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Shock waves and avalanches in type-II superconductor

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Abstract. – The rapid penetration of magnetic flux into a Meissner phase of the type-II superconductor is studied analytically and numerically. A sharp shock wave front of the magnetic induction is formed due to the singularity of the resistivity at the transition from the mixed to the normal state. It is shown that current densities at the front reach high values, of the order of the depairing current density. The effects of the heat dissipation and transport on the motion and stability of the interface between the magnetic flux and flux-free domains are considered. The shock wave magnetic induction and the temperature profiles move with constant velocity determined by the Joule heat produced by the electric current in the normal domain at the flux front. The stability of the shock wave solution is investigated. For a sufficiently small thermal-diffusion constant, a finger-shaped avalanche instability appears.

Introduction. – The dynamics of magnetic-flux penetration into a type-II superconductor and its instabilities have been studied by a variety of techniques over the years. Magneto-optics experiments [1] demonstrate that in a wide range of situations there exists a well-defined interface (front) between the magnetic flux penetrating a sample and the flux-free Meissner state. Improvements to the magneto-optical technique have revealed a wide class of instabilities, including a dendritic instability [2, 3]. It is now widely expected that this instability appears in a critical state of type-II superconductors when a small jump in the external magnetic field triggers the avalanches (see [4], and references therein). The instability develops as a result of heat released by the vortices rolling down through the magnetic-induction landscape. It is triggered when the temperature of the sample is locally increased and overpowers the pinning force, releasing a vortex bundle which in turn further heats the sample releasing more bundles. This positive feedback between flux motion and the Joule heat generation leads to an instability of the critical vortex state, and results in the spontaneous branching of propagating flux avalanches. In these theories pinning plays a crucial role, and so the current density at the front is of the order of the critical current J_c. Theories of this kind developed in recent works [5, 6] describe relatively slow processes in which the current density never approaches the depairing current J_d (at which the superconductivity is destroyed) despite the positive feedback.

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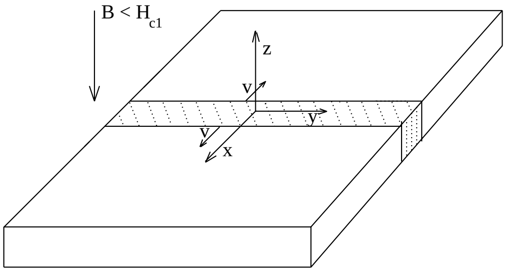

Fig. 1 – Geometry of the problem. The hatched area contains the flux that penetrated the sample during the short initial period when the superconductivity was destroyed at the center ($x = 0$). The arrows indicate the direction of the flux front motion. The direction of the magnetic field B is perpendicular to the xy plane.

In recent experiments, a YBCO superconducting film was rapidly forced out of equilibrium. The superconductivity was destroyed by a femtosecond laser pulse inside a narrow stripe of the sample subjected to a magnetic field perpendicular to the film. The field does not exceed the first critical field H_{c1}, so that initially fluxons cannot penetrate the rest of the sample. Recovery of superconductivity occurs in two stages. After the short pulse has passed, the stripe is cooled and the flux nucleates into a dense system of Abrikosov vortices. On the longer (mesoscopic) time scale the rapidly created vortices are pushed into the superconducting part of the sample sometimes splitting into avalanches [7]. Unlike in the above-mentioned papers, *this instability is formed in the absence of any initial critical state* and therefore apparently is not directly related to pinning. A curious peculiarity of the avalanches is that they exist despite the absence of "mountains".

In the present paper we present a theory of fast flux penetration, and in particular of the creation of the vortex shock wave, and the splitting of its front into avalanches. We predict that the Joule heat released at the flux front can support a constant velocity of the front propagation. The hydrodynamics tangential instability of the flux front destroys the flat front.

