

ON CONJUGATE BANACH SPACES WITH THE RADON-NIKODÝM PROPERTY

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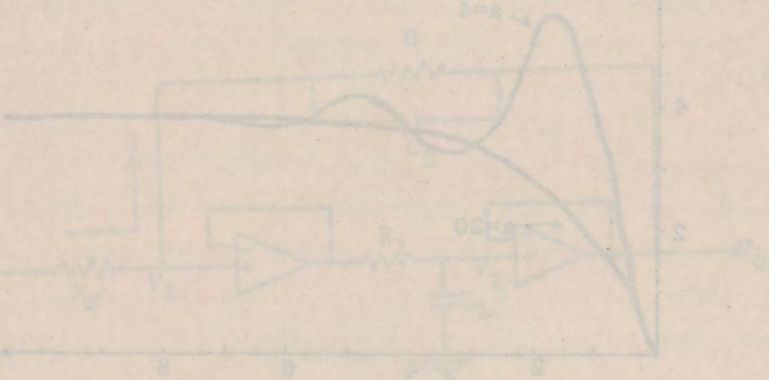


Fig. 2

The operational amplifier we use is of the Philbrick Researches ES8AU type. The step input is 4 volts.

From Fig. 4 we see that the experimental results are very close to the theoretical values.

$$V_1 - V_2 = R_1 I_1 - R_2 I_2$$

$$V_1 - V_2 = \frac{R_1}{sL + R} V_1 - \frac{R_2}{sL + R} V_2$$

$$V_1(1 + \frac{R_2}{sL + R}) = V_2 \frac{R_1}{sL + R}$$

REFERENCES

1. K. Ogata, "Modern control engineering," 1970.
2. R. K. Miller, "Analysis and synthesis of linear active networks," 1969.
3. S. C. Dutta Roy, "A variable second order active filter," *Electronic Engineering*, Feb. 1974.
4. J. Mans and J. K. Sear, "Synthesis of active RC transfer functions using a single op-amp operational amplifier," *Int. J. Control*, Vol. 12, No. 1, 1973.
5. K. Singhal and O. Wing, "A canonical realization of active RC filters," *Int. J. Electronics*, Vol. 27, No. 2, 1969.
6. R. P. Sallen and E. L. Key, "A practical method of designing RC active filters," *IRE Trans. on CT*, CT-2, March 1955.

1. INTRODUCTION AND PRELIMINARIES

A Banach space X is said to have the *Radon-Nikodým Property* (*RNP*) if for each positive finite measure space $(\Omega, \Sigma, \lambda)$ and every λ -continuous vector measure $\mu: \Sigma \rightarrow X$ with finite variation, there exists a Bochner integrable function $f: \Omega \rightarrow X$ such that

$$\mu(A) = \text{Bochner} \int_A f(\omega) d\lambda \text{ for all } A \in \Sigma$$

The classical theorems of Dunford and Pettis [6] and Phillips [12] show that every separable conjugate space and every reflexive Banach space has *RNP*.

Recent developments of the Radon-Nikodým theorem provide a great progress either in the characterizations of Banach spaces with *RNP* or in the related Banach space geometry. For the purposes of this paper, we only list those that will be employed and refer to [17] for a more detailed introduction. The next two results are essentially due to Uhl [18].

Theorem 1.1 *Let X be a Banach space. Then the following statements are equivalent:*

- (i) X possesses *RNP*;
- (ii) every subspace (by a subspace, we refer to a closed infinite-dimensional linear submanifold) of X possesses *RNP*;
- (iii) every separable subspace of X possesses *RNP*.

For a Banach space X , denote by X^* its conjugate space. Observe that if \tilde{Y} is a separable subspace of X^* then there exists a separable subspace Y of X such that \tilde{Y} can be isometrically embedded into Y^* [7, II. 4. 25]. Therefore, as an immediate consequence of Theorem 1.1, one has

Theorem 1.2 *If for every separable subspace Y of X , Y^* is separable. Then X^* has *RNP*.*

The converse of Theorem 1.2 is proved by Stegall [17], i.e.,

Theorem 1.3 *Suppose X^* has *RNP*. Then for every separable subspace Y of X , Y^* is separable.*

We shall use these three theorems to deduce our main results. It seems to be an open question whether a conjugate Banach space X^* has *RNP* whenever the unit ball $B_{X^{**}}$ of X^{**} is weak* sequentially compact. Our result shows that when $B_{X^{**}}$, in its weak* topology, is homeomorphic to a weakly compact subset of some Banach space, or when X^* is isomorphic to a subspace of a weakly compactly generated Banach space (in either case, $B_{X^{**}}$ is weak* sequentially compact) then X^* possesses the *RNP*. This result improves the classical Dunford-Pettis-Phillips Theorem on *RNP*.

