THEORY OF DISTRIBUTED AMPLIFIER USING TRANSMISSION LINES AMD MOS TRANSISTORS

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INTRODUCTION

There are increasing demands for extremely broadband amplifiers. Conventional amplifiers have been shown to have a maximum gainband width product for a given tube or transistor type, no matter how complex the coupling system between stages. Aside from this limitation, any attempt to exceed much more than 50% of the theoretically obtainable maximum will meet practical difficulties. The traveling-wave technique has provided a means for wide-band amplification at microwave frequencies. The traveling-wave tube must be electrically long, and practical limitations make it improbable that such tubes will be available for frequencies much below 1 GC.

The gain-band width product for conventional amplifiers can be expressed as

$$G.\ BW = \frac{g_m}{2\pi(C_g + C_p)} \tag{1}$$

where g_m is the transconductance of the device, C_g and C_p are its input and output capacitances. Parallel connection of the active devices does not help the gain-band-width product since as g_m is increased, capacitances are also increased by the same parallel connection.

A distributed amplifier utilizes sections of inductors and capacitors in both its input and output circuits. These sections serve as filters and artificial delay lines. As the signal flows to the load, g_m's add but C_g's ang C_p's are separated by these sections. The conventional restrictions on gain-band-width are completely removed, thus the high frequency limit being determined entirely by effects within the device proper.

Distributed amplification has been employed in video and broadband amplifiers using vacuum tubes,[1-9] and it has also been used with transistors.[10,11] A problem encountered in the use of transistors with distributed amplifiers is the difficulty in designing fixed input inpedance over the very broad frequency range of these amplifiers.[10] This together with the low input impedance of transistors requires special networks for compensation, limits the impedance of input filters to low values and reduces the gain in cascaded amplification.

This paper describes the development of the characteristics and design equations for distributed amplification using transmission lines and field-effect transistors.

Since the distributed amplification is actually accomplished by applying traveling-wave concepts to the lower frequency region, it is proposed to use a transmission line for distributed amplification instead of traditional lumped inductors and capacitors. When the line is loaded by transistor capacitances, it then behaves partly like π-filter sections.

A transmission line section can be approximated by a π-network having series inductance and shunt capacitance elements, the latter combining with the loading capacitances to form a filter. This result is used in the first step of the design, but this is true only when the line is very short or the frequency is very low. At cutoff frequency, the phase shift per section is as high as 180 degrees, although this should be attributed to both the transmission line and the loading capacitances. On the high frequency side of the amplified band, the line is usually not very short. More exact formulas are calculated and compared with the lumped filter amplifier. Differences in characteristics between the distributed line type and the lumped type are discussed, with the former showing better performance.

The relation between phase shift and frequency is non-linear for the lumped filter as indicated in equation (5), of Chapter II. As a lossless transmission line gives no phase shift distortion, the behavior of the transmission line distributed amplifier is somewhat modified. It will be shown that the filter made of a transmission line and loading capacitances has better phase shift characteristics than lumped filter sections. This then is a method of improving the frequency response, the use of which is not limited to the thin-film case. A more linear phase shift is particularly important in case a large number of stages are to be cascaded.

The impedance of the lumped filter increases appreciably with frequency and near cutoff, a large undesired peak is produced. The transmission line filter has smaller impedance variation over the pass band, therefore a smoother amplitude response can be expected. The cutoff frequency of the transmission line filter is higher than that calculated from its lumped model, the peak is shifted to the right and its frequency coverage is narrowed. By choosing slightly different lengths of transmission line sections for the input and output circuits, it is possible to reduce the peaking effect by the phase shift difference between them near cutoff.

A transmission line is more readily realized than an inductor in a thin-film circuit since the amount of inductance that can be conveniently put iuto thin-film circuit is quite limited. It is difficult to make a pure inductor without introducing an appreciable amount of stray and distriduted capacitance.

The length of the transmission line to be used is important. Unloaded smooth lines will be rather long for the substrate at the frequency of interest, however the loading capacitances of MOS transistors will reduce the length of line required. It is convenient to use silicon monoxide as the dielectric medium which further reduces the line length. Strip sandwich line will be used here because of its adaptation for deposition processes and high isolation characteristic between lines. The almost perfect shield of the sandwich line eliminates the isolation problem between input and output circuits which must otherwise be given careful consideration.

The field-effect transistor has high input impedance and vacuum-tube-like characteristics. For MOS transistorrs, the input resistance is normally 10^9 to 10^{15} ohms, much higher than junction gate field-effect transistors. The MOS transistors are most easily fabricated by thin-film techniques.

Through microelectronic miniaturization, the proposed amplifier will not have those annoying problems encountered by ordinary distributed amplifiers. At frequencies near cutoff, the distributed amplifier is very sensitive to stray capacitances, lead wire inductances, oscillation, etc. This leads to an almost unpredictable behavior of the amplifier near cutoff. By miniaturization stray capacitances are reduced to a minimum and are fixed.

Lead inductance (inside and outside the tubes) in the grid and plate circuits of vacuum tubes has the effect of reducing the cutoff frequency and producing a peak near cutoff. The effect of the cathode lead inductance is even more serious.[1] This inductance, in conjunction with the grid-to-cathode capacitance, produces an input grid conductance which causes a grid loading effect. Self resonance of the lumped coils, attributable to capacitance between coil windings, yields limited amplifier performance at high frequencies.[6] The thin-film amplifier eliminates all these effects by having the shortest leads possible.

CHAPTER II

THEORY

The distributed amplifier principle, first disclosed by Percival,[12] was extensively discussed by Ginzton et. al. in 1948. In the following few years, there was active interest in this principle and numerous accounts of the design and performance were written. Since that time there has been little reference to this method of broadbanding, presumably due to the attention focused upon the then newly developing transistor, which does not seem to be as suitable for distributed amplification as the vacuum tube. As noted in the introduction, there is difficulty in designing a fixed input impedance over the broad frequency range for the transistor. Enloe and Rogers[11] have considered the problem and have succeeded in treating the transistor impedance as one similar to that of a tube for the tube formulas to hold. Beneteau and Blaser[10] have made an elaborate analysis of transistor input impedances and have designed a special network for emitter compensation of each transistor. The MOS transistor, however, can be readily adopted in the design of distributed amplifiers.