Basic equations. – We consider a typical experimental situation, when a relatively thick (with thickness larger than the magnetic penetration depth λ) type-II superconducting film is subjected to a weak external magnetic field ($B < B_{c1}$). The magnetic induction $\boldsymbol{B}$ therefore has only a z component $B_z \equiv B$ and all dependencies on the z coordinate can be neglected. The two-dimensional vortex matter in the hydrodynamics approximation is described by the magnetic induction $B(\boldsymbol{r}, t)$ and the temperature profile $T(\boldsymbol{r}, t)$, where $\boldsymbol{r} = (x, y)$ is a two-dimensional vector (fig. 1). The basic equations are Maxwell's equation and the heat transport equation:

$$\frac{4\pi}{c^2}\frac{\partial B}{\partial t} = \frac{\partial}{\partial x}\left[R\frac{\partial B}{\partial x}\right] + \frac{\partial}{\partial y}\left[R\frac{\partial B}{\partial y}\right]; \qquad C\frac{\partial T}{\partial t} = D\boldsymbol{\nabla}^2 T + \boldsymbol{J}\cdot\boldsymbol{E}(B,T). \tag{1}$$

Here C is the heat capacity and D is the thermal conductivity. In the mixed state the Joule heat is dominated by the motion of the magnetic flux, while in the normal state one has the usual Ohmic resistance losses. As a rule, the nonlinear integral resistivity $R(J, B, T) \equiv E(J, B, T)/J$ in the mixed state of a type-II superconductor is a complicated function of the magnetic field, current and temperature. In this work we will be interested mainly in resistivity at currents much larger than the critical current J_c, when the pinned vortices are released. The vortex resistivity grows quickly above J_c either exponentially or as a power $R \propto J^{\mu}$ with large

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μ. In this relatively low-current regime the dependence on the magnetic induction B is very weak. However, when the current approaches the depairing current J_d, the power μ becomes smaller and the resistivity depends strongly on B. Recently, detailed measurements of the I-V characteristics of Nb films at high current densities of order $10^6\,\mathrm{A/cm^2}$ were performed [8]. Near the depairing current the resistivity has the form

$$R(B,T) = R_n(T)\left(\frac{J}{J^*(T,B)}\right)^{\mu}, \qquad J^*(T,B) = J_d\Delta\left(\frac{B_{c2}(T)}{B}\right)^{\nu/\mu}. \tag{2}$$

Here $R_n(T)$ is the normal-state resistivity and J^* is the depairing current. The upper critical field depends on temperature as $B_{c2}(T) = B_{c2}(0)\Delta$, where we assumed that the dimensionless temperature $\theta = T/T_c$ is not far from 1, so that $\Delta \equiv 1-\theta$ is small. When the current exceeds $J^*(B,T)$, the electric field is continuous, while the resistivity approaches its normal value $R(B,T) = R_n(T)$. Although the resistivity is continuous at the second-order transition to the normal state, its derivative $\frac{\mathrm{d}R}{\mathrm{d}J}$ drops from a finite value to nearly zero in the normal state. Consequently, the nonlinear flux diffusion equation, eq. (1), which contains a derivative $\frac{\mathrm{d}R}{\mathrm{d}x} = \frac{\mathrm{d}R}{\mathrm{d}J}\frac{\mathrm{d}J}{\mathrm{d}x}$, is discontinuous and might therefore have shock wave solutions [9]. In the framework of the time-dependent Ginzburg-Landau equations such solutions were studied by Dorsey [10].

