

SYNTHESIS OF TRANSFER FUNCTION USING ACTIVE RC CIRCUITS

Teng Ching-cheng

(鄧清政)

College of Engineering
National Chiao Tung University

(Received 6 January 1975)

Abstract—A simple active RC circuit is given to synthesize the low-pass type of second order transfer function. The damping ratio is easily adjusted without changing the value of undamped natural frequency. An experimental example is given to illustrate the simple design procedure.

I. INTRODUCTION

Frequently, the performance characteristics of a feedback control system are specified in terms of the transient response to a unit-step input since it is easy to generate and is sufficiently drastic (1). The transfer function used for describing the transient response is always of the lowpass type of second order. i.e. $c(s)/R(s) = \omega_n^2 / (s^2 + 2\xi\omega_n s + \omega_n^2)$, where ξ is the damping ratio, ω_n is the undamped natural frequency. In feedback control systems, we are always interested in the three different cases: the under-damped ($0 < \xi < 1$), critically damped ($\xi = 1$), and overdamped ($\xi > 1$) cases. So, in synthesizing this transfer function, we require the active RC circuits having the important feature that the damping ratio ξ is easily adjustable without affecting the value of undamped natural frequency ω_n . Here, we present a simple active RC circuit to meet the challenging requirement.

II. SYNTHESIS PROCEDURE

There are many network configurations for realizing the low-pass type of second order transfer function (2,3,4,5). But, unfortunately, we have not found any network configuration satisfying the above mentioned requirement. A simple active RC configuration originally proposed by Sallen and Key (6), and modified by Dutta Roy (3) is shown in Fig. 1.

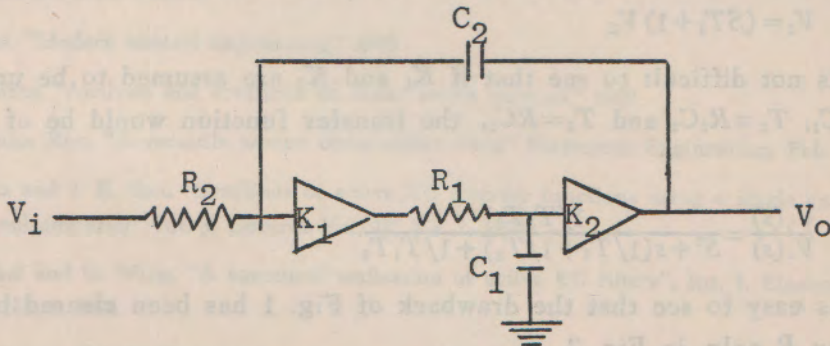


Fig. 1

By analysis, the transfer function is of the form:

$$\frac{V_o(s)}{V_i(s)} = \frac{1/T_1 T_2}{s^2 + s/T_2 + 1/T_1 T_2}$$

where $T_1 = R_1 C_1$, $T_2 = R_2 C_2$ and $K_1 = K_2 = 1$ an unity gain operational amplifier.

Some important circuit features of Fig. 1 are: (1) low sensitivity of Q and ω_0 to active as well as passive components (3); (2) minimum spread in components (note that a minimum of two capacitors is needed); (3) simple design procedure.

The drawback of Fig. 1 is easily understood by inspecting its transfer function. If we are required to adjust the damping ratio, say R_2 . we are also required to change the value of R_2 in order to keep the undamped natural frequency unchanged. Fortunately, this disadvantage can be mitigated by the inclusion of a parallel resistance R with C_2 as shown in Fig. 2.

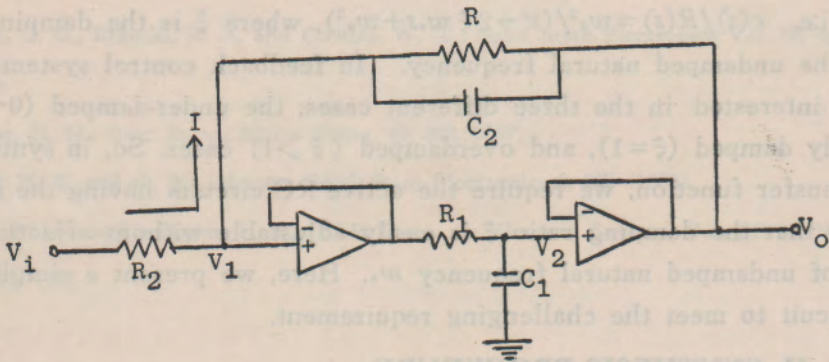


Fig. 2

The circuit analysis can be done by the following equations:

$$IR_2 = V_i - V_1$$

$$I \cdot \frac{R/sc_2}{R+1/sc_2} = V_1 - V_o$$

$$V_1 = (ST_1 + 1) V_2$$

It is not difficult to see that if K_1 and K_2 are assumed to be unity, and $T_1 = R_1 C_1$, $T_2 = R_2 C_2$ and $T_3 = RC_2$, the transfer function would be of the form:

$$\frac{V_o(s)}{V_i(s)} = \frac{1/T_1 T_2}{s^2 + s(1/T_3 + 1/T_2) + 1/T_1 T_2}$$

It is easy to see that the drawback of Fig. 1 has been cleared by adjusting R only, in Fig. 2.

III. EXPERIMENTAL EXAMPLE

Let us use the circuit configuration of Fig. 2 to synthesize the following second order transfer function:

$$V_0(s)/V_i(s) = 64/(S^2 + as + 64)$$

for (1) $a=4$ underdamped case

(2) $a=20$ overdamped case

The value of components are designed to be $R_2=500K$, $C_1=C_2=1 \mu f$, $R_1=32K$, and

$$R = \begin{cases} 500K & \text{for } a=4 \\ 55K & \text{for } a=20 \end{cases}$$

The step response is shown experimentally in Fig. 3.

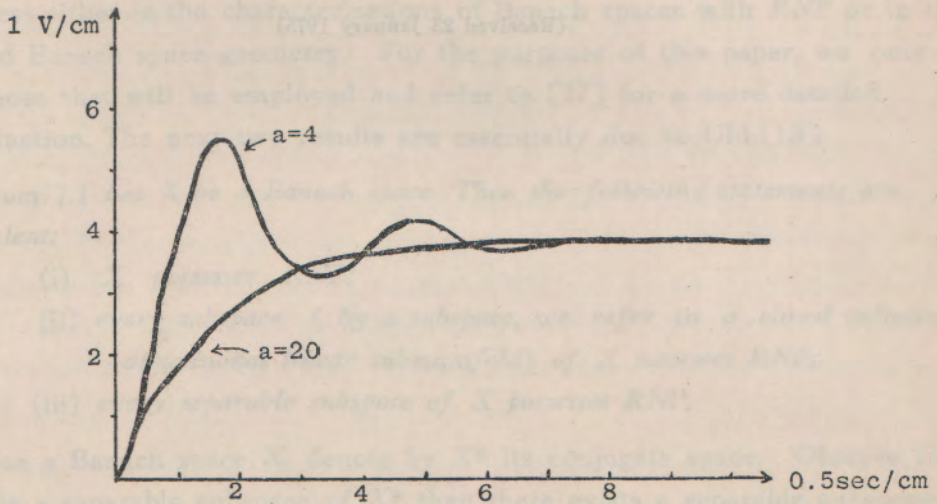


Fig. 3

The operational amplifier we use is of the Philbrick Researches P85AU type. The step input is 4 volts.

From Fig. 4, we see that the experimental results are very close to the theoretical values.

REFERENCES

1. K. Ogata, "Modern control engineering" 1970.
2. S. K. Mitra, "Analysis and synthesis of linear active network" 1969.
3. S. C. Dutta Roy, "A versatile second order active filter" Electronic Engineering, Feb. 1974
4. I. Manna and J. K. Sen, "Synthesis of active RC transfer functions using a single ended operational amplifier" Int. J. Control Vol. 18, No. 1, 1973.
5. K. Singhal and O. Wing, "A canonical realization of active RC filters", Int. J. Electronics Vol. 37, No. 1, 1974.
6. R. P. Sallen and E. L. Koy, "A practical method of designing RC active filters" IRE trans, on CT. CT-2, March, 1955.