Study of Hall Effect

By

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I Introduction

If a conducting material is placed in a magnetic field perpendicular to the direction of current flow a voltage is developed across the material in a direction perpendicular to both the initial current direction and the magnetic field direction. This voltage is called the Hall voltage, after E. H. Hall who first observed the effect in 1879[1]. The Hall voltage arises from the deflection of the moving charge carriers from their normal path by the applied magnetic field

II Basic principles

(a). Free electron model

We first consider a free electron model in which a confined stream of free particles, each having charge e and initial velocity Vx, a magnetic field Hz in the Z direction, the situation is shown in the Fig. 1.

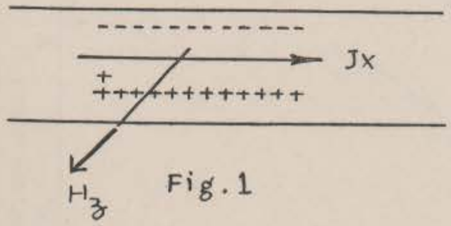
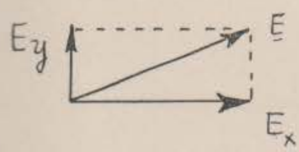


Fig. 1



Lorentz force law tells us

$$\underline{F}=e(\underline{E}+\frac{1}{C}\underline{V}\times\underline{H}) \quad (1)$$

where $\underline{F}$ is the force, e the charge and V the veiocity of the particle; H is the magnetic field intensity and E the electric field intensity. For Goussian system of units, C is the speed of lihgt and a permeability of unity is assumed.

From equation (1) and the Fig. 1, we see that the Hz produces a deflection of charges along the y direction initially. Thus, a charge unbalance is created. And hence an electric field Ey is established. The resulting field is the Hall field, which builds up until a force exerts on a charged particle by the electric field balances the force resulting from the magnetic field.

Consequently, particles of the same charge and velocity are no longer deflected and then reaches a steady state.

At equilibrium condition, the force balance along the y direction must be hold. From equation (1), one obtains

$$Ey+\frac{1}{C}(\underline{V}\times\underline{H})y=0$$

or

$$Ey=-\frac{1}{C}V_xH_z \qquad (2)$$

It will also be recalled that the electric current density Jx can be expressed in terms of charge carriers by the relation

$$J_x=\Sigma i\ n_i\ e\ V_x^i \qquad (3)$$

where n_i is the density of charge carriers having velocity V_x^i. In case of our model, where

$$V_x^i=V_x$$

One obtains

$$Jx=neV_x \qquad \text{where } n=\Sigma i\ n_i \qquad (4)$$

Using equation (4), expresion (2) becomes

$$E_y=J_xH_z/nec=\frac{1}{nec}(\underline{J}\times\underline{H})_y$$

$$=\frac{1}{nec}(\underline{H}\times\underline{J})_y \qquad (5)$$

Now the Hall coefficient is defined by the ratio E_y/J_xH_z

$$R=E_y/J_xH_z=\frac{1}{nec} \qquad (6)$$

Note from (6) that

$R<0$ for conduction by electrons

$R>0$ for conduction by holes

(b) Real solid

In a real solid, we must take into account the distribution of velocity and the interaction of the charge carriers with impurities, defects and lattice thermal vibrations of the solid as well as the band structure of the solid.

Taking all of these into consideration, for n-type semiconductors, it can be found that

$$J_x = \frac{ne^2}{m_e}\left\{\langle\frac{\tau}{1+\omega^2\tau^2}\rangle\Sigma_x - \langle\frac{\omega\tau^2}{1+\omega^2\tau^2}\rangle\Sigma_y\right\}$$

$$J_y = \frac{ne^2}{m_e}\left\{\langle\frac{\omega\tau^2}{1+\omega^2\tau^2}\rangle\Sigma_x + \langle\frac{\tau}{1+\omega^2\tau^2}\rangle\Sigma_y\right\} \qquad (7)$$

where $\tau=\tau(E)$ is the free time between collisions, $\omega=\frac{eH}{mC}$ the cyclotron frequency, n the density of electron, m_e the effective mass of electron in solid and $\langle\ \rangle$ is the average operator using $E^{3/2}\ f_o$ as weighting function, for example

$$\langle\tau\rangle = \frac{\int_0^\infty \tau(E)\ E^{3/2}\ f_o\ dE}{\int_0^\infty E^{3/2}\ f_o\ dE} \qquad (8)$$

From equation (7), one can see many interesting features about R and is given below.