We fitted the I-V curves of Nb and obtained $\mu = 1.5$ with temperature-independent R_n. For Nb at fields of the order of B_{c1} we obtain the best fit $\nu = 1.3$ [8]. The values of other material parameters are: $B_{c2}(0) = 4.43$ T, $R_n = 9.9\,\mu\Omega\cdot\mathrm{cm}$ and $T_c = 8.6\,\mathrm{K}$. These were measured directly. We obtain the best fit for the constant $J_d = 9.2\cdot 10^6\,\mathrm{A/cm^2}$. The power law however holds quite generally. After rescaling, the set of nonlinear coupled equations in the superconducting state reads:

$$\frac{\partial b}{\partial t} = \frac{\partial}{\partial x}\left(\rho\frac{\partial b}{\partial x}\right) + \frac{\partial}{\partial y}\left(\rho\frac{\partial b}{\partial y}\right); \qquad \frac{\partial\theta}{\partial t} = \kappa\boldsymbol{\nabla}^2\theta + \rho j^2, \tag{3}$$

where the dimensionless resistivity and the electric current density are

$$\rho = \frac{R_n(\theta)}{R_n}\left(\frac{b}{\Delta}\right)^{\nu}\left(\frac{j}{\Delta}\right)^{\mu}; \quad j = \sqrt{\left(\frac{\partial b}{\partial x}\right)^2 + \left(\frac{\partial b}{\partial y}\right)^2}. \tag{4}$$

These basic equations are in terms of dimensionless quantities. The dimensionless coordinate, time and magnetic induction are defined using the natural units of length $x^* = cR_n(T = T_c) \equiv cR_n$, magnetic field $B^* = \sqrt{4\pi CT_c}$ and time t^*, $x \to x/x^*$; $t \to t/t^*$; $b = B/B^*$, where

$$t^* = 4\pi R_n\left(\frac{4\pi R_n J_d}{B^*}\right)^{\mu}\left(\frac{B_{c2}(0)}{B^*}\right)^{\nu}; \quad B^* \approx H_{c1}\frac{\lambda^2 k_F v_F}{c\xi}. \tag{5}$$

The flux diffusion equation does not contain parameters, while the heat transfer equation has the dimensionless temperature diffusion constant $\kappa = Dt^*/Cx^{*2}$. In the region in which superconductivity is suppressed by the superconducting current J exceeding the depairing current value $J_d(B,T)$, the normal-state resistivity becomes $R = R_n(T)$. The dimensionless normal-state resistance is defined by $\rho_n(\theta) = R_n(\theta)c^2t^*/4\pi x^{*2}$. In this case the resistivity ρ in eqs. (3) can be replaced by $\rho = \rho_n$.

Structure and evolution of the flux front. – When the boundary conditions are independent of y, the front is straight and the problem becomes one-dimensional. We initially solve a simplified set of equations dropping the diffusion term, so that $\kappa = 0$. This assumption will

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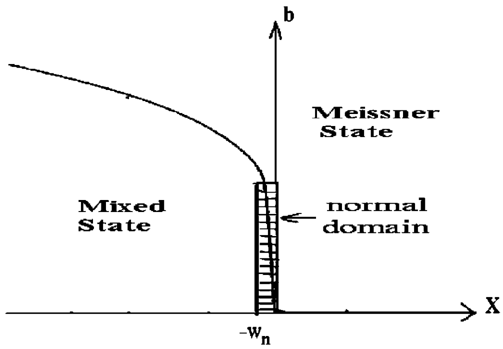

Fig. 2 – Magnetic-induction profile at the front. Three different regions, the mixed, the normal domain and the Meissner state are presented. Here w_n is the width of the normal domain in which the superconductivity is suppressed by excessive current, shown by the hatched area at the front onset.

be supported *a posteriori* by calculating the term's effects and comparing with the numerical solution. The flux front interface consists of two domains, the superconducting domain, where the current density $j < j_d$ (where j_d is the dimensionless depairing current) and the normal domain, where the magnetic-induction profile is so sharp that $j > j_d$. In order to find a solution we have to solve the set of equations (3) in the normal and superconducting areas of the front and afterwards to match them. Looking for a solution in the superconducting area (fig. 2) of the form

$$b_s(X) = A(V)\,|X|^{\alpha}, \qquad \Delta_s(X) = \Delta_{s0} - \Delta_{s1}(V)\,|X|^{\beta}, \tag{6}$$

one obtains

$$\alpha = \frac{\mu+1}{\mu+\nu}; \quad \beta = 2\alpha; \quad A(V) = \Delta_{s0}\alpha^{-\alpha}V^{1/(\nu+\mu)}; \quad \Delta_{s1}(V) = \frac{1}{2}\Delta_{s0}^2\alpha^{-2\alpha}V^{2/(\nu+\mu)}. \tag{7}$$

