SINGLE PARTICLE MODEL FOR SCATTERING OF MICROWAVES FROM PLASMA ELECTRONS*

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Abstract

A simple model of scattering of electromagnetic waves from plasma electrons is given to show a better physical picture of scattering.

I. Introduction

Scattering of electromagnetic waves from plasma electrons has stimulated interest in many fields of science in recent years. The scattering of electromagnetic waves from meteor trails provides a means for VHF long-distence transmission, extending the frequency spectrnm one step further up. Gordon[1] proposed incoherent scattering of radio waves by free electrons as a new technique for space exploration by radar. Bowles[2] first successful experiment of incoherent backscattering of electromagnetic waves from the ionosphere found a very narrow echo spectrum contrary to Gordon's speculation. This discrepency stimulated intense theoretical investigation of the problem by many authors.[3] They concentrated on the calculatlon of the plasma statistical density fluctuation spectrum which describes the power spectrum of the scattering signal. By incorporating the plasma number density fluctuation into the dielectric formulation, Akhiezer[4] showed theoretically that seattering by plasma density oscillation might occur when electromagnetic waves are propagated in a plasma.

We reported[5] that resonance enhancement of scattering occured when the magnetic field confining the plasma was swept through the cyclotron resonances at the probing and scattering signal frequencies.

II. Single particle model

The problem of scattering of electromagnetic waves is approached in general by solving the mathematically complicated Maxwell's equations and those related to the plasma dynamics. In order to preserve some physics of the problem, we present a single particle model of scattering of electromagnetic waves, and introduce the index of refraction to take care of the collective effect of the plasma. We find this simple analysis gives good agreement with Akhiezer's result.

When a plasma is of finite transverse cross-section, one of the characteristic modes of organized plasma oscillation is the longitudinal space charge waves which are electromechanical in nature. The existence of this oscillation (with frequency Ω) lead to a space and time periodic variation of the dielectric constant of the plasma. Thus, when a microwave signal of frequency ω_i is propagated through the plasma, waves with frequencies $\omega_r=\omega_i \pm n\Omega$ will also be re-radiated.

Consider the motion of an electron in a coordinate system given in Figure (1). The longitudinal oscillation of the electron which is excited by the space charge wave $E_s \exp[i(\Omega t-k\cdot z)]$ can be described as:

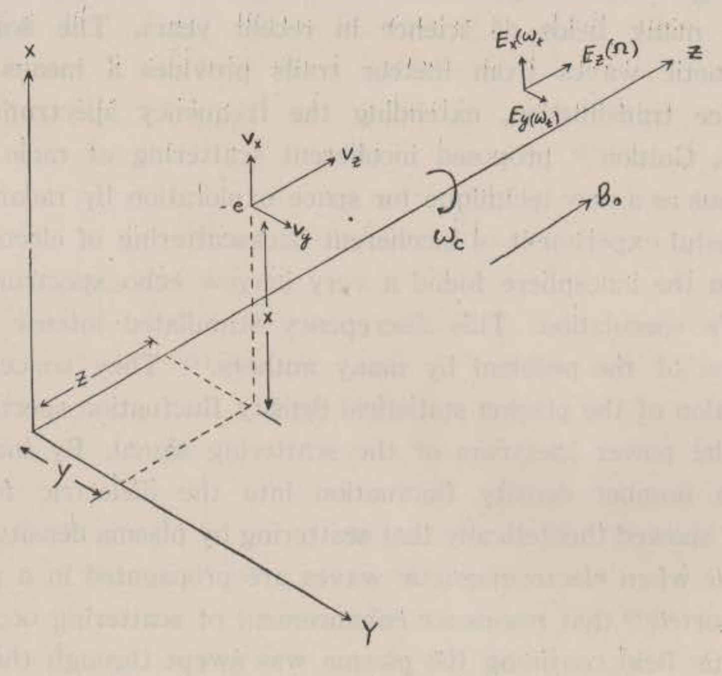

Fig. (1) Single particle Trajectory

$$m\frac{d^2z}{dt^2}=-eE_zCos\Omega t \tag{1}$$

if collisions are neglected.
Solution of equation (1) gives:

$$Z=\frac{eE_z}{m\Omega^2}Cos\Omega t=Z_oCos\Omega t \tag{2}$$

The transverse of the particle is governed by the incident transverse microwave $\underline{E}_t=\underline{E}_o\exp[i(\omega_tt-k_oz)]$ propagating along Z-axis with frequency ω_t and the proagation constant k_o. The transverse equation of motion then can be expressed as:

$$m\frac{d\underline{v}}{dt}=-e(\underline{E}_t+\frac{1}{C}\underline{V}\times\underline{B}_o) \tag{3}$$

where B_o is the confining magnetic field in parallel to the plasma column axis.
In separated components:

$$m\frac{dv_x}{dt}=-eE_x-\frac{e}{c}v_yB_o \tag{4}$$

$$m\frac{dv_y}{dt}=-eE_y+\frac{e}{c}v_xB_o \tag{5}$$

Equation (4) and (5) can be decomposed by transformation into a rotating coordinate defined by:

$$V^+=V_x-iV_y \qquad V^-=V_x+iV_y \tag{6}$$

$$E^+=E_x-iE_y \qquad E^-=E_x+iE_y \tag{7}$$

where "+" corresponds to a right handed wave.
"−" corresponds to a left handed wave.
The decoupled equations become:

$$\frac{dV^+}{dt}+i\omega_cV^+=-\frac{e}{m}E^+ \tag{8}$$

$$\frac{dV^-}{dt}-i\omega_cV^-=-\frac{e}{m}E^- \tag{9}$$

where $\omega_c = \frac{eB_o}{mc}$ is the cyclotron frequency of the particle under the steady confining magnetic field. Equation (9) is the equation of motion of the particle due to the left handed wave excitation, which shows no resonance and is of no particular interest to us.

Solving Equation (8) and restoring the time and space dependent factor, we get:

$$V^{+} = \frac{e}{i\,m}\frac{E_o^{+}}{(\omega_t - \omega_c)}\exp[-i(\omega_t t - k_o z)] \tag{10}$$

where E_o^{+} is the electric field of the right-handed incident wave.

We have neglected the time dependence of z in solving Equation (8) for V^{+} as a first approximation. To couple the longitudinal motion of the particle to its transverse motion, we substitute Equation (2) into (10) and rewrite,

$$V^{+} = \frac{e}{i\,m}\frac{E_o^{+}}{(\omega_t - \omega_c)}\exp[-i\omega_t t]\exp[ik_o z_o \mathrm{Cos}\Omega t] \tag{11}$$

The last term in Equation (11) can be expanded into a series of Bessel functions:

$$\begin{aligned}\exp[ik_o z_o \mathrm{Cos}\Omega t] &= \sum_{N=-\infty}^{\infty} i^n J_n(k_o z_o)\exp[in\Omega t] \\ &= J_o(k_o z_o) + iJ_1(k_o z_o)\exp[i\Omega t] \\ &\quad - iJ_1(k_o z_o)\exp[-i\Omega t] + \cdots\cdots\cdots\cdots\end{aligned} \tag{12}$$

Multiplying the factor $\exp[-i\omega_t t]$ into the Equation (12), the second and third terms give rise to the combination scattering $\omega_t \pm \Omega$. Without losing its generality, we can neglect all the terms in the Bessel series expansion except the combination terms. Then we have,

$$V^{+} = \frac{e}{m}\frac{E_o^{+}}{(\omega_t - \omega_c)} J_1(k_o z_o)\exp[-i(\omega_t \pm \Omega)t] \tag{13}$$

By further substituting the combination relation $\omega_r = \omega_t \pm \Omega$ and taking the time derivative of V^{+}, we find the acceleration of the particle:

$$a^{+} = -i\frac{e}{m}E_o^{+}\frac{\omega_r}{(\omega_t - \omega_c)} J_1(k_o z_o)\exp[-i\omega_r t] \tag{14}$$