A single stage distributed amplifier using vacuum tubes is shown in Fig. 1. Two artificial lines made of filter sections are in the grid and plate circuits. The input signal passes through filter sections in the grid line to reach a grid. The amplified output signal also passes filter sections in the plate line to reach the load. No matter what path the signal may take from input to output, the same number of sections, equal to the number of active elements in the stage, is always passed. If the phase shift per section is the same, the signal will reach the load in phase,

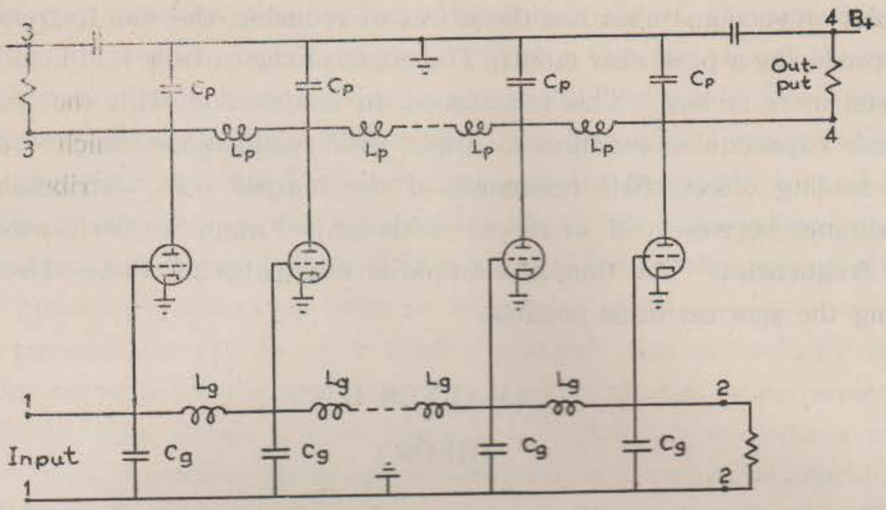


Fig. 1. Basic distributed amplifier using vacuum tubes

having an additive effect. The output voltage is therefore directly proportional to the number of tubes.

The artificial line shown is formed by π-section constant k filters with coil L_g (or L_p) as series element and $\frac{1}{2}C_g$ (or $\frac{1}{2}C_p$) at both ends as shunt elements. The image impedance of the π-network is

$$Z_\pi = \frac{\sqrt{Z_1 Z_2}}{\sqrt{1+\frac{Z_1}{4Z_2}}}, \tag{2}$$

where Z_1 and Z_2 are series and shunt impedances respectively. The cutoff frequency, occurring when Z_π is infinite, is

$$f_c = \frac{1}{\pi\sqrt{LC}} \tag{3}$$

where L, C are the total series inductance and shunt capacitance. The image phase shift per filter section θ at frequency f (angular frequency ω) of the lossless π-network in the pass band is given by

$$\cos\theta = 1 + \frac{Z^1}{2Z^2}, \tag{4}$$

or for the present case,

$$\cos\theta = 1 - \tfrac{1}{2}\omega^2 LC = 1 - 2\left(\frac{f}{f_c}\right)^2 . \tag{5}$$

It should be noted thst the phase shift at the cutoff frequency is 180 degrees. For equal phase shift per section the cutoff frequency of the two artificial lines should be the same. Thus, from (3):

$$L_g C_g = L_p C_p \tag{6}$$

and

$$f_c = \frac{1}{\pi\sqrt{L_g C_g}} = \frac{1}{\pi\sqrt{L_p C_p}}. \tag{7}$$

Formula (2) is then

$$Z_1 = \frac{\sqrt{\frac{L_g}{C_g}}}{\sqrt{1-(\frac{f}{f_c})^2}}, \tag{8}$$

$$Z_2 = \frac{\sqrt{\frac{L_p}{C_p}}}{\sqrt{1-(\frac{f}{f_c})^2}} \tag{9}$$

where Z_1 and Z_2 as before are image impedances of the input and output filter lines respectively. At low frequencies, these image impedances reduce, using (8) and (9), to:

$$R_1 = \sqrt{\frac{L_g}{C_g}}, \tag{10}$$

$$R_2 = \sqrt{\frac{L_p}{C_p}}. \tag{11}$$

If the grid filter chain is terminated at this image impedance, and if the line is dissipationless and without grid loading, the drivingpoint impedance at terminals 1-1 is independent of the number of tubes so connected. In a like fashion, the impedance which the tube plate sees is independent of the numder of tubes. The plate filter sections are terminated at both ends with resistances equal to R_2. The amplified signal in the plate circuit will flow in both directions, being divided equally and absorbed. The impedance connected to terminals 2-2 is called the grid termination. That connecued to terminals 3-3 and 4-4 are called the reverse termination and output termination (or load) respectively. Practically, the reverse termination can be removed without deteriorating effect and doubles the gain.[3]

The effective g_m of this distributed stage may be increased to any desired limit theoretically. Thus, no matter how low the gain of each section is (even if it is less than unity), the output will increase as one desires by merely using a sufficient number of sections. The distributed amplifier stage can be used alone or when sufficient gain has been accumulated in one stage, such stages can be cascaded. For different plate and grid image impedances, matching networks can be utilized between stages. Fig. 2. shows a way of direct coupling. The series inductor combines with shunt capacitance to form a matching network. The series capacitor serves to block the direct current.