The possession of *RNP* by the conjugate spaces of the two specific classes of Banach spaces, the Grothendieck spaces and the Banach spaces X with X^{**}/X separable, is being investigated. For instance, if X is a non-reflexive continuous linear image of $C(S)$ with S an F -space then X^* cannot have *RNP*; if X is a Banach space with X^{**}/X separable then both X^* and X^{**} (and hence X) have the *RNP*.

It is also shown that if a conjugate space X^* possesses the *RNP* and X is weak* sequentially dense in X^{**} then $B_{X^{**}}$ is weak* sequentially compact. Thus, in particular, if X^{**}/X is separable then $B_{X^{**}}$ is weak* sequentially compact.

2. THE RADON-NIKODYM PROPERTY IN X^* AND THE WEAK* SEQUENTIAL COMPACTNESS OF THE UNIT BALL OF X^{**} .

In the terminology of [10], a Banach space X is called *quasi-separable* if for each separable subspace Y of X , Y^* is separable; on account of Theorems 1.2 and 1.3, this concept is equivalent to the possession of *RNP* by X^* . We indicate here that if X is quasi-separable then every continuous linear closed image of X has the same property. For if Z is a continuous linear image of X then Z^* is isomorphic to a subspace of X^* ; Z^* then has *RNP*. Thus by Theorem 1.3, every separable subspace of Z has a separable conjugate. This solves the question proposed by Lacey and Whitley [10] that whether a quotient space of a quasi-separable space is itself quasi-separable.

It is also not known whether a Banach space X is quasi-separable if $B_{X^{**}}$ is weak** sequentially compact. This can be equivalently translated as whether a conjugate space X^* has *RNP* if $B_{X^{**}}$ is weak* sequentially compact. Before proceeding to our discussion, recall that a Banach space X is said to be *weakly compactly generated* (WCG) if it is the closed span of some weakly compact subset of itself. As a result of Amir and Lindenstrauss [1], X is WCG if and only if B_{X^*} in its weak* topology, is affine homeomorphic to a weakly compact subset of some Banach space. A compact Hausdorff space S is *Eberlein compact* if it is homeomorphic to a weakly compact subset of some Banach space. In view of Eberlein's Theorem, S is sequentially compact if it is Eberlein compact. Our result shows that if $B_{X^{**}}$ is Eberlein compact in its weak* topology, or if X^* is isomorphic to a subspace of a WCG space then X^* has *RNP*.

For a subspace $Y \subset X$, set

$$Y^\perp = \{f \in X^* : f(y) = 0 \text{ for all } y \in Y\}.$$

Theorem 2.1 *Let X be a Banach space. Suppose $B_{X^{**}}$ is Eberlein compact in the weak* topology; then X^* possesses the *RNP*.*

Proof. In view of Theorem 1.2, it suffices to show that every separable subspace of X has a separable conjugate space.

Let Y be a separable subspace of X . By Goldstine's theorem, B_Y is weak*-dense in $B_{Y^{**}}$; thus $B_{Y^{**}}$ is weak*-separable. Let $J: Y \rightarrow X$ be the inclusion map. Observe that $J^{**}: Y^{**} \rightarrow X^{**}$ is a weak* isomorphism of Y^{**} onto $Y^{\perp\perp}$ with $J^{**}(B_{Y^{**}}) = B_{Y^{\perp\perp}}$. Hence $B_{Y^{\perp\perp}}$ is weak*-separable. Moreover, $B_{Y^{\perp\perp}}$ is weak* closed in $B_{X^{**}}$, which is Eberlein compact by hypothesis, whence $B_{Y^{\perp\perp}}$ is itself Eberlein compact.

It is well known that a separable Eberlein compact space is metrizable. We have then that $B_{Y^{\perp\perp}}$ is metrizable. This then implies that $B_{Y^{**}}$ is metrizable. Therefore, Y^* is separable; which completes the proof.

Theorem 2.2. *Suppose X^* is isomorphic to a subspace of a WCG Banach space Z ; then X^* possesses RNP.*

Proof. Again, it suffices to show that every separable subspace of X has a separable conjugate space. Let Y be a separable subspace of X . Apply the same argument as in the proof of Theorem 2.1, we see that $B_{Y^{**}}$ is weak*-separable.

