On the Synthesis of a Special Class of
the Communication Network

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Summary

The highway system is a special kind of communication network. In this paper three problems related to the mathematical model of the highway system is discussed and solved.

I. Introduction:

A communication network is a network with nodes representing the stations and the links connecting between any two nodes, the branch capacity is the capacity of each link of which the amount of communication flow can be transferred along the indicated direction denoted by the symbol C_{ij}, the subscript letter i,j is ordered to indicate the direction from node i to j, and the terminal capacity is defind between any two nodes in the network and taking the whole network into consideration denoted by the symbol t_{ij}. In recent years, there are many problems being solved in the field of communication network and some new problems have been posed. Most of the problems are connected with linear programming, here we only consider a special class of communication network namely for the mathematical model of the highway systems.

II. The mathematical model of the highway systems: As considered by the author[1], the mathematical model of highway systems is in generally speaking $C_{ji} \neq C_{ij}$, but the terminal capacity owing to the conservation of moving cars, is evidently $t_{ij} = t_{ji}$. Hakmi in his paper[2] states that some branches are oriented and some are not. But in the author's opinion, we can take all the highways as oriented network without loss of generality, in other words the model of the highway system is an oriented communication network. The highway connected between any two stations i and j is shown in Fig. 1

i

c_{ji} c_{ij} Fig. 1

j
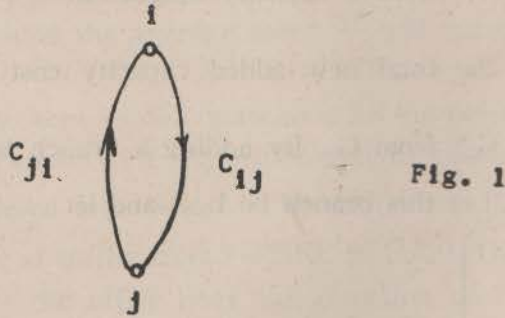
III. Problems related to highway systems:

A. To construct the new specified highway system.

Because of the conservation of cars we know $t_{ij}=t_{ji}$ for every pair of stations i and j and the highway system is in cycling form. Then according to theorem 6 of the author's paper[1] which states: For every cycling oriented communication network in which each directed branch. Its capacity is of the same value, then it will always have an equivalent cycling nonoriented communication network in which every branch capacity is equal to one half of the branch capacity of the cycling oriented communication network. We can change it into non-oriented communication nets, then using the method in Gemory and Hu's paper (3) to reduce it into a mxm square matrix of Ford & Fulkson's type (4) After this is done, we can send it to computer, let it do the remained work for us. When the results come out we get the min–cost new highways to handle the specified traffics.

B. If a highway system already exists because of the increasing traffic requirement to increase the highway system from t_{ij} to $t_{ij}+\triangle t_{ij}$ if we can build new highway capacity between any two stations in the old ones. Prof. Hakimis newly published paper considers telephone problems (5), but the method also can do for the highway system. Hakimi's method is as follows:

$$B = \begin{pmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ b_m \end{pmatrix}$$ a capacity matrix of the graph G

suppose now for every branch capacity b_k goes to $b_k + \triangle b_k$, then B goes to $B + \triangle B$. And the total new added capacity cost is $\sum\limits_{k=1}^{m} C_k \triangle b_k$. We construct a graph $G_1{}^*$ from G_1. By adding a branch between v_i & v_j of capacity $t_{ij} + \triangle t_{ij}$. Let this branch be b_{m+1} and let

$$X^* = \begin{pmatrix} x_1 \\ x_2 \\ \bullet \\ \bullet \\ x_{m+1} \end{pmatrix}$$ where X_k is real may be minus the minus

sign means flows oppose the direction assigned to the branch, then by conservation of cars. $AX^* = 0$ where A is the incident matrix and force

$$X_{m+1} = t_{ij} + \triangle t_{ij}$$

$$x_k = b_k + \triangle b_k$$

$$X_1 = B + \triangle B \quad \text{where } x_1 \text{ consists 1st m component of } X^*.$$

a linear program is formed for min. cost as follows:

$$\begin{cases} AX^* = 0 \\ X_1 = B + \triangle B \\ X_{m+1} = t_{ij} + \triangle t_{ij} \\ \text{min.} \sum\limits_{k=1}^{m} C_k(\triangle b_k) \end{cases}$$

C. Traffic problems arise in city due to its increasing of traffic flows and no street capacity can be added.

Since the natural increase of population and the extension of the region of the city, the traffic increases day by day, due to the limit of street capacity, what can we do in this case? We need to dig in more deeply and consider about the velocity of the cars. We know that the permissible traffic in a street is proportional to the velocity then we define:

$$F = KC\overline{V}$$

F=street traffic capacity per hour

C=branch capacity as usual

$\overline{V}$=Average speed (m/unit time)

k is a constant factor depending on the case involved. When cars encounter red signs or stop signs the average speed $\overline{V}$ will automatically decrease. e.g.