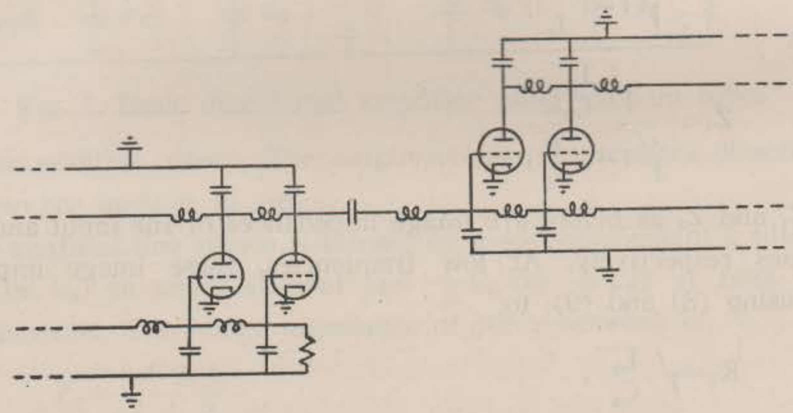


Fig. 2. Two stage distributed amplifier showing direct coupling

The distributed amplifier is the only means available for amplification when the maximum frequency desired is greater than the band width index frequency (where gain is unity) of the active device being used. with conventional circuits, it is usually found to be impractical to achieve much more than 50% of the theoretically aveilable band width. This is because the theoretical limit requires the use of extremely complex coupling circuits, which can hardly be considered practical and which increase the stray capacitance to ground. This is not the case in the distributed amplifier.

The basic filter sections shown in Fig. 1. and Fig. 2. are lowpass structures. The theoretical band is from the cutoff frequency down to zero frequency. In practical circuits, the response at the low frequency end is limited by the decoupling capacitors inserted for d-c biasing purpose.

The distributed anplifier can be made to operate at frequencies down to d-c by using standard d-c amplifier techniques. It is obvious that the principle is equally applicable to band-pass filters since they also have capacitors as their shunt elements as shown in Fig. 3. The grid bias supply for the active device in band-pass applications must go through a separate choke.

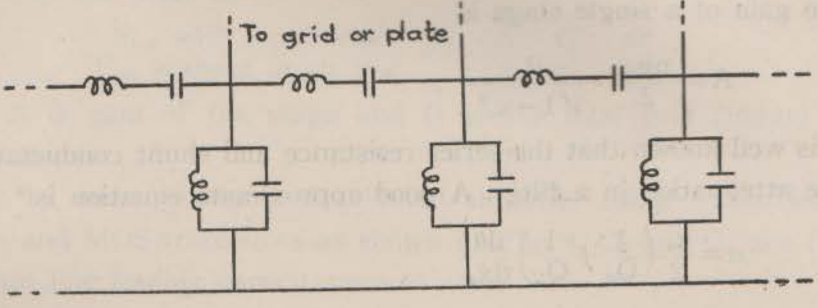


Fig 3. Band-pass filter chains

The voltage gain A for a single distributed amplifier stage is

$$A=\frac{ng_mZ_2}{2} \qquad (12)$$

where n is the number of sections in the stage and the output filter impedance Z_2 is much less than the plate resistance of the tube.

When the stage is cascaded, a transmission coefficient enters if the impedances of input and output circuits are different. It is proportional to the square root of the impedance ratio. When this is included, the gain of the amplifier consisting of n sections per stage and m cascaded stages can be expressed as

$$A=\left[\frac{ng_m}{2}\sqrt{Z_1Z_2}\right]^m \cdot \qquad (13)$$

If the two impedances are equal, then for a low-pass amplifier,

$$Z_1=Z_2=\frac{R_o}{\sqrt{1-x_k^2}} \qquad (14)$$

where

$$x_k=\frac{f}{f_c} \qquad (15)$$

$$R_o = R_1 = R_2 = \sqrt{\frac{L}{C}} = \frac{1}{\pi f_c C}. \tag{16}$$

C is the total shunt capacitance per section. Under this condition, the gain equation becomes

$$A = \left[\frac{ng_m}{2} R_o\right]^m (1 - x_K^2)^{-\frac{m}{2}} \tag{17}$$

and the gain of a single stage is

$$A = \frac{ng_m}{2} \frac{R_o}{\sqrt{1 - x_k^2}}. \tag{18}$$

It is well known that the series resistance and shunt conductance will produce attenuation in a filter. A good approximate equation is[13]

$$\alpha = \frac{x_k}{2}\left(\frac{1}{Q_C} + \frac{1}{Q_L}\right)\frac{d\theta}{dx_k} \tag{19}$$

$$= \frac{1}{4\pi f_C}\left(\frac{G}{C} + \frac{R}{L}\right)\frac{d\theta}{dx_k} \tag{20}$$

where

α = attenuation in nepers per section

x_k = the normalized frequency function

Q_C = the Q of the capacitors

Q_L = the Q of the coils

θ = the phase shift per section in radians

G = the shunt conductance across the capacitance C

R = the series resistance with inductance L

f_C = the cutoff frequency.

The first and second terms are obviously due to shunt and series losses respectively. The attenuation increases rapidly near cutoff as $d\theta/dx_k$ increases. The value of $d\theta/dx_k$ can be obtained from equation (5) and (15) for the low-pass filter case

$$\frac{d\theta}{dx_k} = \frac{2}{\sqrt{1 - x_k^2}} \tag{21}$$

and is approximately equal to 2 for the low frequency case.

The delay time, τ, per section is given by

$$\tau = \frac{d\theta}{d\omega} = \frac{1}{\omega_C}\frac{d\theta}{dx_k} = \frac{2}{\omega_C\sqrt{1 - x_k^2}}. \tag{22}$$

An important cause of grid loading is the effect of the cathode lead inductance L_c in conjunction with the grid-to-cathode capacitance C_{gc}. This grid (input) conductance is given by[14]

$$G=g_m\omega^2 L_c C_{gc}. \tag{23}$$

The ratio of output voltages with and without this input conductance loss can be expressed as[1]

$$\frac{E_{out}\text{ with input loss}}{E_{out}\text{ without input loss}}=1-\frac{A}{4}\frac{G}{g_m}\frac{d\theta}{dx_k} \tag{24}$$

where A is gain of the stage and G is the total grid (input) loading conductance.

Let us now consider a distributed amplifier using transmission line sections and MOS transistors as shown in Fig. 4. C_G and C_D are the gate and drain line loading capacitances.

