituting equations (5), (10) and (14) into equation (13), worm gear surface equations generated by the oversize hob cutter are obtained as follows:

$$x_2 = [-B_o \cos\gamma_h \sin\phi_g - A_o \cos\phi_g]\sin(\phi_h - \psi_o) - (C_o - P_o\psi_o)\cos\phi_g \sin\gamma_h + [-A_o \cos\gamma_h \sin\phi_g + B_o \cos\phi_g]\cos(\phi_h - \psi_o) + S_h \cos\phi_g,$$

$$y_2 = [B_o \cos\gamma_h \cos\phi_g - A_o \sin\phi_g]\sin(\phi_h - \psi_o) + (C_o - P_o\psi_o)\cos\phi_g \sin\gamma_h + [A_o \cos\gamma_h \cos\phi_g + B_o \sin\phi_g]\cos(\phi_h - \psi_o) + S_h \sin\phi_g$$

and

$$z_2 = [-B_o \sin(\phi_h - \psi_o) - A_o \cos(\phi_h - \psi_o)]\sin\gamma_h + (C_o - P_o\psi_o)\cos\gamma_h \quad (15)$$

where parameters ϕ_h and ϕ_g represent the rotation angles of the oversize hob cutter and worm gear, respectively. Parameter P_o is the lead, per radian revolution, of the oversize hob cutter, and ψ_o is the rotation angle of screw motion during the cutting process. Parameters A_o, B_o and C_o are defined similar to those represented in equation (5) by changing the subscript "l" with "o". The surface unit normal vector of the worm gear, represented in coordinate system S_2, can be obtained by applying the vector transformation matrix equation

$$\mathbf{n}_2 = [L_{2o}] \cdot \mathbf{n}_o \quad (16)$$

where $\mathbf{n}_2$ and $\mathbf{n}_o$ are surface unit normal vectors of worm gears and oversize hob cutters, represented in coordinate systems S_2 and S_o, respectively. Unit normal vector $\mathbf{n}_o$ can be obtained from equation (7) by replacing the subscripts "$_l$" with "$_o$". $[L_{2o}]$ is the surface normal vector transformation matrix, transforming from coordinate system S_o to coordinate system S_2, and it can be obtained by deleting the last column and last row of matrix $[M_{2o}]$ represented in equation (14). Therefore, the surface unit normal vector of the produced worm gear surface is expressed by:

$$n_{x2} = [-E_o \cos\gamma_h \sin\phi_g - D_o \cos\phi_g]\sin(\phi_h - \psi_o) - nz_o \sin\phi_g \sin\gamma_h + [-D_o \cos\gamma_h \sin\phi_g + E_o \cos\phi_g]\cos(\phi_h - \psi_o),$$

$$n_{y2} = [E_o \cos\gamma_h \cos\phi_g - D_o \sin\phi_g]\sin(\phi_h - \psi_o) + nz_o \cos\phi_g \sin\gamma_h + [D_p \cos\gamma_h \cos\phi_g + E_o \sin\phi_g]\cos(\phi_h - \psi_o)$$

and

$$n_{z2} = [-E_o \sin(\phi_h - \psi_o) - D_o \cos(\phi_h - \psi_o)]\sin\gamma_h + nz_o \cos\gamma_h. \quad (17)$$

Parameters D_o and E_o are defined similar to those represented in equation (7) by changing the subscripts "$_l$" with "$_o$". During the cutting process, oversize hob-cutter and worm gear surfaces remain in line contact at every moment. Therefore, the equation of meshing between the oversize hob cutter and worm gear should also be developed. According to equation (8), the equation of meshing can be obtained as follows:

