Analysis of H-type Waveguide

by

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Abstract

This report presents a theoretical investigation and the experimental results of a special kind of waveguide, H-guide. Physically, the guide consists of a dielectric strip sandwiched between two parallel metal plates. Electrically it is excited in a mode having no longitudinal current flowing in the metal and most energy is located in the dielectric. The fields in the air decay exponentially as the distance from the dielectric surface increases.

The basic formulas which describe the behavior of H-guide are derived. Expressions which are used for the determination of the cut-off frequency, rate of field decay in the air and attenuation due to heat losses are described. An interesting property of the guide is that the attenuation due to heat losses in the metal decreases as the frequency increases.

In order to find out whether the properties of the H-guide could be realized practically, H-guides have been constructed. Iron has been used for the conducting plate to increase the guide loss so that attenuation measurements could be made more easily. As the frequency is raised, the measured insertion loss drops.

I. Introduction

H-guide is an open type transmission consisting of 2-parallel metal plates separated by a central strip of dielectric. It is intended for the use at frequencies above 30 kmc/sec. At frequencies below 30 kmc/sec the standard rectangular waveguide is satisfactory in use.

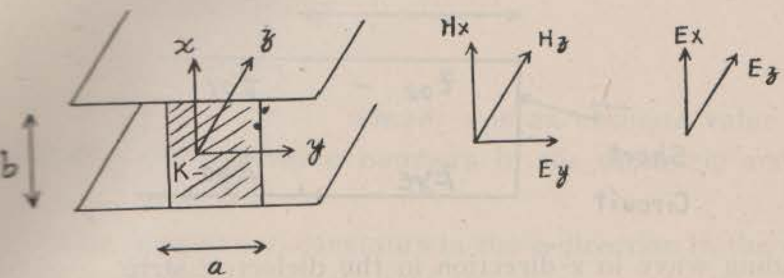
For a uniform metallic waveguide, a low-loss mode is identified by the absence of longitudinal current flow. The H-waveguide is excited in a mode having no longitudinal current flow in the metal. Most of the transmitted energy is located in the dielectric and the field in the air decay exponentially as the distance from the dielectric surface increases. Because of the similarity of the field Pattern inside the H-guide to that ofthe rectangular waveguide, excitation of the proper mode will be accomplished by means of tapered horn transition. An interesting property of of the H-guide that suggests the possibility of achieving low-loss operation is that the attenuation due to heat losses in the metal decreases as the frequency increases.

The behavior of the H-guide will be studied theoretically and experimentally. It includes the following topics.

1. Modes in H-guide
2. Electric and magnetic fields of a sidewise TM-mode.
3. Power distribution
4. Attenuation
5. Calculation for a hypothetical H-guide
6. H-guide construction
7. Measurement of polyfoam dielectric constant
8. H-guide attenuation measurements
9. Conclusion

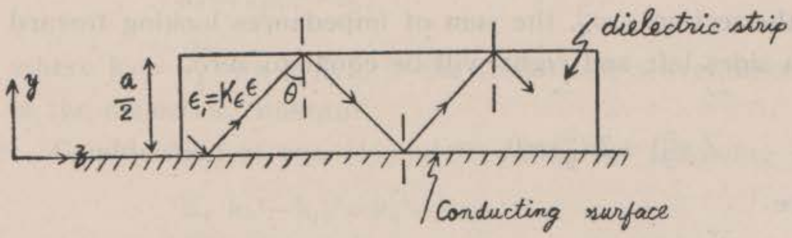
II. Modes in H-guide

with reference to the y-direction of the attached sketch, the modes fall into 2-classes: E and H modes. Again each of these types falls into 2-class according whether the plane $y=0$ as an electric short circuit or an open circuit. In this report, we shall represent the behavior of TM-mode with plane $y=0$ as an electric short circuit. The 3-field components



E_y, H_x and H_z are normally required for parallel-plate waveguide. The two additional components E_x and E_z are necessary to satisfy the boundary conditions at the air-dielectric interface.

1. Path of the wave in yz-plane with plane y=0 as short circuit.



Plane wave obliquely incidents on boundary surfaces

↓

Wave will reflect successively from the conducting surface and the dielectric boundary

↓

When the incident angle, θ, is greater than the critical angle, wave will be totally reflected from the dielectric-air interface

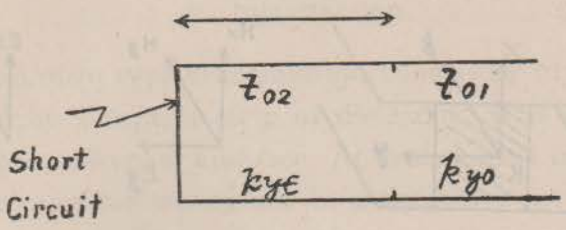
↙ ↘

Wave propagation in z-direction | wave attenuated exponentially in the air region

↘ ↙

Dielectric slab will guide the wave to propagate in z-direction.