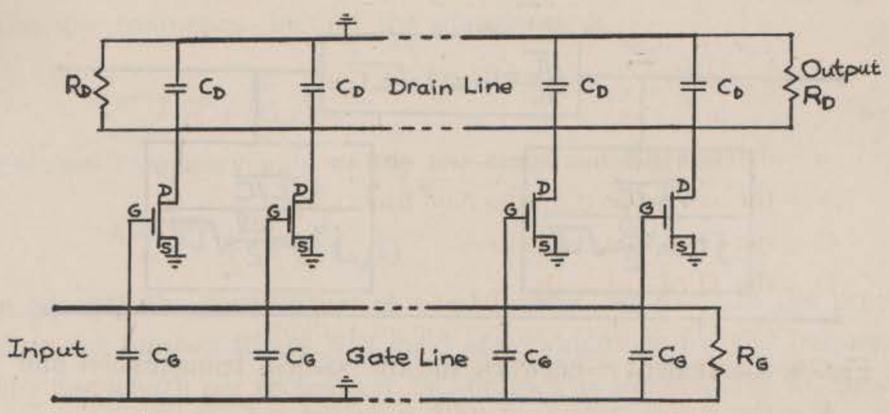


Fig. 4. Basic distributed amplifier using transmission line and MOS transistors

A section of transmission line can be represented by an equivalent π-network as shown in Fig. 5.,

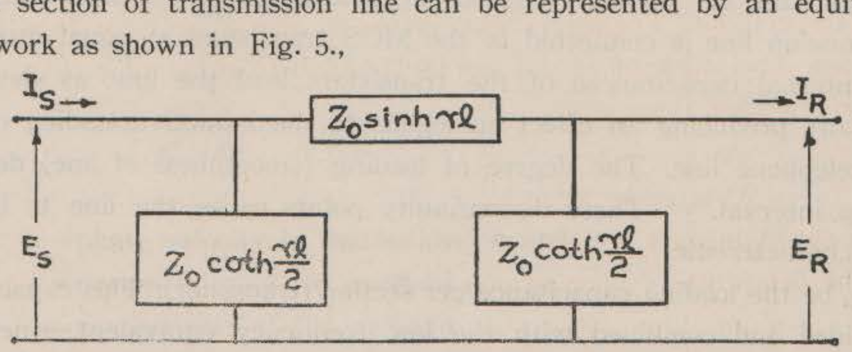


Fig. 5. Equivalent π-network for the transmission line

where Z_o, γ, ℓ, are the characteristic impedance, propagation constant and sectional length of the transmission line respectively. The equivalent circuit will yield the same phase shift, attenuation and terminal voltage current relations for the steady-state condition. The values of the elements in the equivalent circuit are functions of frequency. Thus, Z_o and γ will generally change with frequency. For a lossless line, Z_o is fixed and γ is equal to $j\beta$:

$$Z_o = \sqrt{\frac{L}{C}} \tag{25}$$

$$\gamma = j\beta = j\omega\sqrt{LC}. \tag{26}$$

Here L, C are the inductance and capacitance per unit length of the line. The equivalent network for lossless line section is then as shown in Fig. 6.

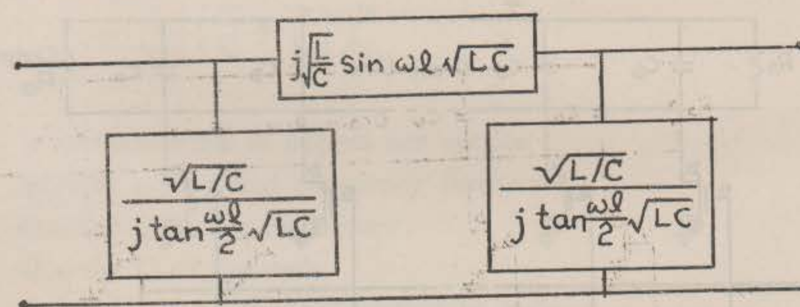
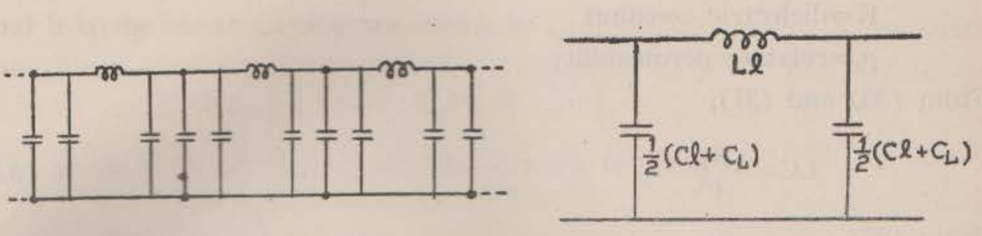


Fig. 6. Equivalent π-network for the lossless transmission line

Near the low frequency end of the pass band of the distributed amplifier where $\sin\omega\ell\sqrt{LC} \approx \omega\ell\sqrt{LC}$ and $\tan\frac{\omega\ell}{2}\sqrt{LC} \approx \frac{\omega\ell}{2}\sqrt{LC}$ the equivalent network can be further reduced to a lumped L, C network. The transmission line is connected to the MOS transistors at equal intervals. The internal capacitances of the transistors load the line, as shown in Fig. 7.(a), producing an effect analogous to the inductive loading on the of a telephone line. The degree of loading (smoothness of line) depends loading interval.[15,16] These discontinuity points cause the line to have a filter characteristic.

Let C_L be the loading capacitance per section (transistor). This capacitance is divided and combined with the low frequency equivalent π-networt, forming still another π-network, as shown in Fig. 7(b).



(a) (b)

Fig. 7. Capacitively loaded line

The cutoff frequency is roughly estimated by

$$f_c = \frac{1}{\pi\sqrt{L\ell(C\ell+C_L)}}\ , \tag{27}$$

since at cutoff frequency the approximation leading to Fig. 7(b) may not be true.

The low frequency limit of the impedance is

$$Z_\pi=\sqrt{\frac{L}{C+(C_L/\ell)}}\ . \tag{28}$$

The low frequency gain of the one-stage amplifier is

$$A=\frac{ng_m}{2}\sqrt{\frac{L}{C+(C_D/\ell)}}\ . \tag{29}$$

In general, the substitution of $L'=L\ell$ and $C'=C\ell+C_L$ in the previous equations for lumped filters will yield approximations for low frequencies.