In the normal domain the equations are simpler, leading to

$$j_n = \frac{\mathrm{d}b_n}{\mathrm{d}X} = j_n\left(X\right) \approx j_{n0}\left(1 - \frac{XV}{\rho_n}\right), \qquad \Delta_n\left(X\right) \approx \Delta_0 + \frac{j_{n0}^2}{V}X. \tag{8}$$

The constants appearing in (7) and (8) are determined by matching the two solutions. The electric current $j_s = \partial b_s/\partial X$ formally diverges as $|X|^{(1-\nu)/(\mu+\nu)}$ at the front for $\nu > 1$. Of course, the divergence is intercepted by the phase transition into the normal state creating the "hot" region of presumably small width w_n (see fig. 2) determined by the condition that the depairing current $j = j_d$ is reached at this point ($X = -w_n$), where

$$j_d = \Delta_{s0}\left(\frac{w_n}{\alpha}\right)^{(1-\nu)/(\mu+\nu)}V^{1/(\nu+\mu)}; \qquad \Delta\left(-w_n\right) = \Delta_0 - \frac{j_d^2 w_n}{V} = \Delta_{s0}\,;$$
$$\Delta'\left(-w_n\right) = \frac{j_d^2}{V} = \beta\Delta_{s1}(V)w_n^{\beta-1}. \tag{9}$$

The front velocity V has to be obtained from the equation $V^2 - V^{(2\mu+\nu)/(\nu+\mu)}j_d/\Delta_0 - \alpha j_d^2/\Delta_0 = 0$, which has a solution for any critical exponents μ, ν. In the special cases $\mu = 0$ and $\mu \approx \nu$, it can be solved explicitly; however, an approximate solution for $j_d/\Delta_0 \gg 1$ is readily found to be $V \simeq (j_d/\Delta_0)^{1+\mu/\nu}$. Using this value, we obtain the width of the normal domain at the front $w_n \approx \alpha(\Delta_0/j_d)^{1+\mu/\nu}$.

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Stability analysis. – If the normal resistance of the sample material is temperature dependent, then the essential dependence of the front velocity on the Joule heat released near the interface leads to the instability of the straight front. Keeping the normal resistivity in the form $\rho_n = \rho_0 + \rho_1 \theta(x,t)$, we look for a solution of the corrugated front in the normal domain as $b = b_n(x-Vt)+\eta(x)\exp[\Omega t+k_y y]$; $\theta = \theta_n(x-Vt)+\zeta(X)\exp[\Omega t+k_y y]$. The leading-order solutions β_n and θ_n for the set of basic equations, eqs. (3), for $\rho_1 = 0$ were obtained above, while corrections to the first order in ρ_1 will not be required in the stability analysis. The stability matrix has one stable $\Omega_1 = -\rho_0(k_X^2 + k_y^2)$ and one marginal $\Omega_2 = 0$ eigenvalue. Strictly speaking, the marginal eigenvalue Ω_2 calls for investigation beyond the linear stability analysis. However, it is stable and, in any case, addition of the ρ_1-term to the resistivity removes the marginality and the degeneracy. To find the corrected eigenvalue Ω_2, one has to diagonalize on the corresponding subspace the operator $\Omega_2 = \rho_1 j_d^2$, which demonstrates the instability for any wave vector.

In fact, it is the well-known hydrodynamics tangential instability of the flux front which is responsible for the front decomposition. Indeed, in this case warmer segments of the front move faster and might destroy the flat front line.

