Propagation of Electromagnetic Waves

in

A Magnetoactive Plasma (I)

by

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Abstract

Microwave plasma diagnostic technique based upon the interaction of plasma with low intensity microwave field have been the subject of extensive development in recent years. We intended to carry on a systematic study of the microwave plasma interaction here at the plasma laboratory, Chiao-Tung University. This is the first report of our study. In this article we shall be dealing with a quasi-transverse mode with propagation vector making 45° with respect to the confining magnetic field, while the electric field of the wave is perpendicular to both the D. C. magnetic field and the propagation vector. A short theoretical derivation of the imaginary part of the index of refraction which predominates the attenuation of the probing wave is given. A computer plotting of the attenuation line shape as function of the confining magnetic field is given, and is found in good agreement to the observed result. The magnitude of the power attenuation is found also in good agreement.

I. Introduction

A plasma imbedded in a strong confining magnetic field is lossy, refractive, anisotropic, non-reciprocal, non-linear, non-homogeneous, dispersive and resonant when the plasma and electromagnetic wave interactions arise[1]. In this article we deal with the interaction of probing microwaves with an infinite, uniform, Lorentz plasma.

Microwave plasma diagnostic techniques based upon the interaction

of plasma with low intensity microwave field have been the subject of extensive development in recent years[2]. The basic advantages of using microwave to study plasma are that the plasmas are not "perturbed" as compared to other methods. The microwave techniques have been well developed such that its signal detecting sensitivity is high enough to use very low intensity microwave signals. Meanwhile, in most cases, the plasma can de probed by a microwave field without introducing any foreign body into plasma.

In this article we shall limit ourselves to the study of a quasi-transverse wave propagating 45° with respect to the D. C. magnetic field while the wave is polarized such that its electric vector is perpendicular to both the propagation vector and the magnetic field.

A short theoretical background of the derivation of the index of refraction was given in part II. Description of experimental set up, results and its interpretation were given in part III & IV respectively.

II. Theory

An infinite, unifrom magneto-active plasma with the confining strong D. C. magnetic field in Z-direction can be represented by a dielectric permittivity tensor ϵ_{ij}[3]. Solving the Maxwell's equations in the dielectric formulation the dispersion relation to each Fourier component can be expressed as

$$\det |\Lambda_{ij}| = 0 \tag{1}$$

where $\qquad \Lambda_{ij} = n^2 \left(\dfrac{k_i k_j}{k^2} - \delta_{ij} \right) + \epsilon_{ij} \qquad\qquad i, j = 1, 2, 3$

$\qquad$ n: index of refraction

$\qquad$ k: propagation constant

$\qquad$ δ_{ij}: Kroneckle delta

Choosing the coordinate as shown Fig. 1, where $\underline{k}$ lay on the y-z plane, the dispersion relation can be greatly simplified to
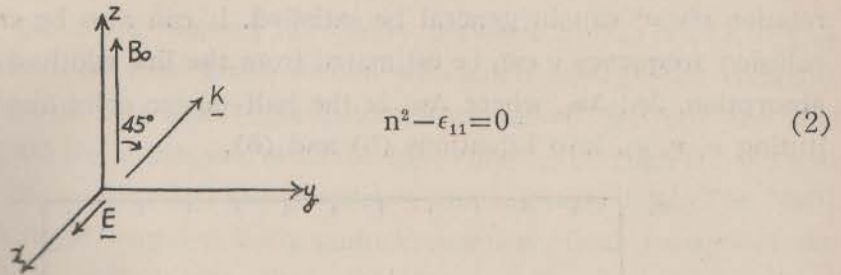
Fig1 Coordinate Used

In the cold plasma, high frequency limit, while including the pheno-
menological collision frequency v, ϵ_{11} can be expressed as[4].

$$n^2 - \epsilon_{11} = 0 \tag{2}$$

$$\epsilon_{11} = 1 - \frac{\omega_p^2(\omega + jv)}{\omega[(\omega + jv)^2 + \omega_c^2]} \tag{3}$$

where $\quad \omega_p : \sqrt{\dfrac{4\pi N_o e^2}{m}}$, electron plasma frequency