The line length per section can be obtained as follows. The LC product, related to the phase velocity of the wave, is a constant:

$$v=\frac{1}{\sqrt{LC}} \tag{30}$$

and

$$v=\frac{1}{\sqrt{\epsilon\mu}}=\frac{c}{\sqrt{K\mu_e}} \tag{31}$$

where

v=phase velocity in meters per second
ϵ=permittivity of the medium
μ=permeability of the medium
c=velocity of light

K=dielectric constant

μ_r=relative permeability.

From (30) and (31),

$$LC=\frac{K\mu_r}{c^2}. \tag{32}$$

From (27),

$$L\ell(C\ell+C_L)=\frac{1}{\pi^2 f_c^2}. \tag{33}$$

Combining (32) and (33), the line length equation is

$$\frac{K\mu_r}{c^2}\ell^2+C_L L\ell-\frac{1}{\pi^2 f_c^2}=0. \tag{34}$$

Since the characteristic impedance of the line Z_o is

$$Z_o=\sqrt{\frac{L}{C}}=vL, \tag{35}$$

it is seen from equation (34) that for higher dielectric constant, permeability, loading capacitance or characteristic impedance, shorter ℓ can be used, as practical considerations such as the size of the amplifier may dictate. It will be seen, however, that longer ℓ gives better performance. In practice, therefore, a compromise will be reached.

Consider now the equivalent network of Fig. 6. The loading capacitors, equal to $C_L/2$, are to be added at both ends. Combining the shunt elements with the shunting capacitors yields a network shown in Fig. 8.,

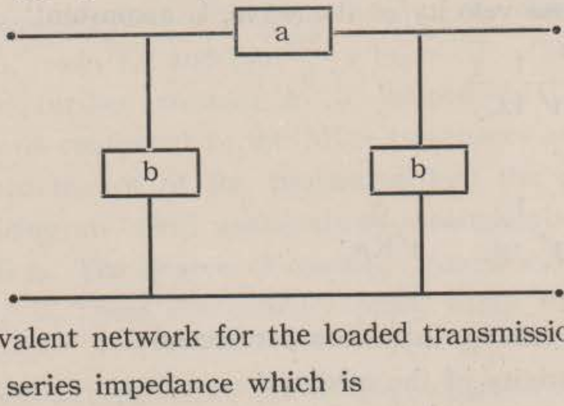


Fig. 8. Equivalent network for the loaded transmission line section

where a is the series impedance which is

$$a=j\sqrt{\frac{L}{C}}\sin\omega\ell\sqrt{LC}, \tag{36}$$

and b is the shunt admittance which is

$$b=j\left(\sqrt{\frac{C}{L}}\tan\frac{\omega\ell}{2}\sqrt{LC}+\frac{\omega C_L}{2}\right). \tag{37}$$

The phase shift per section of this network is given by

$$\cos\theta=1+\frac{Z_1}{2Z_2}=1+ab \tag{38}$$

$$=1-\sin\omega\ell\sqrt{LC}\tan\frac{\omega\ell}{2}\sqrt{LC}-\frac{\omega C_L}{2}\sqrt{\frac{L}{C}}\sin\omega\ell\sqrt{LC} \tag{39}$$

Use of the trigonometric identity $1\text{-}\sin\varphi\tan\frac{\varphi}{2}=\cos\varphi$ yields the equation

$$\cos\theta=\cos\omega\ell\sqrt{LC}-\frac{\omega C_L}{2}\sqrt{\frac{L}{C}}\sin\omega\ell\sqrt{LC}. \tag{40}$$

The actual cutoff frequency occurring when θ is 180°, will not be the lumped cutoff frequency f_c (i. e., the cutoff frequency if the line is replaced by a fixed LC network), but it will be expressed in term of x_k which is a relative measure of frequency with reference to f_c.

Let

$$C_L=C'\ell \tag{41}$$

where C' would be the loading capacitance per meter if it were uniformly distributed. By defining a loading ratio or loading percentage k,

$$k=\frac{C'}{C}, \tag{42}$$

the lumped cutoff frequency can be expressed as

$$f_c=\frac{1}{\pi\sqrt{L\ell(C\ell+C'\ell)}}$$

$$=\frac{1}{\pi\ell\sqrt{LC}\ \sqrt{1+k}} \tag{43}$$

or

$$\sqrt{1+k}=\frac{1}{\pi\ell f_c\sqrt{LC}}$$

$$=\frac{2}{\omega_c\sqrt{LC}\,\ell}=\frac{2}{\beta_c\ell}$$

$$=\frac{2}{\phi} \tag{44}$$

where

ϕ = the angular length of the section of the line at the lumped cutoff frequency, i.e., the phase shift per sectional length of a pure unloaded line at f_c.

ω_c = the lumped angular cutoff frequency.

β_c = the phase shift constant of the line at the lumped cutoff frequency.

Thus, the relation between k and ϕ is

$$\phi=\frac{2}{\sqrt{1+k}} \tag{45}$$

or

$$k=\frac{4}{\phi^2}-1 \ . \tag{46}$$

It is seen that the angular length of the sectional line needed for the distributed amplification is a function of the loading percentage only. It is independent of the cross-sectional geometry of the line, i.e., the shape and spacing, the L/C ratio or the impedance level.

It is interesting to notice that for values ϕ greater than 2, k will be negative. This indicates a limit on the sectional length of the line. The useful sectional angular length of the line is between zero and 2 radians at the lumped cutoff frequency f_c unless the loading capacitance is negative.

Equation (45) shows that in order for the lumped model of Fig. 7 to yield a good approximation, the loading ratio needs to be around a hundred or more, as is the case of telephone inductive loading. This is, however, seldom the case for the distributed amplifier in view of the low value of the input capacitance of the active device.

