GENERALIZED I_C V. S. V_{BE} CHARACTERISTICS OF BIPOLAR TRANSISTORS

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Abstract :

A generalized Ic V.S. V_{BE} Characteristics is derived for bipolar transistor (npn or pnp), which includes all the levels of carrier injection at the emitter and the impurity profiles in the base region. This generalized expression can be reduced to the conventional forms of collcctor current density for both low and high level current injections with arbitrary doping profiles in the base region. The degradations of minority carrier diffusivity (or mobility) due to impurity concentration and high current level injection are also considered. Comparisons of the experimental data and the theoretical results are made. Discussions and applications of the theoretical results are also given.

I INTRODUCTION:

The transport properties of carriers in bipolar transistors had been extensively studied in the previous years. The formulations of transport parameters in bipolar transistors operated in different regions were derived approximately by Ebers-Moll[1], this so called Ebers-Moll model has been the major large-signal model for bipolar transistors. In recent years, the achievements of device technology in the fabrication of bipolar transistors, have made it possible to accurately control the fabricating technique to meet various operating requirements, and to have reproducible characteristics. It was found that there were several important effects [2 3 4 5 6 7] which existed in bipolar transistors, that were beyond the explanations of simplified Ebers-Moll model. Recently, the charge control model has been derived and discussed by several authors[8, 9, 10], it is believed that this simple and accurate charge control model is more suitable for computer network analysis than the Ebers-Moll model. These authors, however, did not give a detailed quantitative description for the effects of the impurity profiles in the base region and the various current injection levels. In this paper, the generalized charge control expressions will be derived, which include the effects of doping profile in the base region, the emitter current injection levels, and the degradations of carrier mobility (or diffusivity)[11] due to impurity concentration and current injection levels. I_C *V.S.* V_{BE} Characteristics of the fabricated bipolar transistors are designed and measured to compare with the theoretical calculations. It is shown that the degradation of carrier mobility (diffusivity) due to current injection level is very important for high current injection. The transition voltage of low current level injection to high current level injection is also derived in term of base doping concentration, which can be used to predict the base doping concentration and the fluctuations of the base doping redistribution after subsequent emitter diffusion and oxidation processes.

(II) THE DERIVATIONS OF CHARGE CONTROL EXPRESSIONS.

The one-dimensional current flow equations of electrons and holes in he semiconductor can be written as

$$j_n = q\mu_n(x)n(x)\mathrm{E}x + qD_n(x)\frac{\partial n(x)}{\partial x} \quad \cdots\cdots (1)$$

$$j_p = q\mu_p(x)p(x)\mathrm{E}x - qD_p(x)\frac{\partial \mathrm{p}(x)}{\partial x} \quad \cdots\cdots (2)$$

where j_p, j_n, $p(x)$, $n(x)$, $\mu_p(x)$, $\mu_n(x)$, $D_p(x)$, $D_n(x)$ are the conventional notations of current density, carrier concentration, mobility, diffusivity for hole and electron, respectively.

For *n-p-n* transistor, electrons are the minority carriers in the base region. The exact solution of the first order differential equation *Eq.* (1) can be written as

$$n(x)=n(x')\ \exp\ (-\int_{x'}^{x} \frac{\mu_n\ (x)}{D_n(x)}\ \mathrm{E}x dx) + \frac{1}{q}\int_{x'}^{x} \frac{j_n}{D_n\ (x'')}\ \exp\ (-\int_{x''}^{x} \frac{\mu_n\ (x''')}{D_n(x''')}\ \mathrm{E}x dx''')\, dx'' \quad \cdots\cdots (3)$$

where x' and x are the arbitrary integration limits. If we set $x'=0$ to be at the edge of emitter junction of the base region and $x=w(V_{CB})$ to be at the edge of the collector junction of the base region. The boundary conditions at the emitter edge and the collector edge of the base region, are $n(0)$ and $n(w)$ (where the effective base width w is a function of the collector-base voltage), respectively. Then, *Eq.* (3) can be written as

$$n(w)=n(0)\exp(-\int_{0}^{w} \frac{\mu_n\ (x)}{D_n(x)}\ \mathrm{E}x dx) + \frac{1}{q}\int_{0}^{w} \frac{j_n}{D_n(x'')}\exp(-\int_{x''}^{x} \frac{\mu_n(x''')}{D_n(x'')}\mathrm{E}x\, dx'')\, dx'' \quad \cdots\cdots (4)$$