Suppose the distance between two stations is 2.5 km permit speed 25 km/hr, then the time need to travel from one station to another is 0.1 hr if one red sign is encountered take time 0.02 hr then $V = \dfrac{2.5}{0.12} = 20.84$ km/hr will reduce the flow of traffic from F=25KC to $\overline{F}$=20.84KC cars/hr. Usually at the beginning of the office hour the direction of the traffic flow is towards downtown and at the time after office the traffic is usually the other way. In this case the only way we can do is to make most two way streets into one way streets as shown in Fig. 2a the total inward capacity toward center station (5) is 8. If we change some of the two way streets to one way as shown in Fig. 2b the inward capacity toward the center station (5) will increase to 14. The one way traffic sometimes not only increases the safety factor but also increases the average speed $\overline{V}$ as shown in example 2.
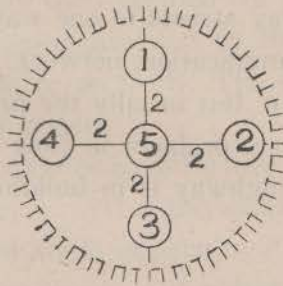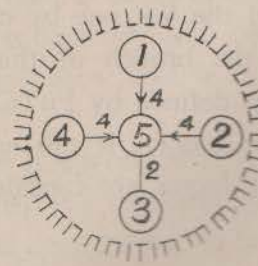
Example 2

Fig. 2a

Fig. 2b
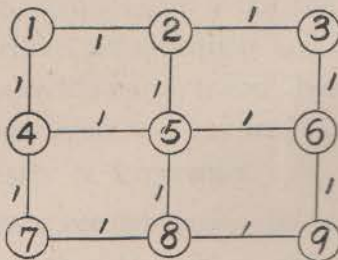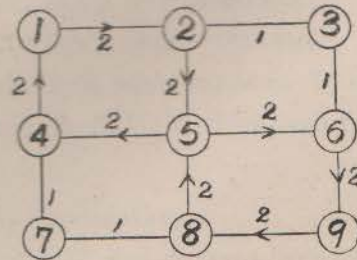

Fig. 3a

Fig. 3b

For simplicity take $K=1$ speed limit 25km/hr and the distance between each station is 1 km. Denote the net flow from station i to station j by $\overline{F}_{ij}$ & $\overline{F}'_{ij}$ respectively for Fig. 3a Fig. 3b,

$$\overline{F}_{24}=1\times25+1\times25\times\frac{8}{9} \qquad \overline{F}'_{24}=2\times25$$

$$=47.2 \text{ car/hr} \qquad\qquad =50\text{car/hr}$$

$$\overline{F}_{12}=45 \text{ car/hr} \qquad\qquad \overline{F}'_{12}=50 \text{ car/hr}$$

$$\overline{F}_{52}=1\times25+2\times20 \qquad \overline{F}'_{52}=60 \text{ car/hr}$$

$$=65 \text{ car/hr}$$

In this example we compared $\overline{F}_{12}$, $\overline{F}_{24}$, $\overline{F}_{52}$ with $\overline{F}'_{12}$, $\overline{F}'_{14}$, and $\overline{F}'_{52}$. We got the conclusion that the communication network in form of Fig. 3b will permit more traffic to flow than in form of fig. 3a. The other way to solve the traffic problem in a crowded city is to increase the height of the bus, increase the volume to handle more people traveling around the city.

IV. Conclusion

In this paper we have stated three problems in highway system and have tried to solve them. The first two will be answered by computation method and the last one by changing two way streets to one way streets. Of course the branch of this kind of communication network still has reliability as defined by Fu (6) as a probability. But usually the probability is somewhat higher, it can not make a serious problem in time of peace and if some factor are considered when the highway is in building.