Since

$$\omega \ell \sqrt{LC}=\frac{f}{f_c}\omega_c \ell \sqrt{LC}$$

$$=\phi x_k \ , \tag{47}$$

the substitution of equations (41), (42) and (45) into equation (40) will yield the result:

$$\cos\theta = \cos(\phi x_k) - \frac{k}{2}\,\phi x_k \sin(\phi x_k)$$

$$= \cos(\phi x_k) - \left(\frac{2}{\phi} - \frac{\phi}{2}\right) x_k \sin(\phi x_k). \qquad (48)$$

The phase shift per section of the transmission line type distributed amplifier θ is plotted in Fig. 9 as a function of normalized frequency x_k with the angular length of section ϕ [also the loading ratio k by virtue of equation (46)] as parameters. When $\phi=2$, the second term vanishes, and represents the unloaded pure line limit which is a straight line (no distortion). When $\phi=0$, the formula reduces to equation (5) which represents the lumped filter case. For other values of ϕ the curves lie between these two limits. It may be noted from Fig. 9 that the phase shift distortion can be reduced to any desired amount by choosing a proper value of ϕ (or k).

The straight line, representing the pure line limit, intersects the $\theta=\pi$ line at $\pi/2$ times the lumped cutoff frequency. The cutoff frequency of the transmission line type distributed amplifier is therefore raised from f_c to a value between f_c and $\frac{\pi}{2} f_c$, given by the equation:

$$\cos(\phi x_k) - \left(\frac{2}{\phi} - \frac{\phi}{2}\right) x_k \sin(\phi x_k) = -1 \qquad (49)$$

or

$$x_k\left(\frac{2}{\phi} - \frac{\phi}{2}\right) = \cot\frac{\phi x_k}{2}\ . \qquad (50)$$

The actual cutoff frequency is plotted in Fig. 10 as a function of ϕ (and k).

The impedance of the section can be found by terminating an unknown admittance Y at one end of the network shown in Fig. 8 and by demanding that the admittance looking from the other end into the network be Y. Therefore,

$$\frac{1}{\dfrac{1}{b+Y} + a} + b = Y \qquad (51)$$

where a and b are given by equations (36) and (37). From equation (51),

$$b + Y = (Y - b)(1 + ab + aY)$$

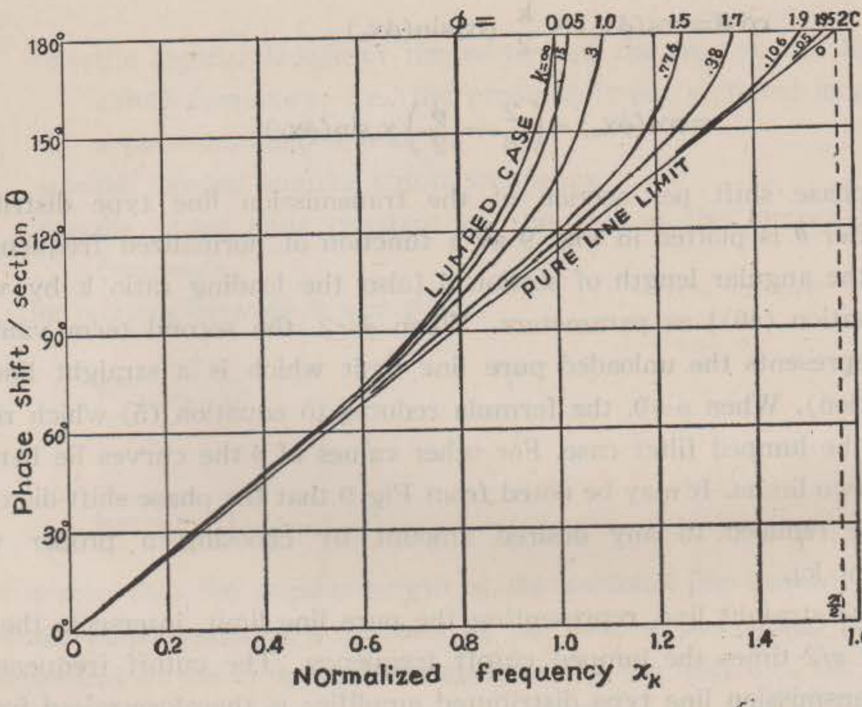


Fig. 9. Phase shift characteristics

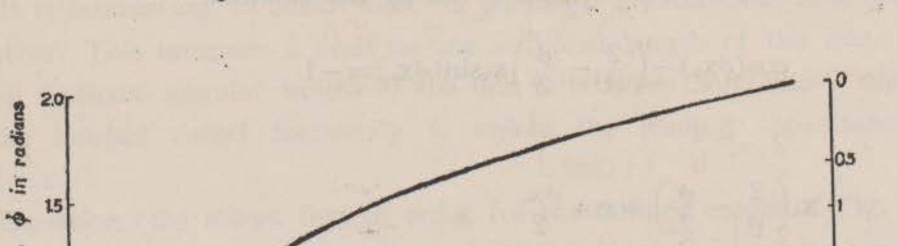


Fig. 10. Normalized cutoff frequency versus normalized line length and loading ratio

or

$$Y=\sqrt{\frac{b(2+ab)}{a}} \tag{52}$$

and

$$Z=\sqrt{\frac{a}{b(2+ab)}} \, . \tag{53}$$

From equation (38),

$$ab=\cos\theta-1. \tag{54}$$

Then,

$$Z=\left[\frac{a}{b}\ \frac{1}{1+\cos\theta}\right]^{\frac{1}{2}} \tag{55}$$

where

$$\frac{a}{b}=\frac{L}{C}\ \frac{\sin\omega\ell\sqrt{LC}}{\tan\frac{\omega\ell}{2}\sqrt{LC}+\frac{\omega C_L}{2}\sqrt{\frac{L}{C}}} \tag{56}$$

$$=\frac{L}{C}\ \frac{\sin(\phi x_k)}{\tan\left(\frac{\phi x_k}{2}\right)+x_k\left(\frac{2}{\phi}-\frac{\phi}{2}\right)} \tag{57}$$

and $\cos\theta$ is given by equation (48). Alternately;

$$Z=\left[\frac{a}{b}\ \frac{1}{1+\cos\theta}\right]^{\frac{1}{2}}$$

$$Z=\left[\frac{a^2}{\cos\theta-1}\ \frac{1}{1+\cos\theta}\right]^{\frac{1}{2}}$$

$$=\frac{\sqrt{-a^2}}{\sin\theta}$$

$$=\sqrt{\frac{L}{C}}\ \frac{\sin\omega\ell\sqrt{LC}}{\sin\theta} \tag{58}$$

$$=\sqrt{\frac{L}{C}}\ \frac{\sin\phi x_k}{\sin\theta} \, . \tag{59}$$

In the pass band;