Eq.(4) is an exact solution in which the effects of diffusion and drift motion of electrons in the base region are included. For normal transistor fabrications, the emitter is heavily doped for the consideration of high emitter injection efficiency, and the majority carrier current is negligible compared to the minority carrier current at every points in the base region i.e., $j_n \gg j_p \simeq 0$ (for *n-p-n* transistor.). So *Eq.* (2) can be written as

$$j_p \simeq 0 = q\mu_p(x)p(x)\mathrm{E}x - qD_p\ (x)\ \frac{\partial p(x)}{\partial x} \quad \cdots\cdots (5)$$

Hence, the built-in electric field in the base region is

$$\mathrm{E}x = \frac{D_p(x)}{\mu_p\ (x)}\ \frac{1}{p\ (x)}\ \frac{\partial p(x)}{\partial x} \quad \cdots\cdots (6)$$

Substitution *Eq.* (6) into *Eq.* (4), and let

$$m = \frac{\mu_n(x)D_p(x)}{\mu_p(x)D_n(x)} \quad \cdots\cdots (7)$$

We have

$$n(w)=n(0)\exp(-\int_{0}^{w} m\ \frac{dp(x)}{p(x)})$$

$$+\frac{1}{q}\int_{0}^{w}\frac{j_n}{D_n(x'')}\ \exp(-\int_{x''}^{w} m\ \frac{dp(x')}{p\ (x')}\ dx')\, dx'' \quad \cdots\cdots (8)$$

In general, m may be the functions of position and injection levels, however, it is still a good approximation to assume m to be a constant when this parameter is slowly varying function of position x i.e., the injection current and the impurity profile does not change rapidly within the length of carrier mean free path in the base region. Under this approximation *Eq.* (8) can be simplified as

$$n(w)p^m(w)=n(0)p^m(0)+\frac{1}{q}\int_0^w \frac{j_n(x)}{D_n(x)}\,p^m(x)\,dx \cdots\cdots\cdots\cdots \quad (8)$$

In modern transistor fabrication, the recombination in the short base region has only a small effect on the junction voltage dependence of the collector current passing from emitter to collector. Hence, the recombination current can be assumed to be negligible in the base region i.e., $j_n=j_c=$constant. The current density can be written as

$$j_c=-\frac{q[n(0)p^m(0)-n(w)p^m(w)]}{\int_0^w \frac{p^m(x)}{D_n(x)}\,dx} \cdots\cdots\cdots\cdots \quad (9)$$

From the quasi charge neutrality condition in the base region, the majority carrier (hole) concentration $p(x)$ in the base region should be equal to the sum of impurity concentration $N(x)$ and the injected minority carrier (electron) concentration in the base region, (i.e)

$$p(x)=N(x)+n(x) \cdots\cdots\cdots\cdots \quad (10)$$

Putting *Eq.* (10) into *Eq.* (9), we have

$$j_c=\frac{-q[n(0)(N(0)+n(0))^m-n(w)(N(w)+n(w))^m]}{\int_0^w \frac{[N(x)+n(x)]^m}{D_n(x)}\,dx} \cdots\cdots\cdots\cdots \quad (11)$$