ω : electromagnetic wave frequency

$\omega_c : \dfrac{eB_o}{mc}$, electron-cyclotron frequency

v : phenomenological electron collision frequency

Combining equations (2) and (3), the dispersion relation becomes:

$$n^2 = 1 - \frac{\omega_p^2(\omega + jv)}{\omega[(\omega + jv)^2 + \omega_c^2]} \tag{4}$$

Let $\quad n = n_r - j\, n_i,$

Expressing the index of refraction separately by its real and imaginary
parts, we obtain

$$n_i^2 - n_r^2 = 1 - \frac{\omega_p^2(\omega^2 + v^2 - \omega_c^2)}{(\omega^2 - v^2 - \omega_c^2) + 4\omega^2 v^2} \tag{5}$$

$$2n_r\, n_i = \frac{\omega v(\omega^2 + v^2 + \omega_c^2)}{(\omega^2 - v^2 - \omega_c^2) + 4\omega^2 v^2} \tag{6}$$

where $\quad n_r$: real part of the index of refraction.

n_i : imaginary part of the index of refraction.

Examination of equations (5) & (6) reveals that resonance absorption
will occur in the vicinity of cyclotron frequency, $\omega^2 = \omega_c^2 + v^2 \doteq \omega_c^2$, since the

relation $v^2 \ll \omega^2$ can in general be satisfied. It can also be shown that the collision frequency v can be estimated from the line width of the resonance absorption, $2v \doteq \Delta\omega_b$, where $\Delta\omega_b$ is the half-power point line width. Substituting ω, v, ω_p, into Equations (5) and (6),
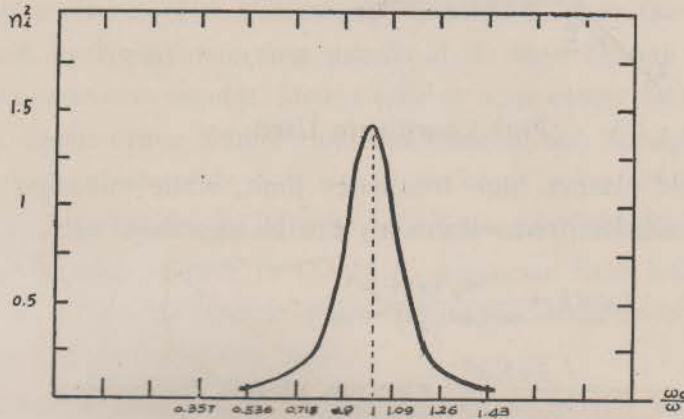

Fig. 2 Computer Calculated resonance line shape.

Figure 2 shows the computed resonance line shape. as a function of confining magnetic field. The collision frequency was obtained a priori from $v = 2.55 \times 10^9 p/_{sec.}$, ω_p from other independent measurement[6].

Right on top of the resonance, $\omega^2 \approx \omega_p^2 + v^2$, Equations (5) & (6) can further be simplified to:

$$n_i^2 \approx \left(\frac{\omega}{4v} - \frac{1}{2}\right) + \frac{\omega_p^2}{8\omega^2} \tag{7}$$

By keeping the pressure constant, we find that the absorption will increase with plasma electron number density.

III. Experimental Set up

The plasma was produced in the positive column of a Helium gas discharge tube, at a pressure of the order of few Torrs with background pressure around 10^{-4} Torr. The cathode was made of tungsten, and could be operated as hot or cold cathode. The diameter of the pyrax glass discharge tube was 6 mm. so that it could be fed through the holes on the narrow side walls of a x-band rectangular wave guide. The discharge could either be pulsed or in D. C. operation. The pulser had a pulse duration of hundreds of microsecond and a repitition rate of 500 to 1000 cps. The peak discharge current could be varried from few milli-ampere to 50

