A Simple Method for Determining the Pretilt Angle of a Vertically Aligned Reflective LCD

Jy-Shan Hsu, Bau-Jy Liang, and Shu-Hsia Chen

***Abstract*—A field-induced birefringence method is demonstrated for determining the pretilt angle of a vertically aligned reflective liquid crystal display. We derive theoretically that the field-induced optical phase retardation is linearly proportional to the square of the applied voltage in the low-voltage regime. The pretilt angle can be deduced from the slope of this linear relation. The optical phase retardation is measured by a simple optical system and the pretilt angle can be determined easily. The experimental results agree well with that obtained theoretically. We also describe the applicable range of the applied voltage and the advantage of our method.**

***Index Terms*—Pretilt angle measurement, vertically aligned (VA) reflective liquid crystal display (R-LCD).**

I. INTRODUCTION

REFLECTIVE LIQUID CRYSTAL DISPLAY (R-LCD) has been widely used because of their low power consumption, light weight, and outdoor readability. The characteristics of R-LCD, such as brightness, contrast ratio, and response time, are determined by the cell gap, twist angle, pretilt angle etc., and the measurements of these parameters are still developing [1]–[4]. Among the modes used in LCDs, the vertically aligned (VA) mode exhibits excellent dark state, which is a nice candidate to be used in the R-LCD, e.g., LC on silicon. The prepolar angle (measured from the substrate normal) of a VA-LCD is usually less than 5° and is crucial for the dark state. Therefore, an accurate measurement method to determine the prepolar angle of a VA-R-LCD is desired.

Several techniques have been proposed and widely applied to measure the pretilt angle [5]–[8] of LC cells, such as the magnetic null method [5], the crystal rotation technique [6], [7], the polarizer rotation method [8], and the phase retardation measurement method [9]. These methods are accurate and reliable for the pretilt angle measurement of the transmissive cells. However, one has to modify these methods in order to measure the reflective cells, which either complicates the setup [5]–[8] or limits itself [9] to the large prepolar angle measurement. The method proposed previously by our group [4] for determining the prepolar angle of VA-R-LCD needs a simulator and manual works to fit the experimental data, which is time consuming and not suitable for the factory.

In this paper, we propose a field-induced birefringence method to determine the prepolar angle of the VA-R-LCD with low prepolar angle. We derive theoretically the field-induced optical phase retardation, which is linearly proportional to the square of the applied voltage and the prepolar angle in the small molecular deformation regime for the small prepolar angle. This linear relation exists not only for the reflective but also for the transmissive VA-LCD. Based on the theoretical analysis, we can determine the pretilt angle of the VA-R-LCD by measuring the phase retardation with a simple optical system. In the experiments, three LC cells were prepared. We measured the phase retardation versus applied voltage of two VA reflective cells and obtained the prepolar angles from the slopes of the fitting lines in the low voltage regime. We also determined the prepolar angle of one transmissive VA cell by this method and the result agrees well with the prepolar angle obtained by crystal rotation technique.

II. THEORY

As we know, when there is no pretilt angle on both substrates of a VA LC cell with hard boundary assumption, the tilt angles of the directors remain zero up to the Fredericks transition threshold voltage. If the cell has nonzero pretilt angle, there is no threshold, and the phase retardation will increase as the applied voltage increases.

To obtain the relation between the phase retardation and the applied voltage, we begin with Frank's elastic free energy density under the continuum theory to calculate the director profile of LC cell under an applied voltage. We define the polar angle $\theta(\mathrm{z})$ as the angle from the normal direction of the cell to the LC director. The free energy per unit surface area is expressed as

$$F = \frac{1}{2}\int_0^d \left\{ (k_{11}\sin^2\theta + k_{33}\cos^2\theta)\left(\frac{d\theta}{dz}\right)^2 + \frac{D_z^2}{\varepsilon_0(\varepsilon_{//}\cos^2\theta + \varepsilon_\perp \sin^2\theta)} \right\} dz \quad (1)$$

where D_z represents the electrical displacement in the z direction, which is uniform throughout the cell, k_{11} and k_{33} are the Frank elastic constants for splay and bend deformations, respectively, $\varepsilon_{//}$ and $\varepsilon_\perp$ stand for the longitudinal and transverse dielectric constants of the LCs, and d is the cell gap. We assume that the directors on the boundaries are fixed with the polar angle θ_0 and we call it the prepolar angle. That is, the pretilt angle is $(90^\circ - \theta_0)$. The relation between $\theta(\mathrm{z})$ and the applied voltage

