FREQUENCY CONTROL
By
PASSTVE MICROWAVE ATOMIC RESONANCE

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Abstract

This article is a study of the Cesium beam resonant response and gives a servo analysis of its application to precision frequency control system.

1. INTRODUCTION

The possibility for controlling frequency by atomic or molecular quantum effects was widely recognized following the Second World war when the first accurate results in microwave spectroscopy were obtained(1). At present, it is generally termed "Atomic Frequency Control" and falls into two broad classes involving active atomic resonators on one hand and passive resonators on the other. The active devices are masers which provide a signal consisting of stimulated emission of microwave energy from the atomic particles. In frequency control by maser, the signal intensity of the direct output is too low for practical application and additional electronic apparatus is required to increase the available power. Ammonia masers and recent atomic Hydrogen masers(2) are examples of active atomic frequency control devices. Hydrogen masers can improve the frequency control with an accuracy in the order of 1×10^{-13}. Passive atomic resonators are designed to stabilize the conventional oscillator by the use of feedback control circuits. The spectral characteristics of the control signal are, of course, influenced by the nature of the feedback circuits as well as the response of the resonator. Rubidium optical absorption cells(3) and Cesium beam frequency standards(4) are examples of passive frequency control devices. The Cesium resonator can achieve frequency control with an accuracy and stability in the order of 1×10^{-12}.

The main task of this article is to study the resonator response and gives a servo analysis of the passive atomic frequency control system based on the Cesium beam frequency standard as an example.

2. CESIUM BEAM RESONATOR

The microwave quantum effects of interest arise from the magnetic nuclear forces caused by the nuclear magnetic hyperfine structure of the electronic ground state of Cesium atoms. (Cesium, like Hydrogen, is an atom with single valance electron.) The ground electronic state does not contribute magnetic effects due to the orbital motion of the electrons. However, there is a magnetic acting between the valance electron and the nucleus which does. It arises from the electron spin dipole moment and the nuclear spin dipole moment. Depending upon the orientation of the electron dipole moment with respect to the nuclear dipole moment, there are slight variations in the energy of the ground electronic states. For Cesium atom, the zero magnetic field electronic ground state is split into two energy levels, namely ($F=4$, $m_f=0$) and ($F=3$, $m_f=0$) as a result of the internal atomic particle configuration. The frequency corresponding to the transition of these two energy levels at zero magnetic field lies in the microwave region, i. e., at 9192631770 c/s.

The Cesium beam resonator is actually a typical beam spectrometer which is shown schematically in Figure 1. The Cesium neutral atoms effused from the oven at the left hand pass through the nonuniform magnetic field of the "state selector" A. It selects and allows the atoms in state $F=3$ passing through a precision designed microwave interaction cavity at its nominal frequency of the transition causes the atoms in state $F=3$ rise to state $F=4$. The atoms which rise to state $F=4$, then pass through the second magnetic "state selector" B and are deflected to strike a collector, a hot wire, which ionizes each impinging atom. The ions pass through a mass spectrometer into an electron multiplier and are converted into output current.

The resonant response of the beam current as a function of applied microwave frequency has been shown to have the characteristics as are illustrated in Figure 2[4]. Because it is an even function about 9192.63…MC, it is possible to use the beam resonance by applying audio phase modulation to control the frequency of crystal oscillstor. Referring to Figure 3, by letting w_m be the angular frequency of the audio phase modulating signal, w_c be the angular frequency of the signal generated directly from the crystal oscillator, and w_s the angular frequency of the signal after the frequency synthesis system, then the actual signal applied to the microwave cavity can be written in the form

$$M(t)=A \cos (w_S t+\theta_m \cos w_m t) \quad (1)$$

and its frequency is

$$w_r=w_S+\theta_m \cos w_m t \quad (2)$$

Let w. be the transition frequency of the resonator and Wi be the frequency difference between the microwave field and the atomic transition, then

$$\begin{aligned} w_i &= w_r-w_0 \\ &= w_S-w_0+\theta_m \cos w_m t \\ &= r+\theta_m \cos w_m t \end{aligned} \quad (3)$$