The power radiated by an oscillating electric dipole per unit solid angle Ω' is given by[6]:

$$\frac{dp_{rad}}{d\Omega'}=\frac{e^2}{4\pi C^3}|\bar{a}(\omega_r)|^2\, n(\omega_r) \tag{15}$$

where $n(\omega_r)$ is the index of refraction of the plasma medium to the right handed scattered orre-radiated signal propagating in parallel to the magnetic field. Substituting Equation (14) into (15), we get:

$$\frac{dp_{rad}}{d\Omega'}=\frac{e^2}{4\pi C^3}\frac{e^2}{m^2}\frac{\omega_r^2}{(\omega_i-\omega_c)^2}J_1^2(k_oz_o)n(\omega_r)(E_o^+)^2 \tag{16}$$

with $k_o \ll k$ the relation $k_oz_o \ll 1$ always holds, hence we expand the first order Bessel function, and neglect any higher order terms:

$$\lim_{k_oz_o \ll 1} J_1(k_oz_o) \doteqdot \frac{k_oz_o}{2}$$

Thus

$$J_1^2(k_oz_o)\approx\frac{1}{4}(k_oz_o)^2=\frac{1}{4}k_o^2\frac{e^2E_z^2}{m^2\Omega^4}\ \text{for}\ k_oz_o\ll 1 \tag{17}$$

Substituting the index of refraction $n(\omega_r)$ explicitly, we have:

$$\frac{dp_{rad}}{d\Omega'}=\frac{E_o^{+2}}{(4\pi)^3}\frac{\omega_p^4\omega_r^2}{n_o^2C^3}\frac{k_o^2}{4k^4}\left(\frac{eE_z}{mV_\varphi^2}n_o\right)^2\frac{1}{(\omega_i-\omega_c)^2}\sqrt{1-\frac{\omega_p^2}{\omega_r(\omega_r-\omega_c)}} \tag{18}$$

where $V_\varphi=\frac{\Omega}{k}$ is the phase velocity of the space charge wave. Recalling that $\underline{E}=-\nabla\varphi$ or $E_z=k\varphi$ and assuming the backscattering, that is $k\doteqdot 2k_o$, we have the final result:

$$\frac{dp_{rad}}{d\Omega'}=\frac{E_o^2\omega_p^4\omega_r^2}{4(8\pi)^3n_o^2C^3}\left(\frac{e\varphi}{mV_\varphi^2}n_o\right)^2\frac{1}{(\omega_i-\omega_c)^2}\sqrt{\frac{\omega_r^2-\omega_r\omega_c-\omega_p^2}{\omega_r(\omega_r-\omega_c)}} \tag{19}$$

By again relating the space charge wave to fluctuation amplitude shown in the appendix, we have:

$$\frac{dp_{rad}}{d\Omega'}=\frac{E_o^2\omega_p^4\omega_r^2}{2(8\pi)^3n_o^2C^3}\langle\overline{\delta n}\rangle^2\frac{1}{(\omega_i-\omega_c)^2}\sqrt{\frac{\omega_r^2-\omega_r\omega_c-\omega_p^2}{\omega_r(\omega_r-\omega_c)}} \tag{20}$$

In order to compare the single particle result to that of Akhiezer's, we reproduce their result here:

$$I(\omega_r)=\frac{4E_o^2\omega_p^4\omega_r^2}{(8\pi)^3n_o^2C^3}\langle\overline{\delta n}\rangle^2\frac{1}{(\omega_i-\omega_c)^2}\sqrt{\frac{\omega_r^2-\omega_r\omega_c-\omega_p^2}{\omega_r(\omega_r-\omega_c)}} \tag{21}$$

Comparing Equation (20) and (21), we find that they are in good agreement except for a missing constant, which can easily be recovered by integrating over the solid angle.

From an electrical engineer's point of view, the resonance peaking of the scattered intensity can be explained in analogy to resonance coupling in a three-terminal pair network, shown in Fig.(2). Inside the dashed line block is the "plasma world", while the others are the "outside world" including microwave circuit and UHF system. The resonance frequencies of the microwave coupling terminals I and II are tuned by the magnetic field B_0. The plasma will receive a strong incident signal when the input terminal I is in resonance with the incident frequency ω_t. The plasma will couple the scattered signal to the outside world strongly when the output terminal II is in resonance with the receiver frequency ω_r. Therefore we observe two enhancement peaks when the gyrofrequency of the magnetic field is in resonance with the microwave frequencies.