$$0\leqq\theta<\pi, \qquad 0\leqq\phi\leqq 2, \qquad 0\leqq x_k\leqq\frac{\pi}{2} \, , \tag{60}$$

the impedance is a positive real number. The low frequency impedance is not $\sqrt{L/C}$, but can be determined by equations (55) and (57) as x_k approaches zero as:

$$Z=\left[\frac{L}{C}\ \frac{\sin(\phi x_k)}{\tan\left(\frac{\phi x_k}{2}\right)+\phi x_k\left(\frac{2}{\phi^2}-\frac{1}{2}\right)}\ \frac{1}{1+\cos\theta}\right]^{\frac{1}{2}}$$

$$=\left[\frac{L}{C}\ \frac{1}{\frac{1}{2}+\left(\frac{2}{\phi^2}-\frac{1}{2}\right)}\ \frac{1}{2}\right]^{\frac{1}{2}}$$

$$=\sqrt{\frac{L}{C(1+k)}}\ . \tag{61}$$

This equation coincides with equation (28). The normalized impedance characteristic is plotted in Fig. 11 with ϕ (and k) as parameters. Comparing with the lumped characteristics in Fig. 11, the transmission line type has a more linear amplitude response and a wider useful frequency range approaching the pure line limit whose impedance is a constant.

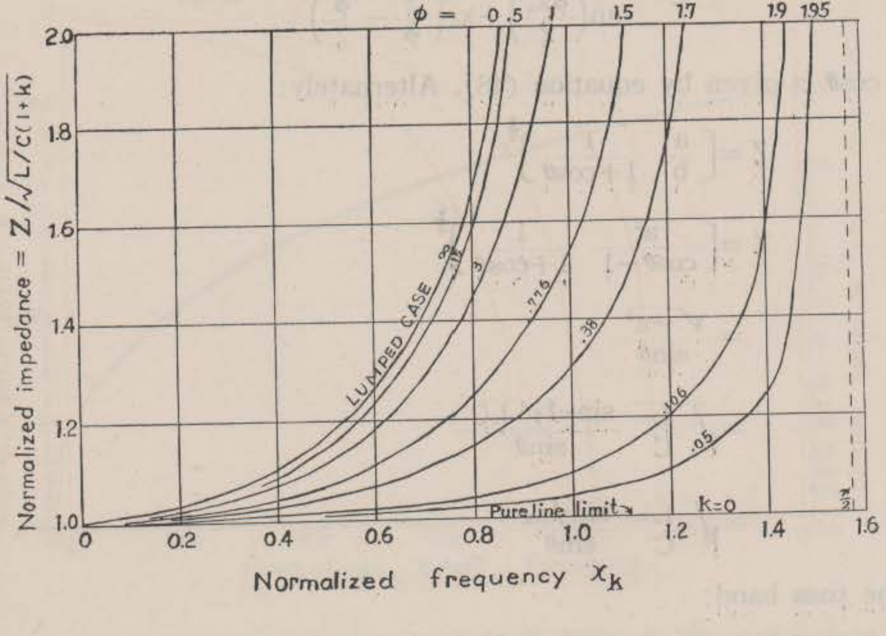


Fig. 11. Normalized Impedance characteristics

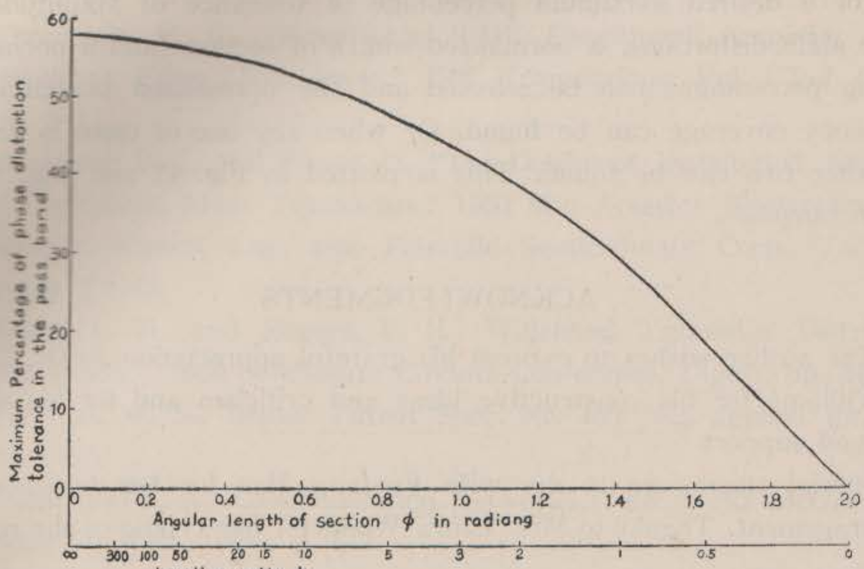


Fig. 12. Percent distortion tolerance versus normalized line length and loading ratio