Let (x_n^{**}) be a weak*-dense sequence in $B_{Y^{\perp\perp}}$ and $J: X^* \rightarrow Z$ be an isomorphism. $J^*: Z^* \rightarrow X^{**}$ is then surjective. By the Open Mapping Theorem, there exists a bounded sequence (z_n^*) in Z^* such that $T^*z_n^* = x_n^{**}$. Denote by W the weak*-closure of $\{z_n^*\}$. By the hypothesis that Z is WCG, B_{Z^*} is then Eberlein compact in the weak* topology and hence W is also Eberlein compact. This together with the separability of W implies that W is a compact metric space in the weak* topology. $J^*(W)$ is then weak* compact and contains $\{x_n^{**}\} \subset B_{Y^{\perp\perp}}$. Hence $J^*(W) = B_{Y^{\perp\perp}}$. Moreover, being a continuous image of a compact metric space, $B_{Y^{\perp\perp}}$ is compact metrizable. Therefore, $B_{Y^{**}}$ is metrizable and Y^* is separable.

It follows immediately from either Theorem 3.1 or Theorem 3.2 that
Corollary 2.3. *If X^* is WCG then X^* has RNP*

Remark. Corollary 2.3 can be proved by use of Theorem 1.2 and the fact that if a Banach space Y is separable and Y^* is WCG then Y^* is also separable. This result improves the classical Dunford-Pettis-Phillips Theorem on RNP, and is well known at present. However, recently H. P. Rosenthal [15] has given a counter-example to the heredity problem for WCG Banach space. Indeed, the Banach space X_R he exhibited has the following properties: (i) X_R is a subspace of a WCG space $L^1(\mu)$ and X_R is not WCG; (ii) X_R is isomorphic to a conjugate Banach space; (iii) the unit ball of X_R^* is Eberlein compact in its weak* topology. Thus our independent proof appears necessary.

Observe that those conjugate Banach spaces X^* with RNP discussed in the above theorems have the property that $B_{X^{**}}$ is weak* sequentially compact. For the converse, we have obtained sufficient conditions to ensure that $B_{X^{**}}$ is weak* sequentially compact whenever X^* has the RNP. In the following theorem, we set for each $A \subset X^{**}$

$$A^\tau = \{f \in X^* : x^{**}(f) = 0 \text{ for all } x^{**} \in A\}$$

and write " \approx " whenever two Banach spaces are isometrically isomorphic. Theorem 2.4. *If X^* possesses the RNP and X is weak* sequentially dense in X^{**} , then $B_{X^{**}}$ is weak* sequentially compact.*

Proof. Let (x_n^{**}) be a sequence in $B_{X^{**}}$. By assumption, X is weak* sequentially dense in X^{**} ; for each x_n^{**} , there exists a sequence $(x_n^k)_k$ in X such that $(x_n^k)_k$ converges to x_n^{**} in the weak* topology of X^{**} .

Let $\tilde{\gamma}$ be the weak* closed subspace of X^{**} spanned by $\{x_n^{**}\}$ and \tilde{z} be the weak* closed subspace of X^{**} spanned by $\{x_n^k\}_{n,k}$. We have then that $\tilde{\gamma} \subset \tilde{z}$ and

$$\tilde{\gamma} = (\{x_n^{**}\}^\tau)^\perp \approx (X^*/\{x_n^{**}\}^\tau)^*$$

$$\tilde{z} = (\{x_n^k\}_{n,k}^\tau)^\perp \approx (X^*/\{x_n^k\}_{n,k}^\tau)^*.$$

Let Z be the closed subspace of X spanned by $\{x_n^k\}_{n,k}$. Observe that Z is weak*-dense in $Z^{\perp\perp}$, whence $Z^{\perp\perp} = Z$. By hypothesis, X^* has RNP; hence Z^* is separable. But

$$Z^* \approx X^*/\{x_n^k\}_{n,k}^\tau \text{ and } \tilde{\gamma} \subset \tilde{z};$$

$X^*/\{x_n^{**}\}^\tau$ is a continuous linear image of $X^*/\{x_n^k\}_{n,k}^\tau$. Thus $X^*/\{x_n^{**}\}^\tau$ is separable. It follows then that the unit ball of $(X^*/\{x_n^{**}\}^\tau)^*$ is weak* sequentially compact.

Moreover, since $(X^*/\{x_n^{**}\}^\tau)^*$ is weak* isomorphic to $\tilde{\gamma}$, the sequence (x_n^{**}) in $\tilde{\gamma}$ has a weak* convergent subsequence. This is equivalent to saying that $B_{X^{**}}$ is weak* sequentially compact.