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V can be obtained by minimizing the free energy density in (1), which is

$$\frac{V}{V_{\text{th}}}=\frac{2}{\pi}\sqrt{(1+\gamma\sin^2\theta_m)}\times\int\limits_{\theta_0}^{\theta_{\text{m}}}\sqrt{\frac{(1+\kappa\sin^2\theta)}{(1+\gamma\sin^2\theta)(\sin^2\theta_m-\sin^2\theta)}}d\theta \quad (2)$$

where θ_m is the polar angle of the midlayer directors, $V_{\text{th}}=\pi\sqrt{k_{33}/\varepsilon_0(\varepsilon_{\perp}-\varepsilon_{//})}$, $\kappa=(k_{11}-k_{33})/k_{33}$, and $\gamma=(\varepsilon_{\perp}-\varepsilon_{//})/\varepsilon_{//}$.

Once we have $\theta(\text{z})$, we can calculate the phase retardation $\Gamma(V)$ from the following equation:

$$\Gamma(V)=\frac{4\pi}{\lambda}\int\limits_0^d\left[\frac{n_e n_o}{\sqrt{n_e^2\cos^2\theta(z)+n_o^2\sin^2\theta(z)}}-n_o\right]dz \quad (3)$$

where n_e and n_o stand for the extraordinary and ordinary refractive indexes of LC material, respectively, and λ is the wavelength of incident light.

In order to obtain an analytic solution, we focus our attention at low applied voltage and small deformation regime. The polar angle of the midlayer directors can be express as $\theta_m=\theta_0+\Delta\theta_m$, where $\theta_0>0$, and $\Delta\theta_m\geq 0$. By series expansion with respect to $\Delta\theta_m$, we have the relation between the applied voltage V and $\Delta\theta_m$

$$\frac{V}{V_{\text{th}}}=\left(\frac{\Delta\theta_m}{\theta_0}\right)^{\frac{1}{2}}\left[\frac{2\sqrt{2}}{\pi}+\frac{(2+3\kappa)\sqrt{2}}{3\pi}\theta_0^2+\ldots\right]+\left(\frac{\Delta\theta_m}{\theta_0}\right)^{\frac{3}{2}}\left[-\frac{5}{3\pi\sqrt{2}}+\ldots\right]+\ldots. \quad (4)$$

When $\theta_0^2\ll|6/(2+3\kappa)|$ and $\Delta\theta_m/\theta_0\ll 2.4$, which is deduced from $(\Delta\theta_m/\theta_0)^{3/2}(5/3\pi\sqrt{2})/(\Delta\theta_m/\theta_0)^{1/2}(2\sqrt{2}/\pi)\ll 1$, (4) is approximately reduced to

$$\Delta\theta_m(V)=\theta_m(V)-\theta_0\approx\frac{\pi^2V^2}{8V_{\text{th}}^2}\theta_0. \quad (5)$$

Substitute (5) into $\Delta\theta_m/\theta_0\ll 2.4$, and we have the limitation of the voltage $\pi^2V^2/8V_{\text{th}}^2\ll 2.4$. It means that when $V^2\ll 1.95V_{\text{th}}^2$, $\Delta\theta_m$ is linearly proportional to V^2 approximately.

Similarly, we derived the relation between the phase retardation and $\Delta\theta_m$ as

$$\Gamma(V)=\Gamma(0)+\frac{4\pi n_o d}{\lambda}\left\{\left(\frac{\Delta\theta_m}{\theta_0}\right)\left[\frac{2\nu}{3}\theta_0^2+\nu\left(\nu-\frac{4}{9}\right)\theta_0^4+\ldots\right]+\left(\frac{\Delta\theta_m}{\theta_0}\right)^2\left[\frac{11\nu}{45}\theta_0^2+\ldots\right]+\ldots\right\} \quad (6)$$

where $\nu=(n_e^2-n_o^2)/n_e^2$. When $\theta_0^2\ll|6/(9\nu-4)|$ and $\Delta\theta_m/\theta_0\ll 2.73$, which is deduced from $(\Delta\theta_m/\theta_0)^2(11\nu\theta_0^2/45)/(\Delta\theta_m/\theta_0)(2\nu\theta_0^2/3)$ (6) is reduced to

$$\Gamma(V)-\Gamma(0)\approx\frac{8\pi n_o d\nu}{3\lambda}\frac{\Delta\theta_m}{\theta_0}. \quad (7)$$