Equation (11) is a generalized expression of charge control equation. The electron current density (or the collector current density) is written in terms of boundary conditions of the base region, in which the effects of impurity profiles in the base region and the carrier injection levels are all included. In the case of low electric field in the base region, the electron and hole can be assumed to be approximately quasi-thermal equilibrium with the host lattice, and the Einstein's relation can be assumed to be valid i.e., $m=1$, then Eq. (11) becomes

$$j_c=-\frac{q[n(0)(N(0)+n(0))-n(w)(N(w)+n(w))]}{\langle D_n^{-1}(x)\rangle[\int_0^w [N(x)+n(x)]dx]} \cdots\cdots\cdots\cdots \quad (12)$$

where

$$\langle D_n^{-1}(x)\rangle=\frac{\int_0^w D_n^{-1}(x)[N(x)+n(x)]dx}{\int_0^w [N(x)+n(x)]dx} \cdots\cdots\cdots\cdots \quad (13)$$

The average minority carrier transit time across the base region is defined as

$$t_B=\int_0^w \frac{dx}{v(x)} \cdots\cdots\cdots\cdots \quad (14)$$

where $v(x)$ is the average electron velocity at position x, this average velocity can be related to the current density j_n by the relation,

$$j_n = +qn(x)v(x) \quad \cdots\cdots (15)$$

Then, the transit time of Eq. (14) may be written as

$$t_B = \frac{Q_n}{|j_c|} \quad \cdots\cdots (16)$$

Where $Q_n = q\int_0^w n(x)dx$ is the total minority carrier charge density injected from the emitter-base junction into the base region. Eq. (12) and Eq. (16) are the charge control relations for the bipolar transistor (n-p-n). Similar expressions may also be developed for p-n-p transistor.

It should be noted that the assumption of $m=1$ is no necessarily true when the electric field in the base region is high. We will discuss it later on.

III EFFECTS OF INJECTION LEVELS AND DOPING PROFILES

From Eq. (12) and Eq. (16), there are still four unknown variables i.e., $n(0)$, $n(w)$ and $N(x)$, which are required to determine. If we assume that the quasi fermi levels are constant in the depletion region of the emitter-base and base-collector junction edges, the electron density at the emitter and the collector edges of the base region can be calculated by using the relationships of the charge neutrality equation in Eq. (10) and the mass action law which relates the applied junction voltage to the carrier concentration at emitter and collector junctions. For emitter junction,

$$n(0)p(0) = n_i^2 \exp\left(\frac{qV_{EB}}{k_BT}\right) \quad \cdots\cdots (17)$$

For collector junction,

$$n(w)p(w) = n_i^2 \exp\left(\frac{qV_{CB}}{k_BT}\right) \quad \cdots\cdots (18)$$

Hence, the relationships for $n(0)$ and $n(w)$ can be written as

$$n(0) = \frac{(n_i^2/N(0))\exp\left(\frac{qV_{BE}}{k_BT}\right)}{\frac{1}{2}+\frac{1}{2}\sqrt{1+\left(\frac{2n_i}{N(0)}\right)^2\exp\left(\frac{qV_{EB}}{k_BT}\right)}} \quad \cdots\cdots (19)$$

$$n(w) = \frac{(n_i^2/N(0))\exp\left(\frac{qV_{CB}}{k_BT}\right)}{\frac{1}{2}+\frac{1}{2}\sqrt{1+\left(\frac{2n_i}{N(w)}\right)^2\exp\left(\frac{qV_{CB}}{k_BT}\right)}} \quad \cdots\cdots (20)$$

Putting Eq. (17)-Eq. (20) into Eq. (12), we get

$$j_c = -\frac{q\langle D_n^{-1}(x)\rangle^{-1} n_i[^2\exp\left(\frac{qV_{EB}}{k_BT}\right) - \exp\left(\frac{qV_{CB}}{k_BT}\right)]}{\int_0^w N(x)\,dx + \frac{Q_n}{q}} \quad \cdots\cdots (21)$$