milli–ampere producing plasma of deusity between $10^{10}/_{cm^3}$ to $2\times10^{10}/_{cm^3}$. Since the discharge tube cross section was small enough, the complete set with discharge tube and wave guides were placed between the pole pieces of a NMR electromagnet, with the tube running through the holes at the center of the pole pieces, and parallel to the magnetic field. The NMR electromagnet could supply a fairly uniforin magnetic field throughout the operating volume which was about the cross sectional dimension of a x–band wave guide. Fig. 3. shows the wave guide with plasma tube running through its side walls. Fig. 4 shows the top view of the wave guide with the direction of wave propagation indicated.
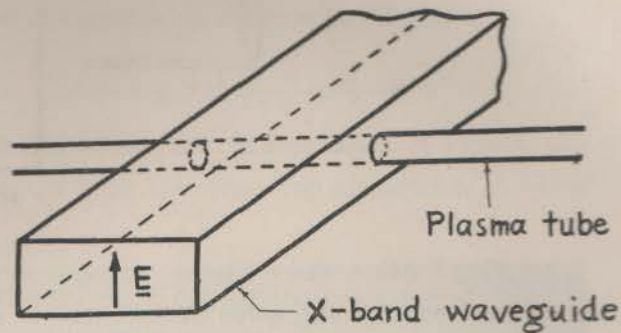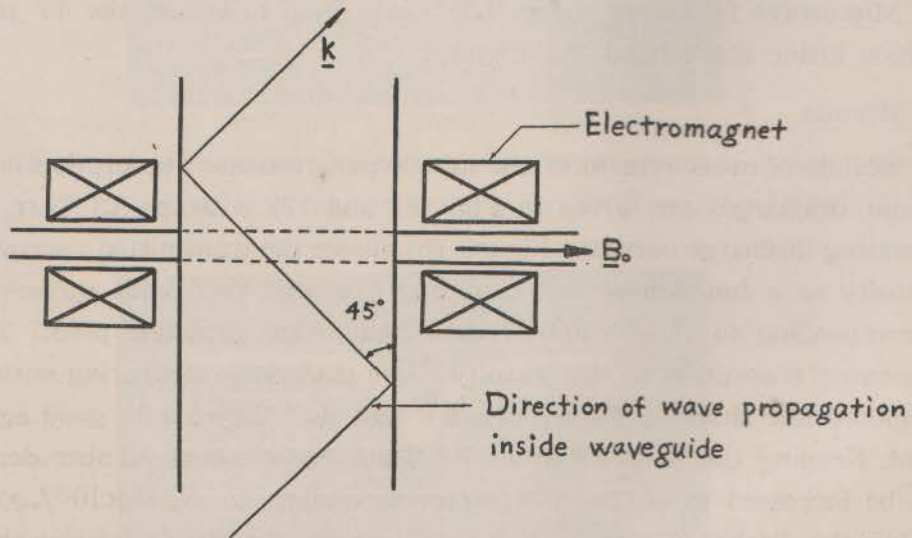
Fig 3   Waveguide section

Fig 4   Waveguide section top view

To prevent any non-linear heating of the plasma by the incident electro-magnetic wave, the probing microwave power level was kept to the order of milli-watt. Fiure 5 shows the microwave circuit used, a directional coupler in the incident side making it possible to measure the transmitted and reflected wave simultaneously.
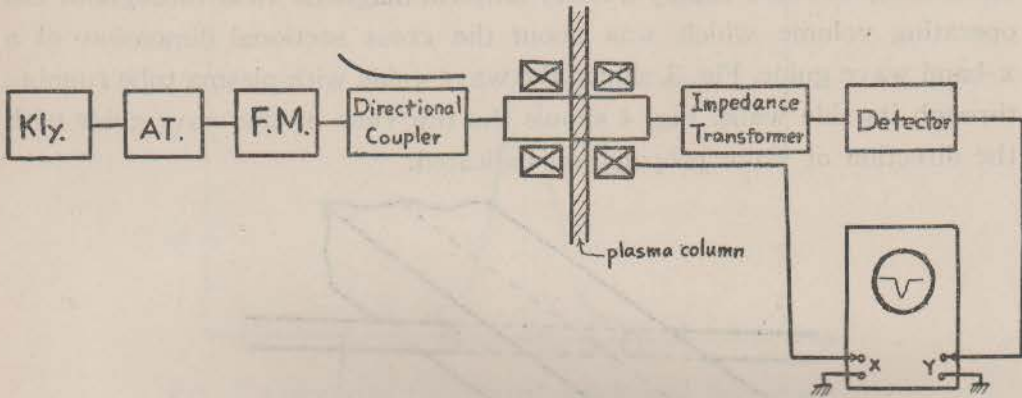

Fig. 5 Microwave Circuit

By feeding the detected microwave signal to the Y-input of an oscilloscope and a signal from the electromagnet power supply into the X-input of the scope, the oscilloscope become a X-Y recorder of minimum time delay.