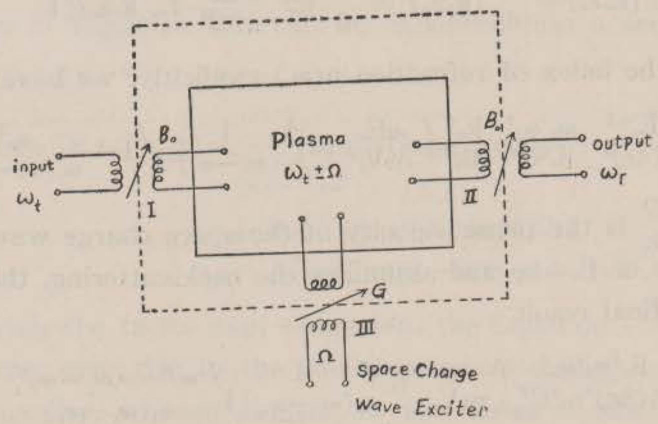

Fig.(2) Circuit analogy of Scattering

III Conclusion

A single particle model of scattering which is in complete agreement to Akhiezer's result has been presented for the purpose of giving better understanding to the physical mechanism of scattering. The space charge wave amplitude has been related to the fluctuation amplitude of the plasma so that abundant theoretical investigation of the plasma fluctuation can be adapted to this work.

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Appendix

Space Charge Waves and Plasma Density Fluctuation

The central theme in the theory of scattering of electromagnetic waves by plasma density fluctuation lies on the calculation of the fluctuation spectral density, more precisely, the statistical average of the auto-correlation function of the Fourier components of the plasma number density fluctuations.

This appendix is presented to investigate the space charge wave in a heuristic way, by considering the space charge wave as a coherent electron number density fluctuation and calculating the relationship between the space charge wave amplitude and the fluctuation amplitude.

We start from the divergence equation:

$$\nabla \cdot \underline{E} = 4\pi\rho \qquad \text{A. 1}$$

where $\underline{E}$ is the electric field of the space charge wave and ρ the space charge density.

In the quasi-static approximation, $\underline{E} = -\nabla\varphi$, we have the Poisson equation:

$$\nabla^2\varphi = -4\pi\rho \qquad \text{A. 2}$$

where φ is the potential associated with the wave. The Laplacian operator can be decomposed into transverse and longitudinal components in cylindrical coordinates with steady magnetic field in the axial direction:

$$\nabla^2\varphi = \nabla_T^2\varphi + \nabla_{//}^2\varphi = -(p^2+k^2)\varphi = -4\pi\rho \qquad \text{A. 3}$$

where p and k are the eigenvalues of the transverse and longitudinal operators ∇_T^2, $\nabla_{//}^2$, respectively.

Considering the ions as a smeared-out positive charge background which does not respond to the space charge weve frequency, the local space charge density ρ can be related to electron number density fluctuation δn by:

$$p = -e\delta n \qquad \text{A. 4}$$

$$\delta n = -\frac{\rho}{e} = -\frac{(p^2+k^2)}{4\pi e}\varphi \qquad \text{A. 5}$$

By substituting the space charge wave dispersion relation[7]:

$$p^2+k^2 = k^2\frac{\omega_p^2}{\Omega^2} \qquad \text{A. 6}$$

into A. 5, we have:

$$\delta n = -\frac{k^2}{4\pi e}\frac{\omega_p^2}{\Omega^2}\varphi$$

$$= -\frac{ek^2}{m\Omega^2}n_o\varphi \qquad \text{A. 7}$$

$$= -\frac{e\varphi}{mV_\varphi^2}n_o$$

where $\omega_p^2 = \frac{4\pi n_o e^2}{m}$ is the plasma frequency, $V_\varphi = \frac{\Omega}{k}$ is the phase velocity of the wave and n_o the average electron number density. Thus we come to the conclusion that the fluctuation amplitude and wave amplitude are related to each other in a simple way:

$$\langle\overline{\delta n}\rangle^2=\left(\frac{e\varphi}{mV_\varphi^2}n_o\right)^2 \qquad \text{A. 8}$$

Once we have related the space charge wave to the fluctuation, the existing theories on scattering of electromagnetic waves from plasma density fluctuations can be applied directly with this minor modification. This fluctuation amplitude of the space charge wave has been used extensively in the single particle model.