If we assume that the injected electron distribution in the short base region (i.e., w is much smaller than the minority carrier diffusion length L_n) can be approximately written as

$$Q_n \simeq \frac{qw}{2}\,(n(0)+n(w)) \quad \cdots\cdots (22)$$

Eq. (22) is equivalent to the assumption of the linear electron distribution in the base region, and is a good approximation only at high current injection levels. But, it is quite clear that Q_n in Eq. (21) is dominant at high injection levels, and is only a second order term which is negligible small compared with the total impurity concentration at low current injection levels. So the linear electron distribution can accurately describe the I-V characteristics of bipolar transistor with different doping profiles and current injection levels quantitatively. From Eq. (19), Eq. (20), the low current and the high current injection limits and the crossover are immediately visible. For example the low current injection level is under the conditions,

$$\frac{2n_i}{N(0)}\exp\left(\frac{qV_{EB}}{2k_BT}\right) \ll 1 \quad \cdots\cdots (23)$$

$$\frac{2n_i}{N(w)}\exp\left(\frac{qV_{CB}}{2k_BT}\right) \ll 1 \quad \text{(where } V_{CB} \text{ is negative sign for reverse biasing)}$$

Then, the collector current density becomes

$$(j_c)_L = -\frac{q\langle D_n^{-1}(x)\rangle_L^{-1}\, n_i^2\left(\exp\left(\frac{qV_{EB}}{k_BT}\right)-\exp\frac{qV_{CB}}{k_BT}\right)}{\int_0^w N(x)\,dx} \quad \cdots\cdots (24)$$

$$\text{where} \langle D_n^{-1}(x)\rangle_L^{-1} = \frac{\int_0^w D_n^{-1}(x)\,N(x)\,dx}{\int_0^w N(x)\,dx} \quad \cdots\cdots (25)$$

The index L represents the low current level injection, the current density $(j_C)_L$ can be easily computed, as the doping profile is known. For high current injection level, the conditions are

$$\left.\begin{aligned} &\frac{2n_i}{N(0)}\exp\left(\frac{qV_{EB}}{2k_BT}\right) \gg 1 \\ &\frac{2n_i}{N(w)}\exp\left(\frac{qV_{CB}}{2k_BT}\right) \ll 1 \end{aligned}\right. \quad (26)$$

Then, the collector current density under these conditions becomes

$$(j_c)_H = -\frac{2qn_i\langle D_n^{-1}(x)\rangle_H^{-1}}{W}\left(\exp\left(\frac{qV_{EB}}{2k_BT}\right)-\exp\left(\frac{qV_{CB}}{2k_BT}\right)\right) \quad \cdots\cdots (27)$$

$$\text{where} \langle D_n^{-1}(x)\rangle_H^{-1} = \frac{\int_0^w D_n^{-1}(x)\,n(x)\,dx}{\int_0^w n(x)\,dx} \quad \cdots\cdots (28)$$

It is clear that the collector current density will change its slope from the low current injection level to the high current injection level, and the impurity profile becomes unimportant at high current injection levels. For the crossover point of the transition from the low current injection level to the high current injection level, is just $N(0)=2n_i \exp\left(\frac{qV_{EB}}{2k_BT}\right)$, which depends on the base doping concentration at the emitter side of the base region. Hence, the measurements of the crossover points can be used to predict the base doping concentration at the emitter side of the base region, and further give the base impurity profile of a certain specific fabrication process. From Eq. (24) and Eq. (27), the generalized collector current density is shown to reduce to the conventional forms of the collector current density for both low and high current injection, respectively, if the electron diffusivity is constant throughout the base region,