$$\{[D_o(C_o - P_o\psi_o) - A_o nz_o]\sin\gamma_n + E_o S_h \cos\gamma_h\}m_{gh} \sin(\phi_h - \psi_o) + \{[-E_o(C_o - P_o\psi_o) + B_o nz_o]\sin\gamma_h + D_o S_h \cos\gamma_h\}m_{gh} \cos(\phi_h - \psi_o) + (B_o D_o - A_o E_o)(m_{gh} \cos\gamma_h + 1) + S_h nz_o m_{gh} \sin\gamma_h = 0 \quad (18)$$

where symbol m_{gh} is the rotation ratio of the worm gear and oversize hob cutter, and it can be expressed as follows:

$$m_{gh} = \left|\frac{\omega_g}{\omega_h}\right| = \left|\frac{\phi_g}{\phi_h}\right| = \frac{z_h}{z_g} \tag{19}$$

where, ω_g and ω_h represent the angular velocities of the worm gear and oversize hob cutter, respectively. Parameters z_g and z_h are the tooth numbers of the worm gear and oversize hob cutter, respectively.

TOOTH CONTACT ANALYSIS OF WORM GEAR SETS

Tooth contact analysis (TCA) of worm and worm gear can be simulated by considering the meshing mechanism of worm gear sets as shown in Fig. 3. For the proposed assembly mechanism, if the worm rotates counterclockwise about its rotation axis through an angle ϕ_1, the worm gear will rotate clockwise about its rotation axis through an angle ϕ_2. Parameter S_w is the shortest distance between worm axis Z_1 and worm gear axis Z_2, and γ_w is the crossed angle of these two axes.

To perform TCA on the worm gear set, position vectors and unit normal vectors of the mating worm gear set must be expressed in the same reference coordinate system $S_w(X_w, Y_w, Z_w)$ as shown in Fig. 3. In the meshing process, worm and worm gear surfaces are continuously in tangency at any contact point. Therefore, the following criteria must be observed at every contact point: (1) position vectors of worm surface $\mathbf{R}_w^1$ and worm gear surface $\mathbf{R}_w^2$ are equal at every contact point. (2) Unit normal vectors of worm surface $\mathbf{n}_w^1$ and worm gear surface $\mathbf{n}_w^2$ are the same at the contact point. This yields that

$$\mathbf{R}_w^1 = \mathbf{R}_w^2$$
$$\mathbf{n}_w^1 = \mathbf{n}_w^2. \tag{20}$$

Based on the relationship among coordinate systems shown in Fig. 3, the position vector and unit normal vector of the worm surface, represented in coordinate system S_w, can be obtained as follows:

$$x_w^1 = B_1 \cos(\phi_1 - \psi_1) - A_1 \sin(\phi_1 - \psi_1),$$
$$y_w^1 = B_1 \sin(\phi_1 - \psi_1) + A_1 \cos(\phi_1 - \psi_1),$$
$$z_w^1 = C_1 - P_1\psi_1 \tag{21}$$

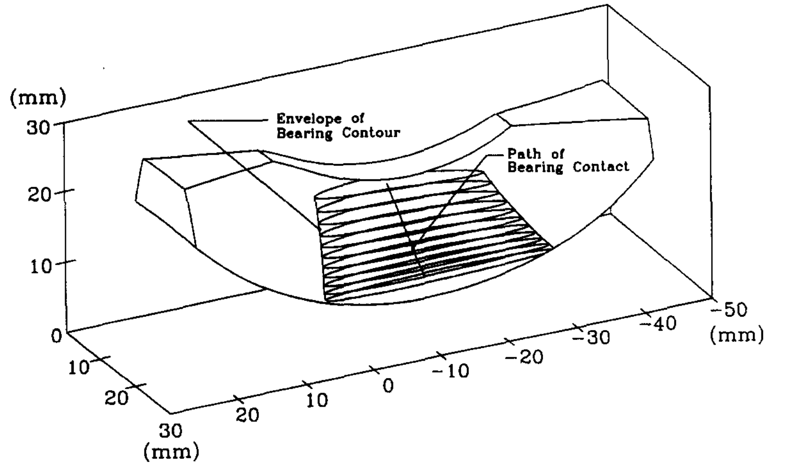

Fig. 5. Envelope and path of bearing contacts with a smaller pitch diameter of 74 mm for oversize hob cutters.