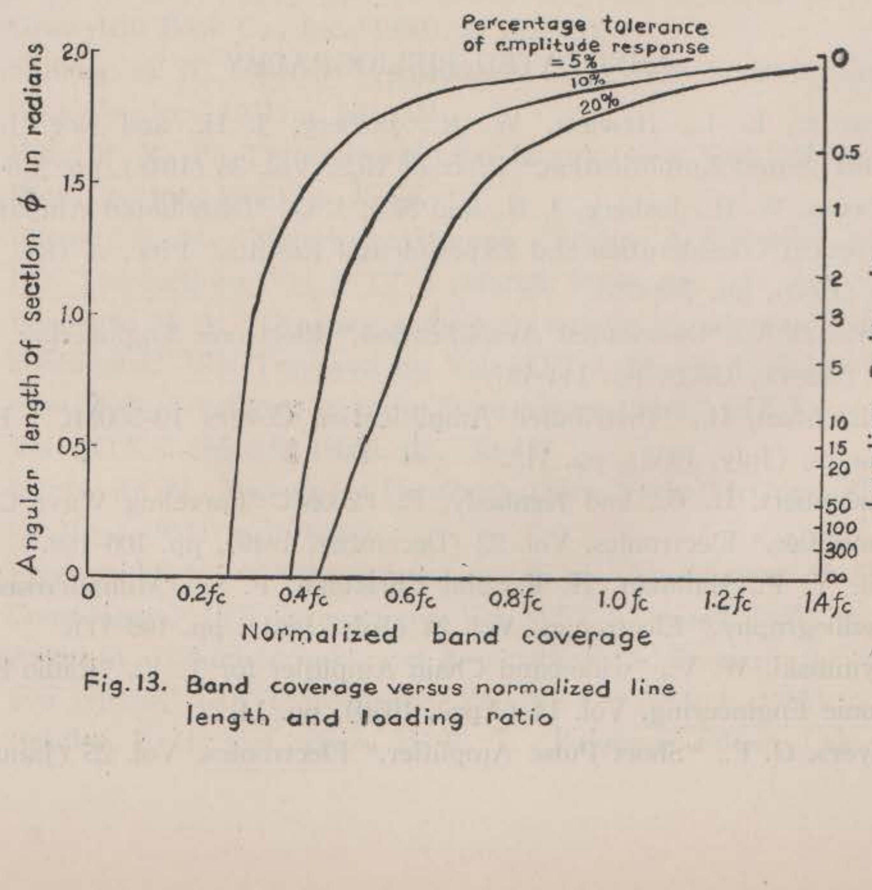


Fig. 13. Band coverage versus normalized line length and loading ratio

For a desired maximum percentage of tolerance of amplitude and phase shift distortions, a normalized length of section (and a normalized loading percentage) can be selected and the normalized percentage of frequency coverage can be found. Or when any one of them is defined, the other two can be found. This is plotted in Fig. 12 and Fig. 13 for design purposes.

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ANNOTATED BIBLIOGRAPHY

1. Ginzton, E. L., Hewlett, W. R., Jasberg, J. H., and Noe, J. D., "Distributed Amplification," Proc. of IRE, Vol. 36 (1948), pp. 956-969.
2. Horton, W. H., Josberg, J. H., and Noe, J. D., "Distributed Amplifiers: Practical Consideration and Experimental Results," Proc. of IRE, Vol. 38 (1950), pp. 748-753.
3. Cormark,A., "Distributed Amplification," Electronic Engineering, Vol. 24 (March, 1952), pp. 144-147.
4. Scharfman, H., "Distributed Amplification Covers 10-360MC," Electronics, (July, 1952), pp. 113.
5. Rudenbert, H. G., and Kennedy, F., "200MC Traveling Wave Chain Amplifier," Electronics, Vol. 22 (December, 1949), pp. 106-109.
6. Yu, Y. P., Hallman, H. E., and Christaldi, P. S., "Millimicrosecond Oscillography," Electronics, Vol. 24 (July, 1951), pp. 106-111.
7. Tyminski, W. V., "wide-band Chain Amplifier for T. V., "Radio Electronic Engineering, Vol. 14 (April, 1950), pp. 14.
8. Myers, G. F., "Short Pulse Amplifier," Electronics, Vol. 25 (January,

1952), pp. 128-131.

9. Thompson, F. C., "Broad-band UHF Distributed Amplifier Using Bandpass Filter Techniques," IRE Transactions, Vol. CT-7 (1960), pp. 8-17
10. Beneteau, P. J., and Blaser, L., "The Design of Distributed Amlifiers Using Silicon Mesa Transistors," 1960 Mid-America Electronics Conference, Kansas City, also Fairchild Semicoductor Corp., Technical paper TP-13.
11. Enloe, L. H., and Rogers, P. H., Wideband Transistor Distributed Amplifiers," 1959 Solid-state Circuits Conference, Digest, pp. 44-45.
12. Percival, w. S., British Patent Spec. No. 460. 592, applied for, July 24, 1936.
13. Guillemin, E. A., Communication Networks (New York: McGraw-Hill Book Co., Inc., 1935), p. 447.
14. Teman, F. E., Radio Engineering (New York, McGraw-Hill Book Co., Inc., 1947), p. 369-371.
15. Johnson, W. C., Transmission Lines and Networks (New York, McGraw-Hill Book Co., Inc., 1950), p. 352-353.
16. Skilling, H. H., Electric Transmission Lines (New York: McGraw-Hill Book Co., Inc., 1951),p. 202-209.
17. King, R. W. P., Transmission-Line Theory (New York: McGraw-Hill Book Co., Inc., 1955), p. 45-46.
18. Barrett, H. M., "Microwave Printed Circuits—A Historical Survey," IRE Transactions, Vol. MTT-3 (March, 1955), pp. 1-9.
19. Begovich, N. A., "Capacity and Characteristic Impedance of Strip Transmission," IRE Transactions, Vol. MTT-3 (March, 1955), pp. 127-133
20. cohn, S. B., "Problems in Strip Transmission Lines," IRE Transactions, Vol. MTT-3 (March, 1955), pp. 119-126.
21. Marcuvitz, N., Waveguide Handbook (New York: McGraw-Hill Book Co., Inc., 1951), p. 263-265.
22. Fromm, W. E., "Characteristics and Some Applications of Stripline Components," IRE Transactions, Vol. MTT-3 (March, 1955), pp. 13-20.
23. Arditi M., "Characteristics and Applications of Microstrip for Microwave Wiring," IRE Transactions, Vol. MTT-3 (March, 1955), p. 45.
24. Bradley, E. H., and white, D. R. J., "Bandpass Filters Using Strip

Line Techniques," IRE Transactions, Vol. MTT-3 (March, 1955), PP. 163-169.

25. Torgow, E. N., and Griemsmann, J. W. E., "Miniature Strip Transmission Line for Microwave Applications," IRE Transactions, Vol. MTT-3 (March, 1955), p. 57.
26. Ramo, S., and Whinnery, J. R., Fields and Waves in Modern Radio (New York: John Wiley & Sons, Inc., 1962), p. 248.
27. Borkan, H., and Weimer, P. K., "An Analysis of the Characteristics of Insulated-Gate Thin-Film Transistors," RCA Review, June 1963, p. 153.