It has been shown that the mobility (or diffusivity) is a function of doping concentration in the base region for both majority carrier and minority carrier. The origin of this dependence comes from the carrier scattering within the base doping impurities. The various scattering mechanisms should be considered in determining the form of D_n or μ_n in Eq. (21) are (1) lattice scattering, (2) scattering by charge impurities, (3) carrier-carrier scattering. These are combined in Eq. (29) to give the total diffusivity for electrons, the first two contributing the low Injection and the last one being responsible for the dependence of the diffusivity on electron concentration. Thus,

$$D_n^{-1} = D_{nl}^{-1} + D_{nh}^{-1} \qquad (29)$$

where D_{nl} is the low level diffusivity for electron, and D_{nh} represents the effect of carrier-carrier scattering. For the low level diffusivity of electron the diffusivity can be written as the following empirical formula at $T=300°K$[12], in MKS units,

$$D_{nl}^{-1} = 275.78\left(1+\frac{350}{N(x)+1.05\times10^{19}}\right) \qquad (30)$$

Carrier-carrier scattering has been considered by Fletcher[13] and Howard and Johnson[14]. According to the latter authors, the diffusivity has the following relationship for D_{nh}, in MKS unit

$$D_{nh} = \frac{12\pi\varepsilon_s^2\left[\frac{\pi}{2q^3}\left(\frac{m_e+m_h}{m_e+m_h}\right)\right]^{\frac{1}{2}}\left(\frac{k_BT}{q}\right)^{5/2}}{n\ell_n\left[\frac{48\pi^2\left(\frac{k_BT}{q}\right)^2}{h^2n}\left(\frac{m_em_h}{m_e+m_h}\right)\right]} \qquad (31)$$

Where m_e, m_e are the effective masses of electrons and holes, respectively; h is the Planck's constant, and ε_s is the dielectric constant of the semiconductor. The average of inverse diffusivity in Eq. (21) can be computed as long as the impurity profile in the base region and the injected carrier concentration are known.

IV. NUMERICAL RESULTS AND EXPERIMENTAL DATA

Numerical results of Eq. (21) are computed for both D_{ne} (without considering the carrier-carrier scattering) and D_n (both impurity scattering

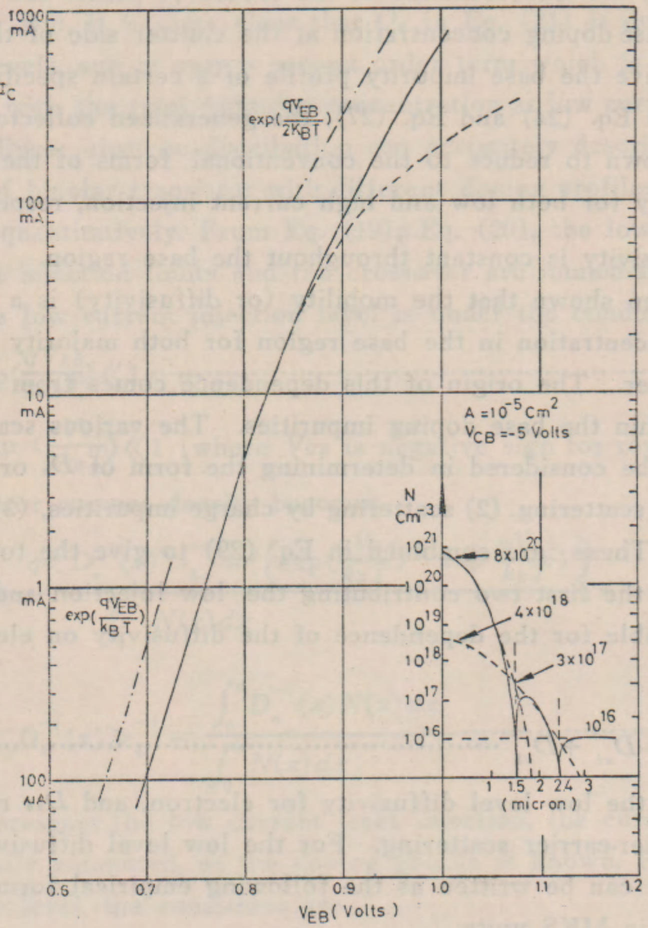

Fig. 1 Theoretical Calculations of I_c V, S, V_{BE}, ——— without carrier-carrier scattering effect, — — —with carrier-carrier scattering effect.