By substituting (5) into (7), we have

$$\Delta\Gamma(V)=\Gamma(V)-\Gamma(0)\approx\frac{\pi^3\left(n_e^2-n_o^2\right)n_o d}{3n_e^2\lambda}\frac{\theta_0^2}{V_{\text{th}}^2}V^2. \quad (8)$$

TABLE I
PARAMETERS USED IN THE SIMULATION

Liquid crystal	MLC-6608	Cell gap	5μm
n_e	1.556	$\varepsilon_{//}$	3.6
n_o	1.4738	$\varepsilon_{\perp}$	7.8
k_{11}	11.5 pN	k_{22}	8.4 pN
k_{33}	18.1 pN	V_{th}	2.19 Volts

We analyze the limitations of the applied voltage from the range of the deformation: one is $\Delta\theta_m/\theta_0\ll 2.73$ and the other is $\Delta\theta_m/\theta_0\ll 2.4$. We should choose the stricter one, which means when $V^2\ll 1.95V_{\text{th}}^2$ $\Delta\Gamma$ is linearly proportional to V^2 and θ_0^2. We use the parameters of LC MLC-6608 (shown in Table I) as an example to find the applicable range. We obtained that θ_0 is less than 25° and V is less than 0.97 V. One can use (8) to determine the prepolar angles in the experiments by measuring the phase retardation.

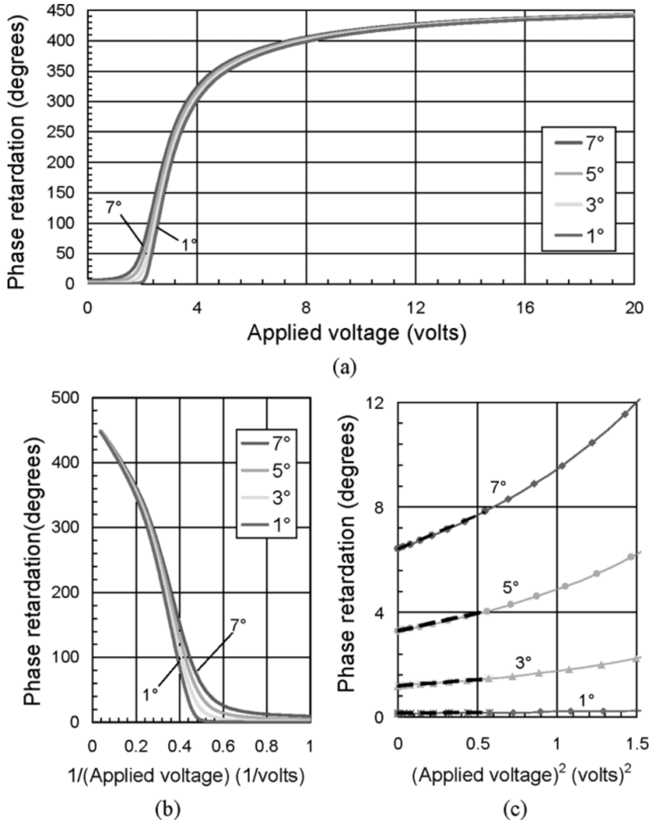
The merit of this method is that the phase retardation is generated by the reorientation of the directors caused by the applied voltage. The phases of other layers, such as indium–tin–oxide (ITO) and alignment layers etc., do not change when we apply a voltage to the cell, therefore, they do not influence the results. Beside, we can use the same optical setup to obtain the cell gap. That is the phase retardation $\Gamma(V)$ in the high-voltage regime can be written as [4], [10], [11]

$$\Gamma(V)=\frac{4\pi(n_e-n_o)d}{\lambda}\left(1-\zeta\frac{V_{\text{th}}}{V}\right) \quad (9)$$

where ζ is a material constant. When $V\rightarrow\infty$, all the directors are reoriented by the field and the maximum phase retardation $\Gamma_{\max}=4\pi(n_e-n_o)d/\lambda$ is achieved. Therefore, we use the extrapolation method to get the maximum phase retardation and the cell gap can be derived from it [4].

In one word, we have designed a field-induced birefringence method to determine the prepolar angle by simply measuring the phase retardations with respect to the applied voltages. We get the cell gap by extrapolating the curve of the phase retardations versus the inverse of the applied voltage in the high voltage regime. By using this cell gap, we determine the prepolar angle from the slope of the linear part of the phase retardation versus the square of the applied voltage in the low voltage regime.