From equation (7), set $J_y=0$ we can find R in the form

$$R = -\frac{\gamma}{nec} \qquad (9)$$

still in the same form as the expression (6).

Following features may be considered as features about γ or R:

1. γ depend on $\gamma(E)$ actually $\gamma(E)$ depends on the type of scattering. It can be proved that

$$\gamma_\ell \propto E^{3/2} \text{ for lattice scattertng}$$

$$\gamma_i \propto E^{3/2} \text{ for impurity scattering} \qquad (10)$$

2. If H is small, or $\omega\tau \ll 1$, by (7), it is found thet

$$\gamma=\frac{\langle\tau^2\rangle}{\langle\tau\rangle^2}\geq 1 \tag{11}$$

3. For $\tau_\ell \propto E^{-1/2}$, it is found that

$$\gamma=\frac{3\Pi}{8}=1.18>1 \tag{12}$$

4. For $\tau_I \propto E^{3/2}$ by the same reason, it can also be found that

$$\gamma=\frac{\langle\tau^2\rangle}{\langle\tau\rangle^2}=\frac{315\pi}{512}=1.93 \tag{13}$$

5. For a degenerate case

$$\gamma=\frac{\langle\tau^2\rangle}{\langle\tau\rangle^2}=\frac{[\tau(E_f)]^2}{[\tau(E_f)]^2}=1 \tag{14}$$

6. For large magnetic field

$\omega^2\tau^2\gg 1$, equation (7) can be written as

$$\tau_x=\left\{\frac{1}{\omega^2}\langle\frac{1}{\tau}\rangle\Sigma_x-\frac{1}{\omega}\Sigma_y\right\}\frac{ne^2}{m_e} \tag{15-a}$$

$$\tau_y=\left\{\frac{1}{\omega}\Sigma_x+\frac{1}{\omega^2}\langle\frac{1}{\tau}\rangle\Sigma_y\right\}\frac{ne^2}{m_e} \tag{15-b}$$

equations (15-a), (15-b) and the relation that $\omega\tau\gg 1$ tells us that equation (15-a) is approximately

$$\tau_x=\frac{ne^2}{m_e}\left(-\frac{1}{\omega}\Sigma_y\right) \tag{16}$$

using (16) and $\omega=\frac{eH}{m_eC}$, we find that

$$R=\frac{-1}{neC}$$

Note that as H becomes larger and larger,

then $\quad \tau=1$ (17)

7. Because the m_e appears in equation (7), it is appearent that γ is a function of the shape of the constant energy surfaces.

III Electrric field, Hall angle and Hall mobility

For the Hall sample in the magnetic field, the Hall field is quickly established, When the Hall field is presented, the resultant field lies at some angle Q to the x axis. The angle is the Hall angle.

$$\theta = \tan^{-1}\frac{E_y}{E_x}$$

Use definition (6) and (9), it can be foundthat

$$\tan\,\theta = R\sigma H \qquad (18)$$

where σ is the conductivity of the sample. Now, return to equation (7), it can be written in the form

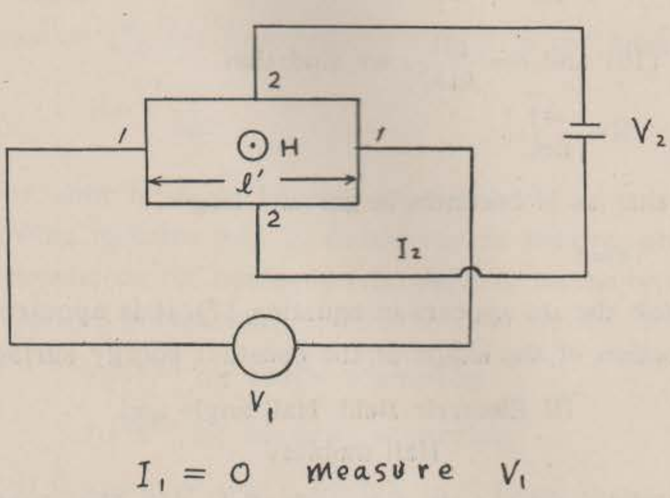
$$\tau_x = \sigma_{xx}\ E_x + \sigma_{xy}\ E_y$$
$$\tau_y = -\sigma_{xy}\ E_x + \sigma_{xx}\ E_y \qquad (19)$$