and carrier scattering), and are shown in Fig. (1) for comparisons. The impurity profile of double-diffused transistor is also shown at the bottom right corner for reference. Figure (2) shows that the diffusivity is quite strong

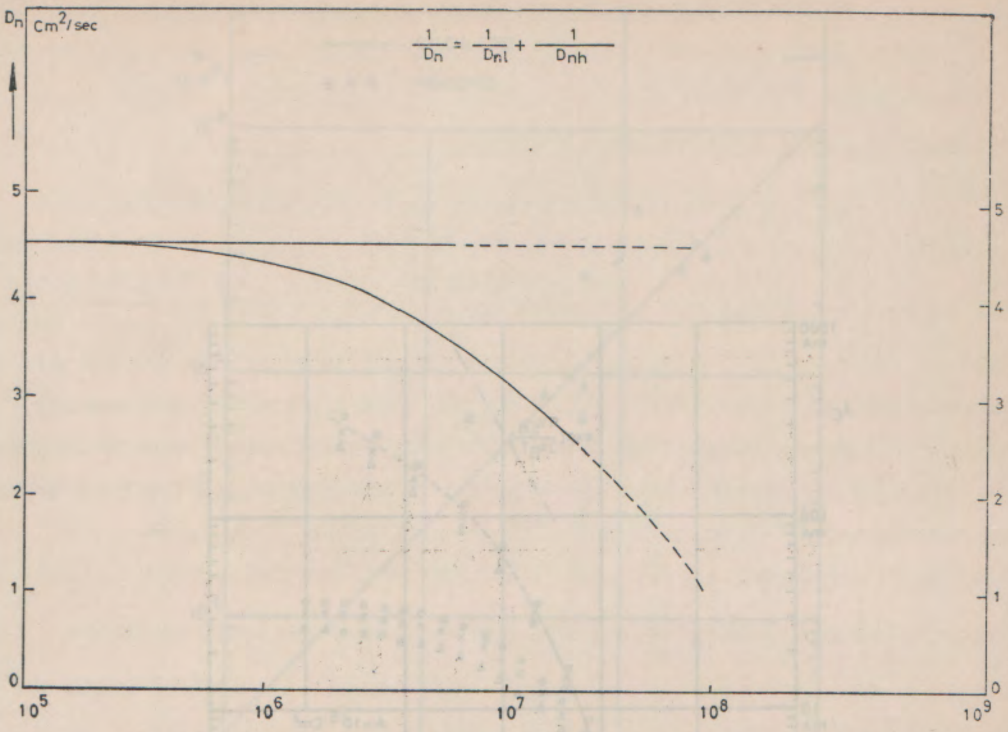

Fig 2, Effects of the injected current on the electron diffusivity.

function of the current injection level i.e., the diffusivity decreases as the current injection increases. In order to compare the theoretical results with the experimental data, the planar technologies were used to fabricate the double-diifused bipolar transistor. The staring material is $n/n+$ silicon wafer with n-expitaxial layer thickness of 6 $\mu \pm 15\%$ and sheet resistivity of 0.6 Ω-cm$\pm 15\%$ is used. The standard planar transistor processes are employed which consists of base and emitter diffusion, preohmic contact, and metalization. The sheet resistance of base diffusion is measured by the four point probe method. The junction depth is estimated by using the angle lapping and staining techniques. The I_C *V.S.* V_{EB} characteristic curves are obtained by using the Tektronics 576 curve tracer with pulse signal applied to the base terminal. These curves are then plotted in semi-logarithm papers, and the crossover voltages (or Knee voltages) are obtained for different kinds of base doping diffusion. Fig. (3) and Fig. (4) show the comparisons of