Microwave frequency $\omega/2\pi = 9.3^{g.c.}$ was used to ensure the 45° propagation inside the x-band. wave gnide.

IV. Results

Results of measurement of the microwave resonance absorption in the helium discharges are given in Fig. (6) and (7) with p≈3.3 Torr. and increasing discharge currents. Figure (6) shows the transmitted microwave intensity as a function of the confining magnetic field with $\omega_p/2\pi \approx 1^{g.c.}$ (corresponding to $N_e \approx 2 \times 10^{10}/cm^3$) and helium gas pressure p≈3.3 Torr. Resonance absorption in the vicinity $\frac{\omega_c}{\omega} = 1$ is obvious, comparing with the computed line shape in Figure (2), we find that they are in good agreement. Keeping the pressure at p=3.3 Torr, the electron number density can be increased to $\omega_p/2\pi \approx 1.7^{g.c.}$ (corresponding to $N_e = 4 \times 10^{10}/cm^3$) by raising the discharge current. Figure (7) shows the absorption line shape. By comparing their line width, we find that the collision frequencies are

almost constant as expected with magnitude slightly higher than that given by equation (6). We believe this is a reasonable agreement to the results of Hirshfield[5] and others since our background pressure is higher than others.

Taking the imaginary part of the index of refraction $N_i \approx 1.168$ from Figure (2), we estimate the absorption level by assuming the plasma as a slab of 3 mm. width.

Assuming
$$E \propto \exp[-\frac{\omega}{c}n_i x - j\frac{\omega}{c}n_r t]$$

The power absorption in db can be obtained as:

$$P_r = 20 \log_{10} exq(-\frac{\omega}{c}n_i d) \approx 0.59^{db}$$

Calibrating the absorption power level of Figure (7) with a precision attenuator, we find that the observed absorption $P_r \approx 0.56^{db}$ which is again
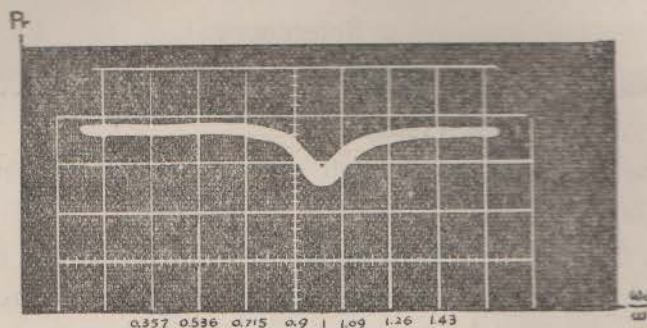

Figure 6. Resonance absorptivn of microwaves as a function of Confining magnetic field, p=3.3 Torr, $N_e = 2 \times 10^{10}/_{cm^3}$
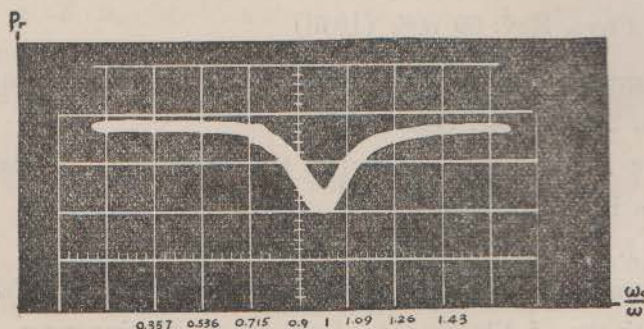

Figure 7. Resonance absorption of microwaves as a function of Confining magnetic field, p=3.3 Torr, $N_e = 4 \times 10^{10}/_{cm^3}$

a good agreement. The real part of the index of refraction N_r has also been computed, we find that the abrupt change of N_r near cyclotron resonance has been smoothed out due to the rather high collision frequency. No significant reflection is observable at the vicinity of cyclotron resonance by displaying the directionsl coupler output in parallel to the transmitteb signal on a dual beam scope.

Comparing the maximum absorption levels of Figure (6) and (7) confirms the prediction of equation (7). that the absorption increases with the plasma density.

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Acknowledgments

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