It should be noted that till now we have considered the $\kappa = 0$ case only. In this case, the normal domain in the front shows instability with respect to small temperature fluctuations with arbitrary wave vectors. Dispersion appears for the case of a non-zero heat diffusion coefficient. In fact, however, these small fluctuations cannot destroy the straight line front. It becomes unstable due to large-amplitude fluctuations. Let us consider the evolution of the instability. Due to diffusion along the flux front interface, the instability develops under the condition $ut_0 > \sqrt{\kappa t_0}$, where $t_0 \simeq 1/(\rho_1 j_d^2)$ is the characteristic time of the instability evolution. This requirement allows us to determine the critical velocity of the fluctuation when the avalanche is developed: $u > u_c = j_d\sqrt{\kappa\rho_1}$. In metals and alloys the normal-state resistivity practically does not depend on temperature in the relevant temperature range. This means that $\rho_1 = 0$ and, consequently, no instability is expected. The threshold in the fluctuation velocity u_c (which is proportional to the Joule heat released in the front) means that only a large temperature fluctuation can provide essential Joule heat to destroy the front.

In order to study the development of instability for arbitrary κ, the set of equations (3) has been solved numerically. The resistivity in the normal domain at the front was chosen in the following model form: $\rho_n(\theta) = \rho_0\{1+\alpha[\theta(X,y,t)-\theta_0]\}$, where the initial temperature is perturbed in the area $\theta(X,y,t=0) = 0.88$ for $4 < y < 5$, $0 < X < 1$ while $\theta(y,t=0) = \theta_0 = 0.7$ for $y \notin [4,5]$, $X \notin [0,5]$. We chose $\alpha = 14.5$, $\kappa = 0.05$ and 2.5. The results are presented in figs. 3, 4 where the profile of the magnetic induction and avalanches obtained numerically support our theoretical predictions.

Discussion. – To summarize, we considered the formation, stability and evolution of an unstable normal domain forced onto a type-II superconductor subjected to weak magnetic field. On the mesoscopic time scale, when dissipation controls the dynamics, a sharp flux front is formed. Strong screening currents significantly exceeding the critical current J_c flow in the mixed state. For such strong currents the vortex matter resistivity R has the form $R \propto B^\nu J^\mu$. We predict that when $\nu > 1$ both the moving flux and the temperature profile form sharp singular shock waves. Strong screening currents in the vortex matter approaching the depairing current J_d cause destruction of superconductivity. An area of material adjacent to the interface between the Meissner state and the mixed state of the size (returning to dimensional units) $W_n = \frac{cB^*(1-T_0/T_c)}{4\pi\nu J_d}$ becomes normal. Here $B^* = \sqrt{4\pi C T_c}$, C is the heat capacity and T_0 is the temperature of the cool superconductor. The stable superconductor-normal interface is formed due to the combined effect of the nonlinear magnetic-flux dynamics and thermal effects. The condition $\nu > 1$ is independent of μ and has the following physical

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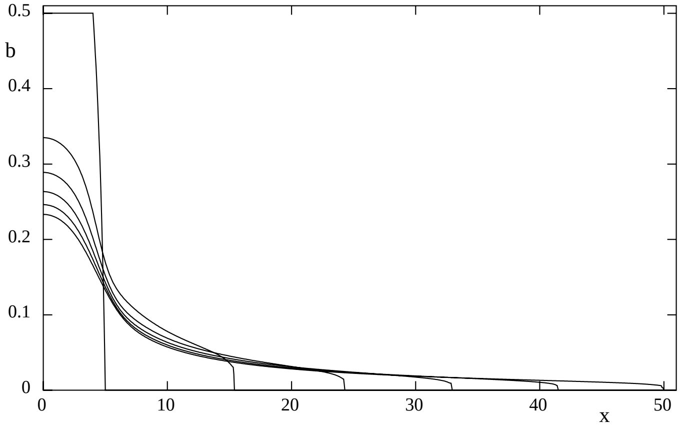

Fig. 3 – The evolution of the magnetic induction for the flux conserving boundary condition. The value of the Joule heat released in the normal domain. Ξ_n was kept fixed at $\Xi_n = 0.5$.

meaning. It is well known that above the critical current the resistivity is proportional to the number of vortices (the flux-flow Bardeen-Stephen formula), $R \propto B$ ($\nu = 1$). The condition for the formation of the normal domain is therefore that the dependence of the flux-flow resistivity on magnetic induction at currents close to the depairing currents is stronger than linear. For Nb, this happens at least for small fields.