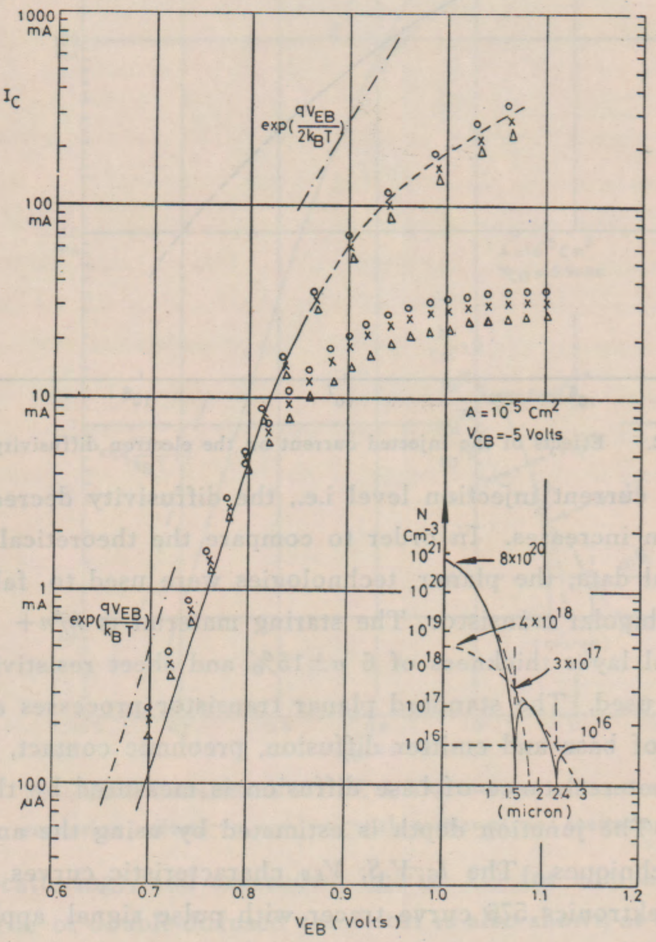

Fig 3, **Comparisons of theoretical result with the experimental data (with and without series resistance correction)**

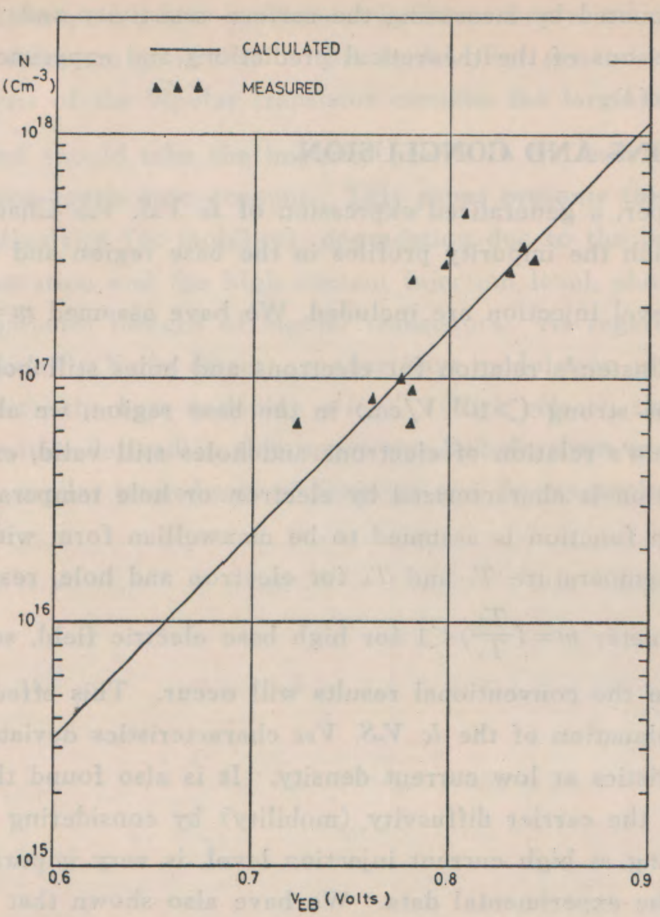

Fig 4. Comparisons of the measured crossover voltage and doping concentration at the emitter side of the base region with the theoretical prediction.