Note that $W = eH/m_o c$, and compare equation (7) with equation (19), then

σ_{xy} is an odd function of H and

σ_{xx} is an even function of H

It is interesting to find that the same characteristic in a rather different way. Let us see the Fig. 2, consider the n-type semiconductor



$I_1 = 0$ measure V_1

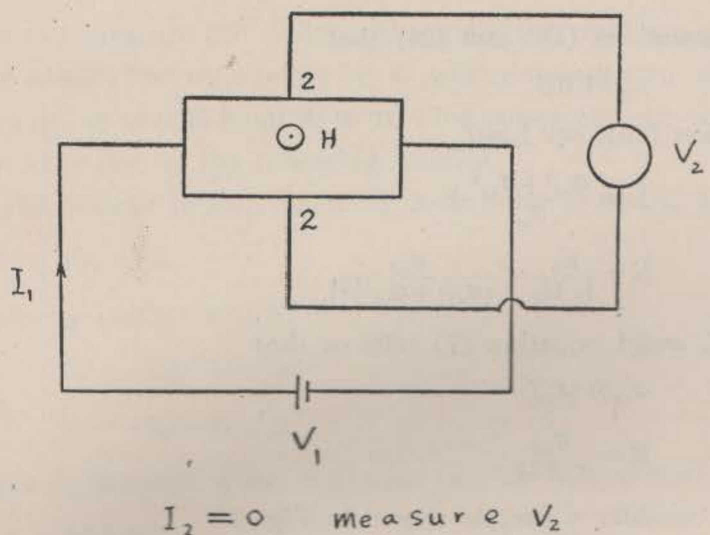


Fig. 2 Hall sample with H applied in a direction which points out of thepaper. Thickness of the sample is assumed to be b.

By definition

$$V_1 = Z_{11}\ I_1 + Z_{12}\ I_2$$
$$V_2 = Z_{21}\ I_1 + Z_{22}\ I_2 \qquad (20)$$

For $I_1 = 0$

$$Z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0} = \frac{-RJ_2H_y\ell'}{I_1}$$

or

$$Z_{12} = -\frac{RH_y}{d} \qquad (21)$$

For $I_2 = 0$, similiarly

$$Z_{21} = \frac{RH_y}{d} \qquad (22)$$

So

$$Z_{12} = -Z_{21} \qquad (23)$$

Equation (23) tells us that a Hall effect unit can be used as a non-reciprocal electrical element, which has been call a "gyrator".

It is usually convenient to defined a Hall mobility for conduction by a single type of carrier, by the relation

$$\tan\ \theta = \frac{\mu_H H}{C} \qquad (24)$$

Note from equations (18) and (24) that

$$\mu_H = R\sigma C \quad (25)$$

From equation (19), set $J_y = 0$

$$J_x = \frac{\sigma_{xx}^2 + \sigma_{xy}^2}{\sigma_{xy}} E_y$$

$$R = \frac{E_y}{J_x H_z} = \frac{\sigma_{xy}}{(\sigma_{xx}^2 + \sigma_{xy}^2) H_z} \quad (26)$$