where $r=w_S-w_0$

Investigating the response curve near w_0, the frequency response curve nearby w_0 can be fitted by a parabola

$$I_b=B\ (1-w_i^2) \quad (4)$$

By Eq. (3), if the microwave freqency is above the transition frequency, i. e., r is positive then

$$\begin{aligned} I_b &= B[1-(r+\theta_m \cos w_m t)^2] \\ &= B[(1-r^2-\theta_m^2/2)-2\theta_m r \cos w_m t+\theta_m^2/2 \cos 2w_m t] \end{aligned} \quad (5)$$

It shows that the fundamental and second harmonics will appear in the output. If the microwave frequency is below the transition frequency, i. e., r is negative, the output beam current will be the same as in Eq. (5), and the fundamental and second harmonic currents also appear. If the microwave frequency is tuned exactly to the transition frequency, then r is zero and

$$I_b=B(1-\theta_m/2+\theta_m/2 \cos 2w_m t) \quad (6)$$

Only the second harmonic appears in the beam output. From this fact, the frequency difference r can be controlled by the fundamental frequency ontput signal.

3. FREQUENCY CONTROL LOOP

Based on the physical property stated above, the over-all control loop can be completed as in Figure 4, the simplest system diagram. The output signal from the Cesium Cesium beam resonator is fed to an AC amplifier tuned at the fundamental frequency w_m of the modulating signal (and eliminates the second harmonic output). The output of the AC amplifier is then proportional to the frequency difference r between the microwave input and the atomic transition. As the signal is compared with the original audio signal in the precision synchronous detector, the DC output of the detector is in turn proportional to the variable r, After amplifing it to a certain level, it is used to control the voltage of the VARACTOR in the crystal oscillator circuit, reducing the frequency error to zero.
It is easy to see that the system described in Figure 4, is actually a typical second order position control loop[5]. The crystal frequency output w_e is the control parameter and the over-all simplified sevo diagram is given as in Figure 5. The open-loop transfer function is

$$G(s)=\frac{w_s(s)}{r(s)}=K_1K_2K_3\frac{F(s)}{s}=K\frac{F(s)}{s} \tag{7}$$

where $K=K_1K_2K_3$. It is then possible to write the closed-loop transfer function for the system as

$$Y(s)=\frac{w_e(s)}{w_o(s)}=\frac{G(s)}{1+n\ G(s)}$$

$$=\frac{KF(s)/s}{1+n\ KF(s)/s}=\frac{KF(s)}{s+n\ KF(s)} \tag{8}$$

For the case of RC filter used, the transfer function F(s) may be shown to be

$$F(s)=\frac{1/T_1}{s+1/T_1},\ \text{where } T_1=RC \tag{9}$$

Using this to Eq. (8), yields

$$Y(s)=\frac{K/T1}{s(s+1/T_1)+n\ K/T_1}=\frac{K_0}{s^2+s/T_1+nK_0} \tag{10}$$

where $K_0=K/T_1$.
The denominator of Eq.(10) is of the form

$$D(s)=s^2+2\xi w_n\ s+w_n^2 \tag{11}$$

where $$\xi=\frac{1}{2w_n\ T_1} \tag{12}$$

is the ratio of actual damping to critical damping, and $w_n=\sqrt{n K_0}$ is the natural (undamped) frequency of the system. The roots are of course, the poles of Eq.(10) and are

$$S_1=-w_n\xi+jw_n\sqrt{1-\xi^2}=-a+jw_d \tag{13}$$

$$S_2=-w_n\xi-jw_n\sqrt{1-\xi^2}=-a-jw_d \tag{14}$$

where

$$a=w_n\xi \text{ and } w_d=w_n\sqrt{1-\xi^2} \tag{15}$$

is the damping frequency. The closed-loop transfer function Y(s) may be written in terms of its poles as

$$Y(s)=\frac{w_c(s)}{w_0(s)}=\frac{K_0}{(S-S_1)\ (S-S_2)} \tag{16}$$

As it is examined from its root-locus plot in Figure 6, it is easy to see that it never bebomes unstable regardless of the value of the gain,K_0.