The above result will be used in Section 4 to prove that if X^{**}/X is separable then $B_{X^{**}}$ is weak* sequentially compact.

3. THE RADON-NIKODÝM PROPERTY IN GROTHENDIECK SPACES AND THEIR CONJUGATES.

A Banach space X is said to be a *Grothendieck space* if every weak* convergent sequence in X^* is weakly convergent. Let S be a compact Hausdorff space, and $C(S)$ the space of continuous functions on S . The original result of A. Grothendieck [9, p. 168] shows that if S is *Stonian* (i. e., every open set has an open closure), then $C(S)$ is a Grothendieck space. It is unknown whether the conjugate space of a non-reflexive Grothendieck

space can have RNP. If $C(S)$ is a Grothendieck space, then since S contains no nontrivial convergent sequence [2] the set of cluster points of an infinite subsets of S is perfect; thus $C(S)$ contains a subspace isomorphic to ℓ^1 [13], and therefore, $C(S)^*$ doesn't have the RNP.

S is called an F -space if every pair of disjoint open $F\sigma$ subsets of S have disjoint closures. By a result of Seever [16], if S is an F -space then $C(S)$ is a Grothendieck space. It is easily verified that a continuous linear closed image of a Grothendieck space is itself a Grothendieck space.

Furthermore, most of the known Grothendieck spaces are continuous linear images of $C(S)$ with S an F -space. For such Grothendieck spaces, we have

Theorem 3.1. *Let X be a non-reflexive continuous linear image of $C(S)$ with S an F -space. Then X^* cannot have RNP.*

Proof. X is a non-reflexive continuous linear image of $C(S)$ with S an F -space. By a result of Rosenthal [14], ℓ^∞ is then a continuous linear image of X . Hence X^* contains a subspace isomorphic to $(\ell^\infty)^*$. But $(\ell^\infty)^*$ doesn't have RNP; thus X^* cannot have RNP either. (Theorem 1.1).

Remark. Recall that the norm $\|\cdot\|$ of a Banach space X is said to be smooth if for each $x \in X$, $\|x\| = 1$, there exists a unique support functional $f_x \in X^*$. X is said to be smooth if it has an equivalent smooth norm. If $\|\cdot\|$ is an equivalent smooth norm of X , the map $Z: x \rightarrow f_x$ of $\{x \in X: \|x\| = 1\}$ into $\{f \in X^*: \|f\| = 1\}$ is "norm to weak*" continuous [8]. In case that Z is "norm to weak" continuous, X is then called *very smooth* by Diestel and Faires in [5], where they have shown that *if a Banach space X is very smooth then X^* possesses the RNP*. Obviously, if X is a smooth Grothendieck space then X is very smooth; thus X^* possesses RNP. However, we conclude from Theorem 3.1 that if the Grothendieck space X is a continuous linear image of $C(S)$ with S an F -space X is not smooth.

Proposition 3. 2. *Suppose X is a weakly sequentially complete Banach space and X^* has RNP; then X is reflexive*

Proof. Let (x_n) be a bounded sequence in X , and Y the closed subspace spanned by $\{x_n\}$. Since X^* has RNP, Y^* is separable by Theorem 1.3. Let (y_n^*) be a dense sequence in Y^* . A standard diagonal process then shows that there exists a subsequence (x_{n_k}) such that $(y_n^*(x_{n_k}))_k$ converges for each $y_n^* \in Y^*$. By the boundedness of (x_{n_k}) , $(y^*(x_{n_k}))_k$ then converges for every $y^* \in Y^*$, i.e., (x_{n_k}) is a weak Cauchy sequence in X . But X is weakly sequentially complete by assumption; thus (x_{n_k}) converges weakly in X . By Eberlein's Theorem, this means that every bounded set of X is weakly

conditionally compact. Hence X is reflexive.

Corollary 3.3. (i) *If X^* is a Grothendieck space which has RNP then X is reflexive.*

(ii) *If Y is a Grothendieck space and Y^{**} has RNP, then Y is reflexive.*

Proof. (i) By assumption, X^* is a Grothendieck space. It is then clear that X^{**} is weakly sequentially complete, which in turn implies that X is weakly sequentially complete. But X^* is also assumed to have RNP; thus X is reflexive by proposition 3.2. (ii) Y is a Grothendieck space, hence Y^* is weakly sequentially complete. Apply proposition 3.2 to Y^* .