For weak H, $\omega\tau \ll 1$ equation (7) tells us that

$$\sigma_{xx} \gg |\sigma_{xy}| \quad (27)$$

So

$$R = \frac{\sigma_{xy}}{\sigma_{xx}^2 H_z}$$

Next let us consider a sample shown in Fig. 3

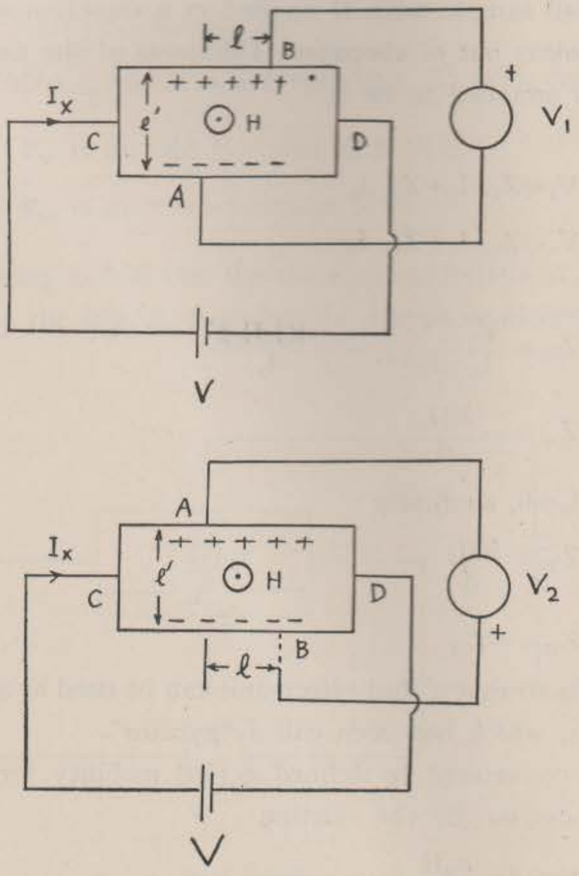


Fig. 3 H field is pointed out of the paper.

If we can not measure the voltage V_{ab} under the condition that AB is perpendicular to CD, the voltage V_1 or V_2 will contain a term due to the potential drop in the sample from A to B. This is not the cne we require, but it can be take out by the following method.

Assume the voltage drop from point A to point B due to I_x

is $$V_{AB}=I_xZ.$$

where Z is clearly independent of H

$$\begin{aligned} V_1&=-I_xZ+RJ_xHl' \\ V_2&=-I_xZ-RJ_xHl' \end{aligned} \tag{28}$$

A useful result is obtained when equations of (28) is combined

$$\frac{V_1-V_2}{2}=RJ_xHl' \tag{29}$$

$$R=\frac{V_1-V_2}{2J_xHl'} \tag{30}$$

Equation (30) takes out the difficulty that it is almost impossible to make A point and B point on the same potential surface when the magnetic is not applied. The importance of it is easily seen.

The above analysis applies equally well for p-type semiconductor. It can be shown that for p-type semiconductor material

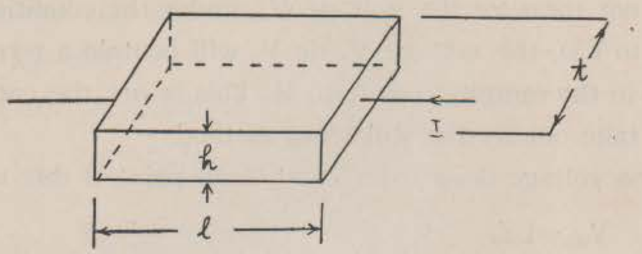
$$R=+\frac{\gamma}{pe}. \tag{31}$$

The variation of γ is exactly the same as that for electrons in the n-type material.

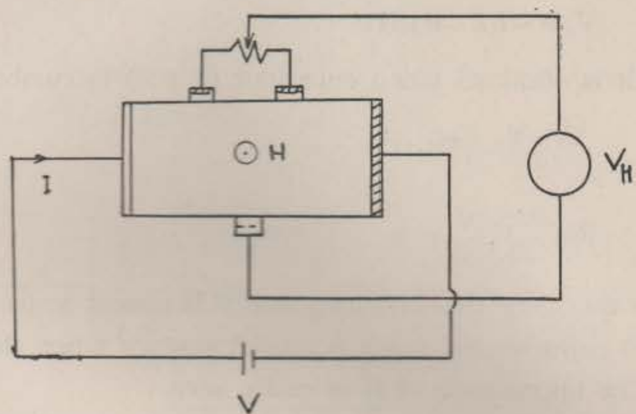
IV Experiment

We use three samples denoted respectively as sample A, sample B and sample C. The type and magnitude are given in the following table and the corresponding figure.

	Sample A	Sample B	Sample C
l	10.79mm	10.72mm	13.40mm
t	3.35mm	3.83mm	2.58mm
h	0.18mm	0.44mm	0.22mm



For these samples, we connect each sample as in the following figure, and placed it in the magnetic field



When B=0, we must adjust R so that $V_H=0$, then we apply the magnetic field and then the Hall voltage is established.
First we set I remain constant varying H and recording V_H. After the process is finished, we keep H remains constant and varying I and recording V_H.

The above procedure is done for each of the three samples.

Use
$$R=\frac{3\pi}{8}\frac{1}{n(\pm e)C}=\frac{E_y}{J_xH_z} \tag{32}$$

the number density n can be found. However, in laboratory, usally it is more convenient to use the formula

$$R=10^8\frac{V_Hh}{H_I} \tag{33}$$

where V_H is the Hall voltage in volt; h is the thickness of the sample in cm, H the magnetic field intensity in gauss and I the current flowing through the sample in ampere.