4. NOISE POWER SPECTRA

The above analysis is based on the case of ideal crystal oscillator. Actually the crystal oscillator after a long term operation, still exhibits a small fluctuation in frequency called "flicker noise" which has been shown to have $\frac{1}{|w|}$ type power spectra. Here the noise generated in electron multiplier and phase detector and noise in VARACTOR in the crystal oscillator are going to be studied.

By letting $V_1(w)$ be the Fourier transform of the noise generated in the electron multiplier and detector, and $V_2(w)$ an equivalent noise in VARACTOR, then the power density of $V_2(w)$ can be written as

$$S_2(w)=\frac{h}{|w|} \tag{17}$$

In order to preserve the dimensions of volt-second for the addition network, it is convenient to assume that the output of the substraction network is $-\beta\dot{r}$, where β has the dimensions of volt-seconds. If the system of Figure 5 is assumed tobe zero frequency difference, then the equation governing the operation of the system can be expressed in frequency domain, as

$$[V_2(w)+\frac{V_1(w)-\beta\dot{r}(w)}{jw\tau_1}]n=\dot{r}(w) \tag{18}$$

where $\tau_1=1/K_0$. This leads to a power spectral density for $\dot{r}$ given by

$$S_{\dot{r}}(w)=\frac{C^2S_1(w)+n^2\tau_1{}^2\ h\ |w|}{w^2\tau_1{}^2+(\beta n)^2} \tag{19}$$

where C is a constant. For the case $w\ \tau_1$ becomes the order of (βn) for small w of order 10 sec^{-1},

$$S_{\dot{r}}(w)=\frac{1}{\alpha 2}\ S_1(w),\quad \alpha=\frac{\beta n}{C} \tag{20}$$

which gives the best prediction of exhibited noise in the system.

5. CONCLUSION

The frequency control using the passive atomic resonator, such as the Cesium beam resonator has been shown to be stable, independent of the over-all loop gain. The noises exhibited in itself are dominated by the noise generated from the phase detector and multiplier.

6. REFERENCES

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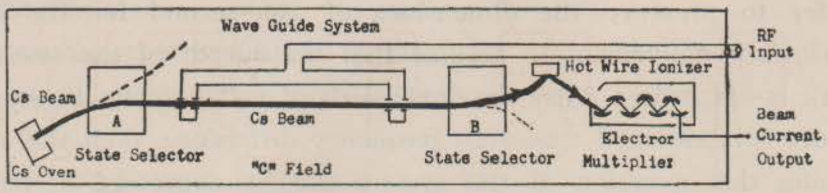