In order to illuminate this idea, we use the commercial simulator DIMOS to obtain the director profile of the LC cell with varied prepolar angles and applied voltages. Table I shows the parameters of the LC used in this simulation. We accumulate the phase retardation layer by layer and plot them in Fig. 1(a). To calculate the cell gap, we replot the phase retardations versus the inverse of the applied voltage and the results are shown in Fig. 1(b). We derive the maximum phase retardations and the corresponding cell gaps from the intercept of the extrapolation lines in the high voltage regime. Fig. 1(c) shows the phase retardations versus the square of the applied voltages. The dash lines are linear fitting curves. We calculate the prepolar angles with (7) by using the slopes of the fitting lines in the different applied voltage ranges and the results are in Table II. Theoretically, the smaller the fitting voltage range, the more accurate the fitting prepolar angles will be. However, the experimental data



(a)

(b) (c)

Fig. 1. Phase retardations obtained from the simulated director profiles for a prepolar angle from 1° to 7°. (a) With respect to the applied voltage. (b) With respect to the inverse of the applied voltage. (c) With respect to the square of the applied voltage. The linearly fitting curves are plotted in dash lines for each prepolar angles.

TABLE II

PREPOLAR ANGLES OBTAINED IN FIG. (1c) WITH DIFFERENT FITTING RANGES OF THE SQUARE OF THE APPLIED VOLTAGE. THE UPPER LIMIT OF THE APPLIED VOLTAGE IS V_{upper} AND θ_{0s} AND θ_{0e} ARE THE PREPOLAR ANGLES USED IN THE SIMULATION AND CALCULATED FROM THE SLOPES IN FIG. (1c), RESPECTIVELY

V_{upper}^2 θ_{0e} θ_{0s}	0.2 (volts^2)	0.3 (volts^2)	0.4 (volts^2)	0.5 (volts^2)
1°	1.029	1.047	1.062	1.076
3°	3.082	3.128	3.164	3.211
5°	5.089	5.187	5.260	5.332
7°	7.159	7.234	7.321	7.420
9°	9.168	9.259	9.358	9.479
Error (%)	< 2.9%	< 4.7%	< 6.5%	< 7.7%

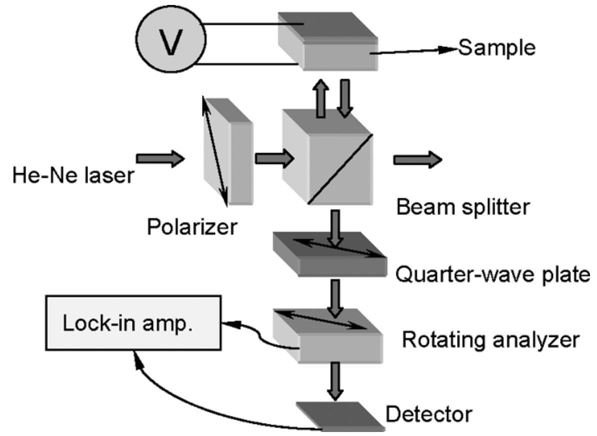


Fig. 2. Optical setup used for measuring the phase retardation of a reflective VA-LC cell.

fluctuate and contain errors, so it is inappropriate to have too small a voltage range for fitting. Table II shows the calculated prepolar angle and corresponding errors by using this method.

III. EXPERIMENT

With the proposed method, we measured two VA reflective cells and one transmissive VA cell in the small prepolar angle range. Fig. 2 shows a schematic diagram of our reflective phase retardation measurement setup. The light from a He-Ne laser (0.6328 μm) passes through the linear polarizer and beam splitter, then impinges on the LC cell with the polarization at an angle of 45° to the rubbing direction. The ordinary and extraordinary waves experience different phases in the LC cell and the light is reflected and leaves the cell with elliptical polarization. Then it passes through the beam splitter again and a quarter wave plate, which is in the same alignment with the input polarizer. The light is transferred to the linearly polarized light and goes through a rotating analyzer, then hits on the optical detector. The lock-in amplifier analyzes the signals and the output phase retardations are delivered to the computer, which is programmed to average the row data in order to raise the accuracy. Finally, we have the phase retardation as a function of applied voltage.