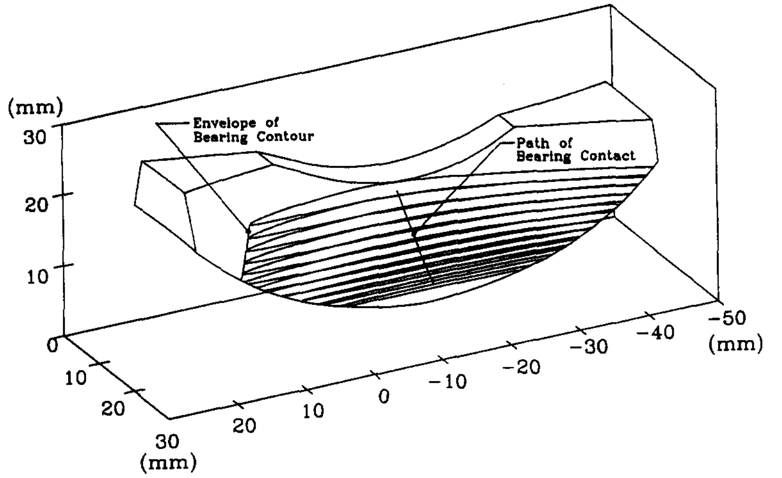

Fig. 6. Enveloped and path of bearing contacts with a smaller pitch diameter of 73.6 m for oversize hob cutters.

and

$$nx_w^1 = E_1 \cos(\phi_1 - \psi_1) - D_1 \sin(\phi_1 - \psi_1),$$
$$ny_w^1 = E_1 \sin(\phi_1 - \psi_1) + D_1 \cos(\phi_1 - \psi_1),$$
$$nz_w^1 = nz_1. \tag{22}$$

Similarly, the position vector and unit normal vector of the worm gear surface, represented in coordinate system $S_w(X_w, Y_w, Z_w)$, are obtained as follows:

$$x_w^2 = x_2 \cos \phi_2 + y_2 \sin \phi_2 - S_w,$$
$$y_w^2 = [-x_2 \sin \phi_2 + y_2 \cos \phi_2]\cos \gamma_w - z_2 \sin \gamma_w,$$
$$z_w^2 = [-x_2 \sin \phi_2 + y_2 \cos \phi_2]\sin \gamma_w + z_2 \cos \gamma_w \tag{23}$$

and

$$nx_w^2 = nx_2 \cos \phi_2 + ny_2 \sin \phi_2,$$
$$ny_w^2 = [-nx_2 \sin \phi_2 + ny_2 \cos \phi_2]\cos \gamma_w - nz_2 \sin \gamma_w,$$
$$nz_w^2 = [-nx_2 \sin \phi_2 + ny_2 \cos \phi_2]\sin \gamma_w + nz_2 \cos \gamma_w. \tag{24}$$

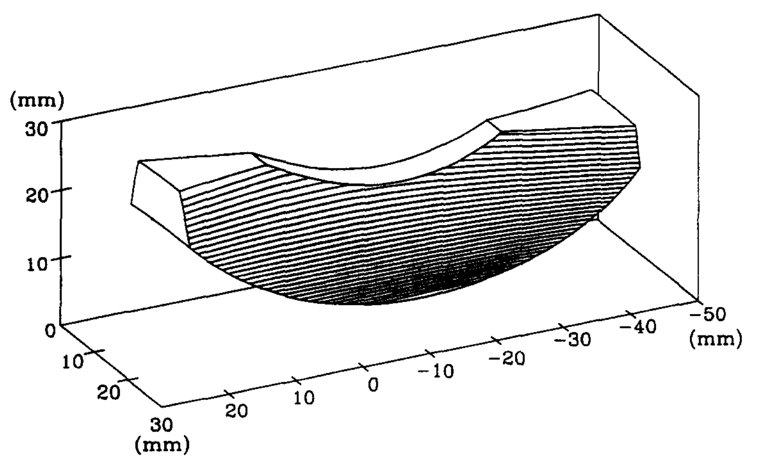

Fig. 7. Contact lines on the worm surface.