The conductivity for n-type sample

$$=\text{neu}=\mu.\frac{3\pi}{8}\ \frac{1}{R} \tag{34}$$

Equation (33) and (34) tell us that the mobility μ can be found if R and σ are known by experiment.

Use the formula

$$\mu=\frac{e\tau}{m^*} \tag{35}$$

τ is found.

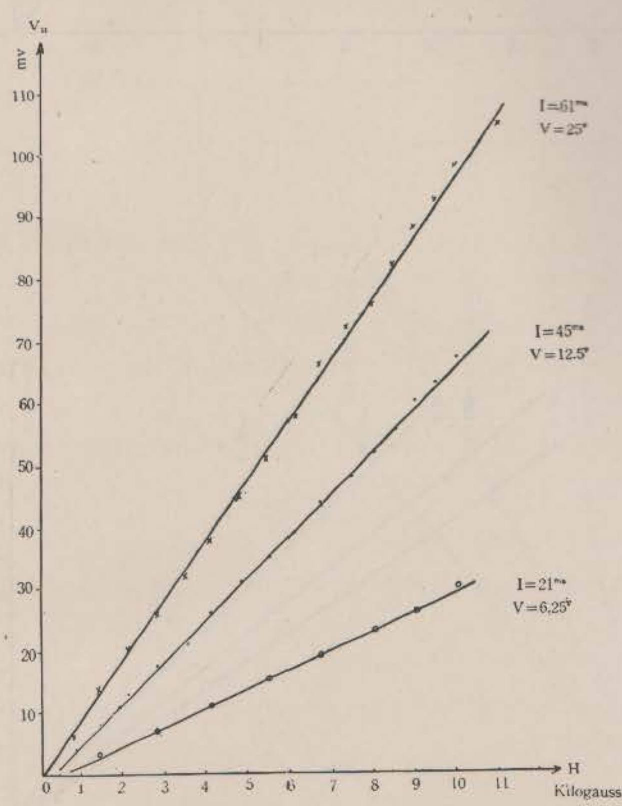


Fig. E-1 Sample A. Keep I Constant and Varyies H.

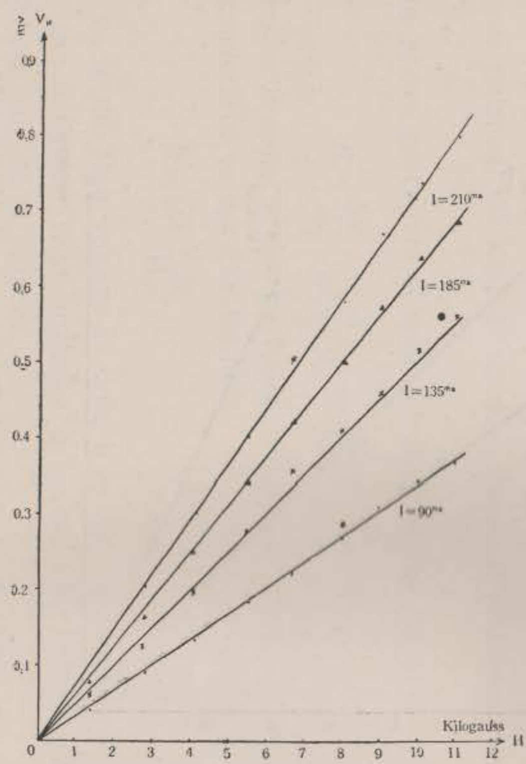


Fig. E-3 Sample C. Keep I Constant and Vary H.