We used two ITO-coated glasses, one of which was with aluminum under ITO as a reflector, to fabricate one VA reflective cell. With the polymer JALS-2021 as the homeotropic alignment layers, we used rubbing method to produce pretilt angle. We combined the substrates by using 4-μm spacers mixed in the adhesive without being spread in the cell. Then we filled it with LC MLC-6608. The measured phase retardation is shown in Fig. 3(a), the relation between the measured phase retardation and the inverse of the applied voltage is shown in Fig. 3(b). From the intercept of the extrapolation, the maximum phase retardation is 328° and the calculated cell gap is 3.51 μm. In order to get the prepolar angle, we replot the phase retardations versus the square of the voltage again as shown in Fig. 3(c). From the slope of the linearly fitting curve, the prepolar angle of the cell is 4.3°.

The other VA reflective cell was provided by ERSO/ITRI of which the manufacture processes are similar to that of the LCD factory. The 4–μm spacers were spread uniformly in the cell, and then filled with LC MJ011 675. By using the proposed method, we obtain the cell gap and the prepolar angle to be 3.89 μm and 0.8° respectively.

To verify the reliability of the above theory, we fabricated a transmissive cell since our theory is applicable for it. Besides, we can confirm the prepolar angle of the cell by the conventional crystal rotation method [6]. However, the He-Ne laser

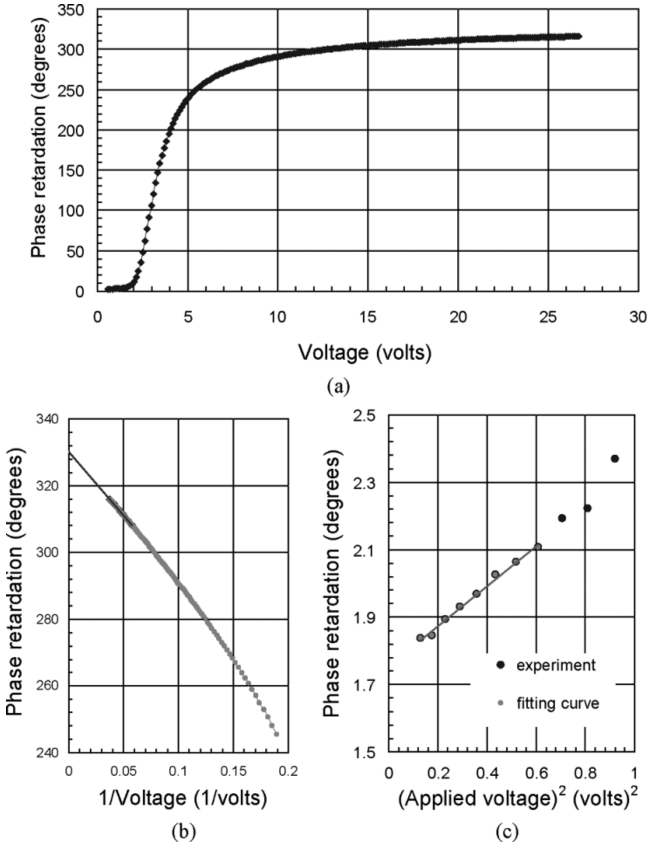


Fig. 3. Measured phase retardation of the VA reflective cell. (a) With respect to the applied voltage. (b) With respect to the inverse of the applied voltage. (c) With respect to the square of the applied voltage. The solid lines in (b) and (c) are the linearly fitting curves to get the cell gap and the prepolar angle, respectively.

passes through the transmissive cell only once instead of twice, we modify (8) to

$$\Delta\Gamma(V) \approx \frac{\pi^3\left(n_e^2 - n_o^2\right)n_o d}{6n_e^2\lambda}\frac{\theta_0^2}{V_{\mathrm{th}}^2}V^2. \tag{10}$$

The cell gap is determined to be 11.26 μm and the prepolar angle is 3.98° when the fitting range is $0 \sim 0.5$ volts^2. The prepolar angle measured by the crystal rotation method is 3.99° which agrees excellently with the one obtained by our field-induced birefringence method.

IV. CONCLUSION

We have proposed a simple method to determine the prepolar angle of a nontwisted homeotropic LC cell with negative anisotropic nematic LCs, which the conventional measurement methods are not applicable. A simple equation is deduced from the relationship between the prepolar angle and the phase retardation as a function of applied voltage. In the experiments, we have applied this method and determined the prepolar angles of two reflective VA cells. We also compared the method with the crystal rotation method by measuring one transmissive VA cell and the result is satisfactory. This method enables us to monitor the reflective VA cell in the manufacture, aging, and stability processes, which is important for the study of the LC displays.

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