It is unknown whether a weakly sequentially complete Grothendieck space must be reflexive. However, the following corollary holds:

Corollary 3.4. *Let X be a Grothendieck space. Assume that X is both weakly sequentially complete and smooth; then X is reflexive.*

Proof. By assumption, X is a smooth Grothendieck space. It then follows from the preceding remark that X^* possesses the RNP. But X is also weakly sequentially complete; therefore, X is reflexive.

Remark. If a Banach space X has an equivalent Fréchet differentiable norm, then X is very smooth [8]; thus X^* has RNP. In other words, a non-reflexive weakly sequentially complete Banach space has no equivalent Fréchet differentiable norm.

Let X, Y be Banach spaces. We shall denote by $B(X, Y)$ the Banach space of bounded linear operators from X to Y , and $W(X, Y)$ the subspace of weakly compact operators.

Theorem 3.5. *If X is a Grothendieck space and Y is isomorphic to a conjugate Banach space with the RNP, then $B(X, Y) = W(X, Y)$.*

Proof. Without loss of generality, we may assume that $Y = Z^*$. Consider now the Banach space $B(X, Z^*)$. Observe that the map $T \rightarrow T^*|Z$ is an isometric isomorphism from $B(X, Z^*)$ onto $B(Z, X^*)$ such that $T \in W(X, Z^*)$ if and only if $T^*|Z \in W(Z, X^*)$. Thus it suffices to show that $B(Z, X^*) = W(Z, X^*)$.

Let $T \in B(Z, X^*)$ be given. Assume that (z_n) is a bounded sequence in Z . We shall prove that (Tz_n) has a weakly convergent subsequence. Denote by W the closed subspace spanned by $\{z_n\}$. W^* is then separable for Z^* has RNP. Thus the argument in the proof of Proposition 3.2 shows that there exists a weak Cauchy subsequence (z_{n_k}) of (z_n) , which in turn implies that (Tz_{n_k}) is a weak Cauchy sequence in X^* . But X is assumed to be a Grothendieck space; thus X^* is weakly sequentially complete. Therefore,

(Tz_{nk}) converges weakly. Our argument concludes that every bounded subset of Z has a weakly conditionally compact image by Eberlein's Theorem; hence T is weakly compact.

4. THE BANACH SPACE X WITH X^{**}/X SEPARABLE

In this section, we give examples of Banach space X such that both X^* and X^{**} (and hence X) have RNP. The Banach space X we are considering has the property that X^* is WCG and $B_{X^{***}}$ is weak* sequentially compact. Theorem 4.1. *Let X be a Banach space such that X^{**}/X is separable. Then both X^* and X^{**} has RNP.*

Proof. In view of Theorem 1. 2, it suffices to show that every separable subspace of X (resp. X^*) has a separable conjugate space.

Let Y be a separable subspace of X . Note that Y^{**}/Y is isomorphic to a subspace of X^{**}/X [3, p. 908]. By hypothesis, X^{**}/X is separable, so is Y^{**}/Y . It follows then that Y^{**} and hence Y^* is separable.

Assume Z is a separable subspace of X^* . It is known that there exists a separable subspace W of X such that Z is isometrically isomorphic to a subspace of W^* . Z^* is then a continuous linear image of the separable space W^{**} . Thus Z^* is separable.

Remark. It is obvious that if both X^* and X^{**} have RNP then every separable subspace of X has a separable second conjugate. Indeed, if Y is a separable subspace of X , Y^* is then separable since X^* has RNP. But Y^{**} is isometrically isomorphic to a subspace of X^{**} ; Y^{**} has RNP. Thus by Theorem 1.3, Y^{**} is separable. Note that the given hypothesis doesn't necessarily imply that X^{**}/X is separable. As a counter-example, we refer to [11, p. 124].

Together with the result of Theorem 2.4, we obtain

Corollary 4.2 *Suppose X^{**}/X is separable. Then $B_{X^{**}}$ and $B_{X^{***}}$ (and hence B_{X^*}) are sequentially compact in their respective weak* topologies*

Proof. Since X^{**}/X is separable, X^* and X^{**} have RNP by Theorem 4.1. Also a result of [11, p. 123] shows that X^* (resp. X^{**}) is weak* sequentially dense in X^{**} (resp. X^{***}). Thus $B_{X^{**}}$ (resp. $B_{X^{***}}$) is weak* sequentially compact by Theorem 2.4. Moreover, since B_{X^*} is a continuous linear image of $B_{X^{***}}$ in the respective weak* topologies, B_{X^*} is then weak* sequentially compact.