Since $|\mathbf{n}_w^1| = |\mathbf{n}_w^2| = 1$, by substituting equations (21)–(24) into equation (20) the following five independent equations can be obtained:

$$B_1 \cos(\phi_1 - \psi_1) - A_1 \sin(\phi_1 - \psi_1) - x_2 \cos\phi_2 - y_2 \sin\phi_2 + S_w = 0, \quad (25)$$

$$B_1 \sin(\phi_1 - \psi_1) + A_1 \cos(\phi_1 - \psi_1) + [x_2 \sin\phi_2 - y_2 \cos\phi_2]\cos\gamma_w + z_2 \sin\gamma_w = 0, \quad (26)$$

$$C_1 - P_1\psi_1 + [x_2 \sin\phi_2 - y_2 \cos\phi_2]\sin\gamma_w - z_2 \cos\gamma_w = 0, \quad (27)$$

$$n_{yw}^1 n_{zw}^2 - n_{zw}^1 n_{yw}^2 = 0 \quad (28)$$

and

$$n_{xw}^1 n_{zw}^2 - n_{zw}^1 n_{xw}^2 = 0. \quad (29)$$

The above system equations contain seven unknowns θ_1, θ_o, ψ_1, ψ_o, ϕ_1, ϕ_2 and ϕ_h. We may consider the worm rotation angle ϕ_1 as an input, and other size unknowns can thus be determined. By solving the system equations (25)–(29) and the equation of meshing (18) simultaneously, coordinates of every instantaneous contact point can be obtained.

SURFACE SEPARATION TOPOLOGY

In the matting process, worm and worm gear surfaces are tangent to each other at every instantaneous contact point. Therefore, a common tangent plane to the matting surfaces can be found at any contact point. In order to measure the separation value between two surfaces, surface coordinates of the worm and worm gear must be transformed to the common tangent plane, and the separation value of two mating surfaces can be measured along the normal direction of the tangent plane. Based on the prescribed separation value, the contour of contact ellipses can be obtained by applying a contouring algorithm.

NUMERICAL EXAMPLE

A worm gear set with the transmission ratio of 1:43 is chosen here for the demonstrative example. The dimension of the worm gear set to be used in this example are as follows: (1) tooth number of worm and hob cutter is 1; (2) module of worm and hob cutter is 7.564 mm/teeth; (3) shaft angle γ_w is 90°; (4) worm pitch diameter $d_1 = 73.470$ mm; (5) pressure angle $\alpha_1 = 20°$; (6) diameter of grinding wheel for worm and hob cutter is 350.00 mm; (7) hob pitch diameter $d_1 = 76.438$ mm. The oversize hob cutter is designed based on the concept that the tooth widths of worm and hob cutter are equal in their normal sections on the pitch cylinder. In order to satisfy this criterion, the setting angle γ_h of the oversize hob cutter must be set larger than that of the worm (i.e. angle γ_w) as shown in Fig. 3(a), and it can be obtained by

$$\gamma_h = \gamma_w + (\beta_1 - \beta_o) = 90°24'1'' \quad (30)$$

where β_1 and β_o are the lead angles of the worm and oversize hob cutter, respectively, and $\beta_1 = 6°6'10''$ and $\beta_o = 5°42'9''$ are chosen in this paper. The theoretical assembly shortest center distance can be obtained from equation (11) by