The interface moves with constant velocity, which is completely determined by the Joule heat released in the normal domain at the front and hence on the normal resistivity of the sample. The flux front velocity for $\mu = 0$ (in dimensional units) is $V = \frac{cR_n J_d}{(1-T_0/T_c)B^*}(\frac{B^*}{B_{c2}(0)})^\nu$. Taking, for example, material parameters of the optimally doped YBCO, $J_d = 10^8\,\mathrm{A/cm^2}$, $R_n = 2 \cdot 10^{-6}\,\Omega \cdot \mathrm{cm}$, $C = 1\,\mathrm{J/cm^3K}$, one obtains for the flux front velocity $V \approx 10^5\,\mathrm{cm/s}$, which is in a good agreement with experimental data [11]. Note, however, that the value

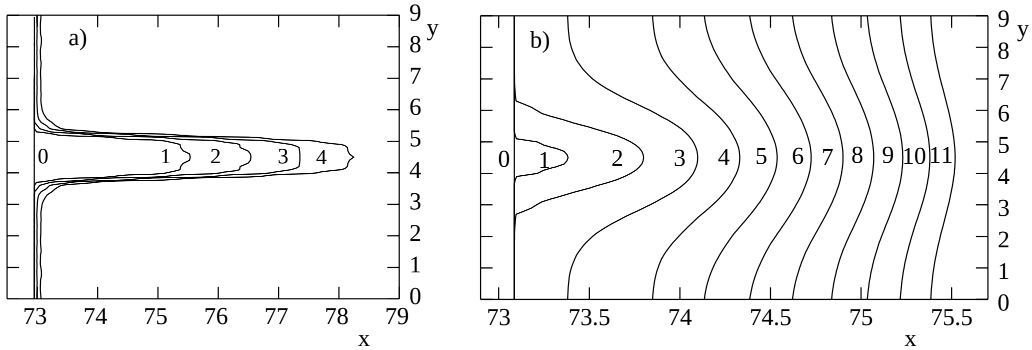

Fig. 4 – Evolution of the magnetic flux front pattern for different values of the heat diffusion constant. The perturbation is triggered by the initial temperature inhomogeneity at the front area. a) Small heat diffusion constant $\kappa = 0.05$. Development of the avalanche instability. Five snapshots (intervals of $\Delta t = 0.05\ t^*$) of a finger-shaped instability in magnetic induction are shown from left to right. b) Large heat diffusion constant $\kappa = 2.5$. Evolution of magnetic-flux pattern. Twelve snapshots (intervals of $\Delta t = 0.125\ t^*$) show that the initially developed small fluctuation dissipates away.

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strongly depends on the exponents μ and ν. The width of the normal stripe is $0.5\,\mu$m.

The physical reason for the avalanche instability is very similar to a well-known hydrodynamic instability, when different layers of the liquid move with different and parallel velocities. In fact it is the positive feedback between excessive local temperature at the front and Joule heat released there that is responsible for avalanches. The instability develops for the avalanche with velocities exceeding the critical value $U > U_c = \frac{J_d}{C}\sqrt{D\frac{\mathrm{d}R_n}{\mathrm{d}T}|_{T_c}}$, where D is the heat diffusion constant. Taking $D = 30\,\mathrm{J/(cm\,s\,K)}$, one estimates the avalanches velocity as $2.6 \cdot 10^5$ cm/s which is in a good agreement with those observed experimentally.

* * *

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