Corollary 4. 3. *Suppose X is non-reflexive and X^{**}/X is separable. Then neither X nor X^* is weakly sequentially complete*

Proof. Follows from Theorem 4.1 and Proposition 3.2.

Remark. Corollary 4.3 is motivated by the problem whether there exists a "small" Grothendieck space. Indeed, let X^{**}/X be separable. Suppose X^* (resp. X^{**}) were weakly sequentially complete; then X (resp. X^*) would be a Grothendieck space. Corollary 4.3 shows that the hypotheses we just made are absurd. However, the following question seems to be open: If $X \oplus R = X^{**}$ with R a reflexive Banach space (see [4] for a detailed study of such Banach spaces X), can X^* be weakly sequentially complete? A positive answer would enable us to provide a non-reflexive Grothendieck space which doesn't contain a subspace isomorphic to c_0 .

As a final result, we further prove that when X^{**}/X is separable X^* is indeed WCG.

Lemma 4.4 *Let Z be a WCG subspace of a Banach space Y such that Y/Z is separable. Then Y is WCG.*

Proof. Y/Z is separable, hence there exists a separable subspace $W \subset Y$ such that $Z+W$ is dense in Y . But both W and Z are WCG; thus Y is WCG.

Theorem 4.5. *Suppose X^{**}/X is separable. Then X^* is WCG.*

Proof. It is known that, under the given hypothesis, there exists a separable subspace Z such that X/Z is reflexive [11, p. 121]. We have then that Z^\perp is reflexive and X^*/Z^\perp is separable. It follows from Lemma 4.4 that X^* is WCG.

REFERENCES

- (1) Amir, D., Lindenstrauss, J., "The structure of weakly compact sets in Banach spaces", *Ann. of Math.* **88**, (1968), 35-46.
- (2) Amir, D., "Continuous functions spaces with the bounded extension property", *Bull. Res. Council Israel Sect. F.* **10F**, (1962), 133-138.
- (3) Civin, P., Yood, B., "Quasi-reflexive spaces", *Proc. Amer. Math. Soc.* **8**, (1957), 906-911.
- (4) Clark, J. R., "Coreflexive and somewhat reflexive Banach spaces", *Proc. Amer. Math. Soc.* **36**, (1972), 421-427.
- (5) Diestel, J., Faires, B., "On vector measures". To appear.
- (6) Dunford, N., Pettis, B. J., "Linear operators on summable functions", *Trans. Amer. Math. Soc.* **47**, (1940), 323-392.
- (7) Dunford, N., Schwartz, J. T., *Linear Operators, Part I: General Theory*, New York: Interscience, 1958.
- (8) Giles, J. R., "On a characterization of differentiability of the norm of a normed linear space", *J. Austral. Math. Soc.* **12**, (1971), 106-114.
- (9) Grothendieck, A., "Sur les applications lineaires faiblement compactes d'espaces du type $C(K)$ ", *Canad. J. Math.* **5**, (1953), 129-173.
- (10) Lacey, E., Whitley, R. J., "Conditions under which all the bounded linear maps are compact", *Math. Ann.* **158**, (1965), 1-5.
- (11) McWilliams, R. D., "On certain Banach spaces which are W^* -sequentially dense in their second duals", *Duke Math. J.* **37**, (1970), 121-126.
- (12) Phillips, R. S., "On weakly compact subsets of a Banach spaces", *Amer. J. Math.* **64**, (1943), 108-136.
- (13) Pelczynski, A., Semadeni, Z., "Spaces of continuous functions (III) (Spaces $C(\Omega)$ for Ω without perfect subsets)", *Studia Math.* **18**, (1959), 211-222.
- (14) Rosenthal, H. P., "On complemented and quasi-complemented subspaces of quotients of $C(S)$ for Stonian S ". *Proc. Nat. Acad. Sci. U.S.A.* **60**, (1968), 1165-1169.
- (15) Rosenthal, H. P., "The heredity problem for weakly compactly generated Banach spaces". To appear.
- (16) Seever, G., "Measures on F-spaces", *Trans. Amer. Math. Soc.* **133**, (1968), 267-280.
- (17) Stegall, C., "The Radon-Nikodym property in conjugate Banach spaces". To appear.
- (18) Uhl, J. J., Jr., "A note on the Radon-Nikodym property for Banach spaces", *Rev. Roumaine Math. Pures Appl.* **17**, (1972), 113-115

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