the experimental data and the theoretical predictions. Fig. (3) shows the typical fluctuations of the characteristic curve with the same sheet resistivity control of the base diffusion but with the different run. From this figure, the experimental data points are quite different from the theoretical calculations with considering the degradation of diffusivity at the higher currents. It should be noted that the collector current of theoretical curve is calculated by assuming V_{EB} to be the emitter-base junction voltages, but the experimental data are measured with V_{EB} to be the emitter-base termina voltages. Hence, the experimental data should be corrected by taking the series resistance of emitter and base contacts into account, especially, at high current levels. The corrected experimental curves are also shown in Fig. (3), in which the emitter-base terminal voltages are deduced by voltage drops in the emitter lead resistance of 80 milliohms. The crossover voltages are also deduced from the measured characteristic curves (without correction), and the doping concentration at the emitter edge of the base region are estimated by measuring the surface resistivity and junction depth. Comparisons of the theoretical predictions and experimental data are shown in Fig. (4).

V DISCUSSIONS AND CONCLUSION.

In this paper, a generalized expression of I_C *V.S.* V_{BE} Characteristics is derived, in which the impurity profiles in the base region and the effect of high current level injection are included. We have assumed $m=\frac{\mu_n(x)D_p(x)}{\mu_p(x)D_n(x)}=1$, i.e., the Einstein's relation for electrons and holes still holds. If the electric field is strong ($>10^3$ V/cm) in the base region, we also assume that the Einstein's relation of electrons and holes still valid, except that the Einstein's relation is characterized by electron or hole temperature (i. e., the distribution function is assumed to be maxwellian form with characteristic temperature T_e and T_h for electron and hole, respectively.), then, the parameter $m=(\frac{T_h}{T_e})<1$ for high base electric field, so some deviations from the conventional results will occur. This effect might be the partial explanation of the I_C *V.S.* V_{BE} characteristics deviated from the ideal characteristics at low current density. It is also found that the degradation of the carrier diffusvity (mobility) by considering the carrier-carrier scattering at high current injection level, is very important factor to interprete the experimental data. We have also shown that the crossover voltage measurements can be used to estimate the base doping concentration at the emitter edge of the base region. Hence the direct measurements of the I_C *V.S.* V_{BE} can be used to check the base doping fluctuations of a certain fabrication processes. It should be noted that the onset of the high current injection depends purely on the doping concentration in the base region, i.e., the lower of the base doping concentration will give the lower of the crossover voltage and more easier of the diffusivity degradation. It is also known that the current gain and cutoff frequency will fall off at high currents possibly due to the base widening (Kirk's effect) and space charge limited current operation mode.[15] In general, the transit time across the base region is proportional to the square of the base width, and is inversely proportional to the diffusivity of carrier. If the carrier diffusivity is degradated at high current, then the transit time will increase and cut-off frequency will decrease, except the effects of the base widening and the space charge limited current. This paper is mainly concerned with a one-dimensional model. Although lateral current effects take place in actual transistor, but the experimental samples are different from the bipolar transistor within I.C. (in which lateral current flow path are quite important)

Nevertheless, the quantitatiue agreement between theoretical results and experimental data, are good test of this model. For general computer network analysis of the bipolar transistor circuits, the large signal modelling is required, and should take the impurity profile in the base region and the current injection levels into account. This paper presents the important features of diffusivity (or mobility) degradation due to the base impurity doping concentration and the high current injection level, should have some aids to the computer designs of bipolar transistors. As regard to the base width modulation[2] (Early effect), conductivity modulation in base region[3] (Webster effect), the base widening effect[5] (Kirk effect), the charge control expressions derived in this paper can include them easily by following the similar procedures of Gummel and Poon's analysis[9].

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