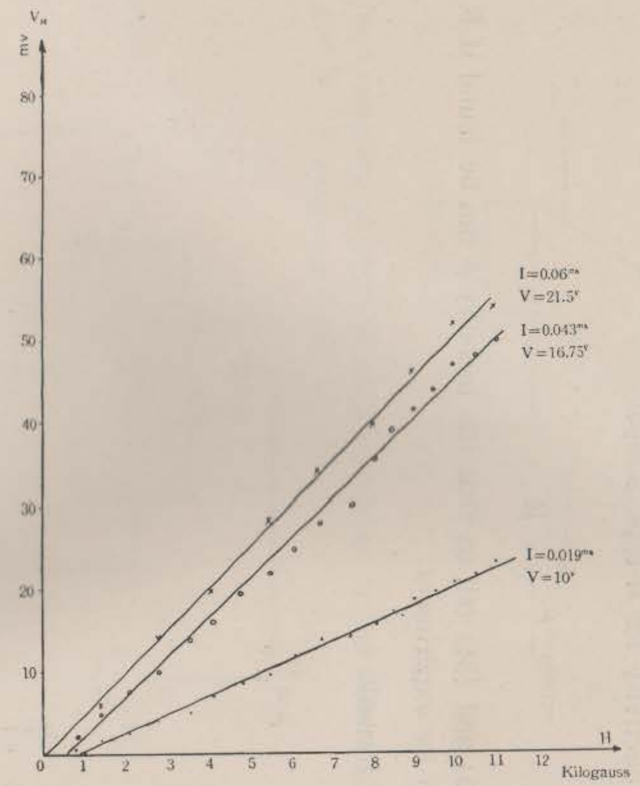


Fig. E-2 Sample B. Keep I Constant and Vary H.

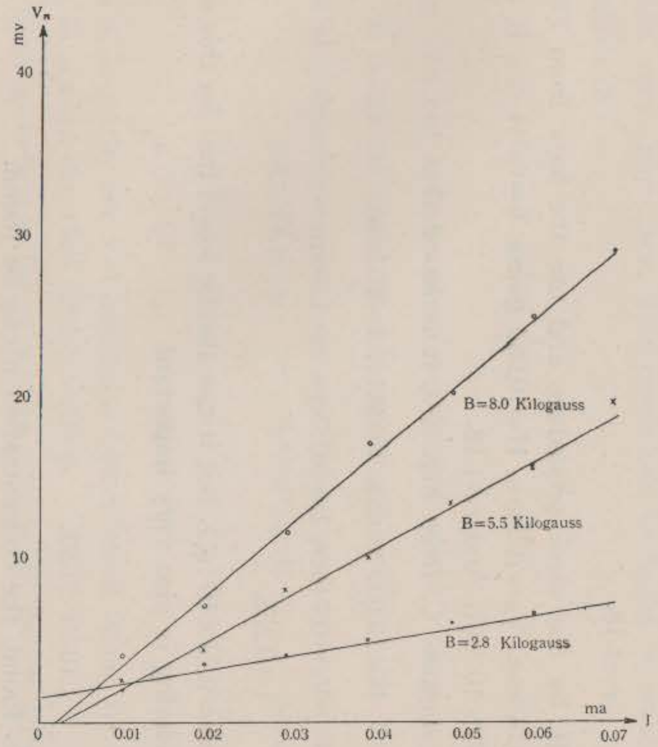


Fig. E-5 Sample B. Keep B Constant and Vary I.

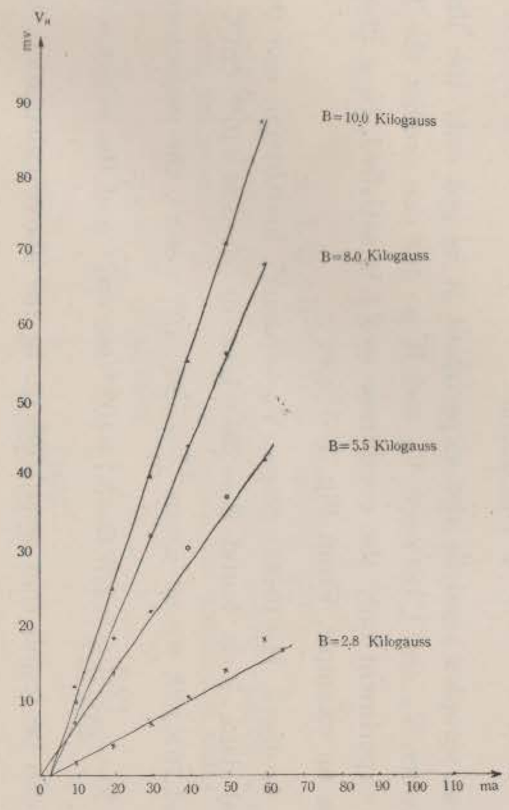


Fig. E-4 Sample A. Keep B Constant and Vary I.