$$S_w = r_1 + r_2 = 199.383 \text{ mm}$$

$$S_h = r_o + r_2 = 200.867 \text{ mm}. \quad (31)$$

Based on the developed mathematical model of the ZK-type worm gear set and surface separation topology algorithm, a TCA computer simulation program is developed to simulate the envelope and path of bearing contacts, as shown in Figs 4–6. The effects of cutter setting modification and assembly errors can also be simulated by this TCA program. The separation value of the bearing contours shown in Figs 4–6 is chosen as 0.006 mm which is the particle diameter of coating for rolling test experiments. Figure 4 is a three-dimensional diagram of surface separation topology of a worm gear cut by an oversize hob cutter with a pitch diameter of 76.438 mm. Figures 5 and 6 are the surface separation topologies of worm gear cut by hob cutter with pitch diameters of 74 and 73.6 mm, respectively. The area of bearing contacts is increased while the pitch diameters of hob cutters are decreased from 76.438 to 73.600 mm. Bearing contacts become line contacts when the pitch diameter of hob cutters is decreased to 73.47 mm, as shown

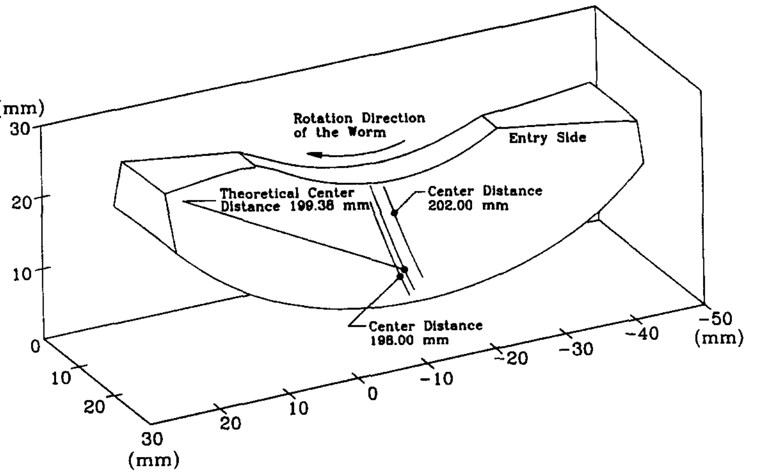

Fig. 8. Contact paths under different assembly shortest distances.

in Fig. 7, which equals to the pitch diameter of the worm. The variations of the contact paths under different assembly shortest distances are shown in Fig. 8. It is found that the contact path has a little bias under the nominal (theoretical) shortest distance of 199.38 mm, and the change of the shortest distance results in the shift of contact path. The shortest assembly-center distance is a design priority rather than a manufacturing procedure. That is if the worm gear set is designed to have a deviation in its assembly-center distance, the same center distance deviation should also be considered between the oversize hob cutter and worm gear during the cutting process.

CONCLUSION

In this paper, a mathematical model of the ZK-type worm gear set has been developed, in which the worm gear is cut by an oversize worm-type hob cutter. If an oversize hob cutter is reground to a smaller pitch diameter, the contact contour of the generated worm gear set will be spread over a thinner and longer contact area, which looks like the path of contact line as shown in Figs 6 and 7. This result matches the fact that when the pitch diameter of the oversize hob cutter is decreased to equal that of the worm, line contact will occur. In this case, the worm gear set becomes a conjugate kinematic pair rather than a pseudo-conjugate gear pair. If a larger center distance is given, the contact path will be shifted upward and move toward the entry side of worm gear surfaces, as shown in Fig. 8. Conversely, if a smaller center distance is given, the contact path will be shifted downward and move toward the exit side of worm gear surfaces. Because the lubrication oil film is much more easily formed if the contact path is located toward the exit side, a smaller assembly center distance is suggested in the designing of a worm gear set.

The effects on the shifting and dimensions of bearing contacts can also be obtained by applying the developed computer programs. The proposed analysis procedures and the developed computer programs are most helpful in designing and analysing of the worm gear set as well as in designing grinding wheels and oversize hob cutters.

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