Fig. 1 Cesium Beam Resonator

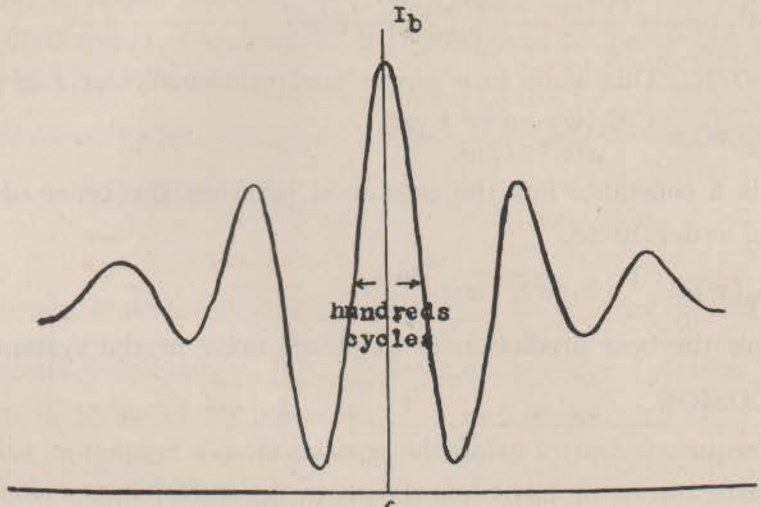

Fig. 2 BeAM CURRENT RESPONSE

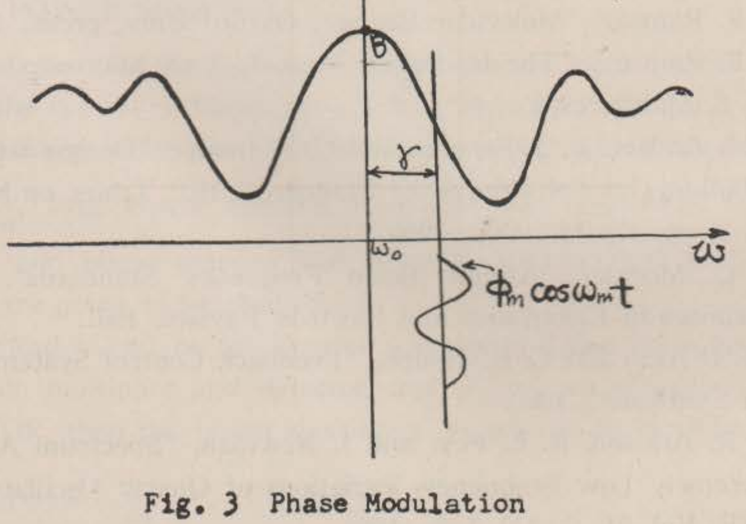

Fig. 3 Phase Modulation

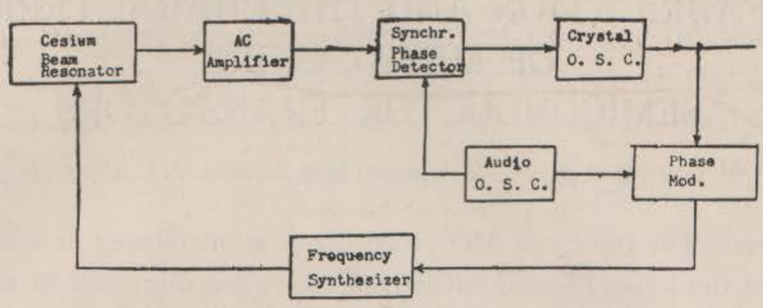

Fig. 4 Atomic Frequency Control

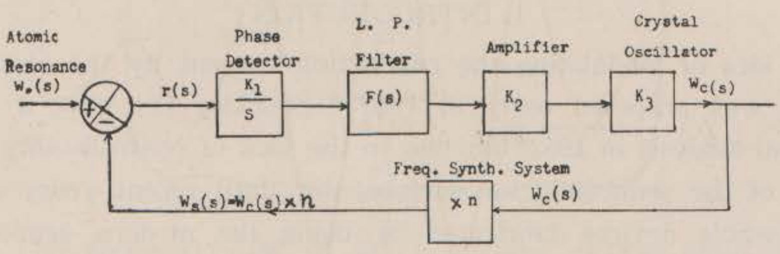

Fig. 5 Servo Diagram

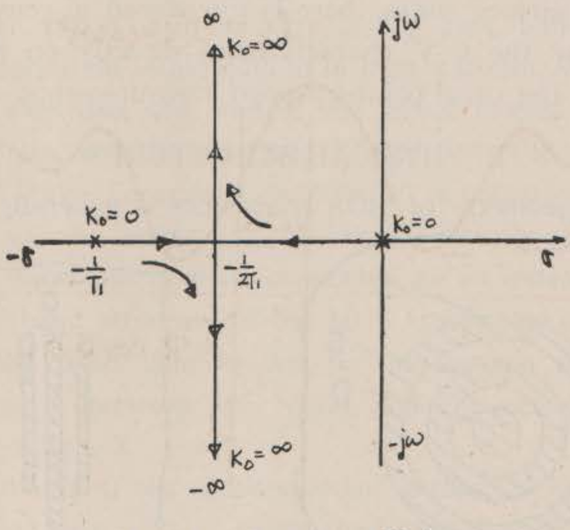

Fig. 6 Root Locus Plot