V Discussion

For these samples used in the experiment it is got only the linear relation detween V_H and I, between V_H and B. so one can predict the fact that in the experiment only the condittion $\omega\tau \ll 1$ is satisfied. Now choose sample A as an example. From Fig. E-1, get

I=45ma, H=10,000 gauss, V_H=66.5mV, h=0.018cm use these experimental data, it is found $R_H=266=1/ne$ or $n=2.34\times10^{16}$ cm^{-3}. By $\omega=eB/m^*C=1.76\times10^{12}$, $\sigma=$ne u$=ne\frac{\tau e}{m}=5.93\times10^{24}\ \tau$ sec^{-1}, the resistance of the sample $R=\frac{12.5}{45\times10^{-3}}=278\ \Omega=3.1\times10^{-10}$ sec cm^{-1} and the relation $R=\frac{1}{\sigma}\frac{\ell}{A}$. It can be found that $\omega\tau=0.538<1$. In the above calculation, only approximate result is got.

From the above calculation, we conclude that

$$\gamma=\frac{\langle\tau^2\rangle}{\langle\tau\rangle^2}>1$$

In this sample, it can easily be proved that lattice scattering is dominant, so wo have

$$\gamma=1.18.$$

For sample B, by the same procedure and use the bata from Fig. E-2 V=10V, I=0.019ma, V_H=20.5mv, H=10,000 gauss, h=0.044 cm, it can be found $\omega\tau=5.72\times10^{-2}\ll1$. And $\gamma=1.18$.
However for sample C, from Fig. E-3, it is found that

H=10,000 gauss, V=0.4v, I=210ma,

V_H=0.74mv, h=0.022cm, e=1.34cm

and t=0.258.

These result $\omega\tau=9.73\times10^{-2}\ll1$. But it can also be found that for this sample, impurity scattering is also very important

so

$$1.18<\gamma<1.93.$$

Ajter γ is found, the concentration n can be found.

For sample A

$$266=\frac{\gamma}{ne}=\frac{1.18}{ne} \quad \text{so } n=2.76\times10^{16}cm^{-3}$$

For sample B

$$n=1.32\times10^{13}\times1.18=1.56\times10^{13}cm^{-3}$$

For sample C, n can not be found because γ is not exactly known.

Now, the mean free time can be found. For sample A, use the equations

$$\sigma=n\frac{\tau e^2}{m}=7\times10^{24}\tau$$

and $$3.1\times10^{-10}=\frac{1}{\sigma}\ \frac{1.079}{0.335\times0.018}$$

it is found

$$\tau=2.54\times10^{-13}\ \text{sec.}$$

Similarly, for sample B, it is found that $\tau=2.75\times10^{-14}$ sec and for sample C, $\tau\sim10^{-13}$ sec.

From these results, the order of magnitude of τ is roughly 10^{-14} sec. So it is a very short time. This result can be found in many books using other methods[2].

Using $$\frac{V}{I}=R\ ,\ R=\frac{1}{A}$$

We can find out ρ, there are many papers where ρ vs. n curve are presented. So n can be found. By the relation $R_H=\frac{-\gamma}{ne}$, n is found and this method is independent of the former. So a check is available.

	SAMPLE A	SAMPLE B	SAMPLE C
n_a	3×10^{16}	10^{12}	6×10^{18}
n_b	2.76×10^{16}	1.35×10^{12}	1.33×10^{18}

Where n_a is got by the V-I method, a better result using the 4-point probe and n_b is got dy the Hall effect method.

From Fig. E-6, it is found that as n increases the mobility μ decreases.

VI Conclusion

The Hall voltage is larger for the semiconductor with higher resistivity. In other words semiconductor of higher impurity concentration has smaller Hall voltage. In our three samples, only $\omega\tau \ll 1$ is satisfied. If we want to detect the variation of γ with H field, then H must be greater, 20,000 guass for sample A. For sample B and sample C, the H field at least must be greater than 100,000 guass.

The variation of γ with n can be determined if we have known concentration samples. Use $R = -\gamma/ne$ if R is known, γ is determined. In our three samples (all are n-type Si semiconductors), since γ is not known, so we determine only the order of magnitude of the concentration.

References

The Hall effect is of fundamental importance in the research of solid state physics. there are many books in which this topic has been treated very thoroughly.

1. S. N. Levine, Quantum physics of Electronics.
2. R. A. Smith. Semiconductor.
3. W. Shockley, Electrons and Holes in Semiconductor.
4. C. Kittel, Introduction to Solid State Physics.