2. Transmission line repressentation of the guide in y-direction



Standing wave in z-direction in the dielectric strip

↓

Voltages varnish on the line at

$y=0$ and $y=\frac{a}{2}$

↓

Line section between $y=0$ and $y=\frac{3}{2}$ $y=\frac{a}{2}$ may be treated as a resonance cavity

↓

At the section $y=\frac{a}{2}$, the sum of impedances looking toward both sides left and right will be equal to zero.

$$\overleftarrow{Z}\left(\tfrac{a}{2}\right)+\overrightarrow{Z}\left(\tfrac{a}{2}\right)=0$$

Since:

$$\overrightarrow{Z}\left(\tfrac{a}{2}\right)=Z_{01}$$

$$\overleftarrow{Z}\left(\tfrac{a}{2}\right)=\text{Input impedance of line 2}$$

$$=Z_{02}\frac{Z_L+jz_{02}\tan k_{y\epsilon}\frac{a}{2}}{Z_{02}+jz_L\tan k_{y\epsilon}\frac{a}{2}}$$

$$=jz_{02}\tan k_{y\epsilon}\frac{a}{2}$$

Then:

$$Z_{01}+jz_{02}\qquad\tan k_{y\epsilon}\frac{a}{2}=0$$

3. Relationship among propagation constants:
   (a) y-component of the wave number in the air region
   wave damped exponentially in y-direction in the air region

↓

$$e^{-jk_{yo}y}=e^{-\alpha y}$$

↓

$$jk_{yo} = \alpha$$
$$\downarrow$$
$$k_{yo} = -j\alpha$$

where: α is an absolute value

(b) Relation between wave numbers in the dielectric and that in the air region:-

The propagation constants in the z-direction in the dielectric and in the air region must be the same:

$$k_{z\epsilon} = k_{z0} = k_z \qquad \cdots\cdots\cdots\cdots \quad (1)$$

The relationships among the various propagation constants for the two regions $y > \frac{a}{2}$ and $0 < y < \frac{a}{2}$ are

$$\left.\begin{aligned} k_z^2 &= k_o^2 - k_{yo}^2 \\ k_z^2 &= k_\epsilon k_0^2 - k_{y\epsilon}^2 \end{aligned}\right\} \cdots\cdots\cdots\cdots \quad (2)$$

where $k_0 = \frac{2\pi}{\lambda} = \omega\sqrt{\mu_0 \epsilon_0}$ is the free space wavenumber and k_ϵ is the dielectric constant.

Combination of eqs. (1) and (2) yields the following relation:

$$K_\epsilon k_0^2 - k_{y\epsilon}^2 = k_0^2 - k_{y0}$$
$$(K_\epsilon - 1) k_0^2 = k_{y\epsilon}^2 - k_{y0}^2$$
$$= k_{y\epsilon}^2 + \alpha^2$$

Let: $k_{y\epsilon} \frac{a}{2} = P$

$$jk_{y0}\frac{a}{2} = \alpha \frac{a}{2} = q$$

then: $(K_\epsilon - 1) k_0^2 = \left(\frac{p}{\frac{a}{2}}\right)^2 + \left(\frac{q}{\frac{a}{2}}\right)^2$

$$(K_\epsilon - 1) k^2 \left(\frac{a}{2}\right)^2 = p^2 + q^2$$
$$\uparrow$$

Equation of circles for constant parameters k_o

4. Graphical solution for TM-wave:

(a) Determination of wave numbers in the dielectric and air region:

(1) Relation between k_{yo} and $k_{y\epsilon}$ expressed as tangent curves:

$$\therefore Z_{01} + jz_{02} \tan k_{y\epsilon} \frac{a}{2} = 0$$

$$\left.\begin{aligned} Z_{01} &= \frac{k_{yo}}{\omega\epsilon_o} \\ Z_{02} &= \frac{k_{y\epsilon}}{\omega K_\epsilon \epsilon_o} \end{aligned}\right\} \text{For TM-wave}$$

$$\therefore \frac{k_{yo}}{\omega\epsilon_o} + j\frac{k_{y\epsilon}}{\omega K_\epsilon \epsilon_o} \tan k_{y\epsilon}\frac{a}{2} = 0$$

$$-j\alpha + j\frac{k_{y\epsilon}}{K_\epsilon} \tan k_{y\epsilon}\frac{a}{2} = 0$$

$$-\alpha\frac{a}{2} + \frac{k_{y\epsilon}}{k_\epsilon}\frac{a}{2} \tan k_{y\epsilon}\frac{a}{2} = 0$$

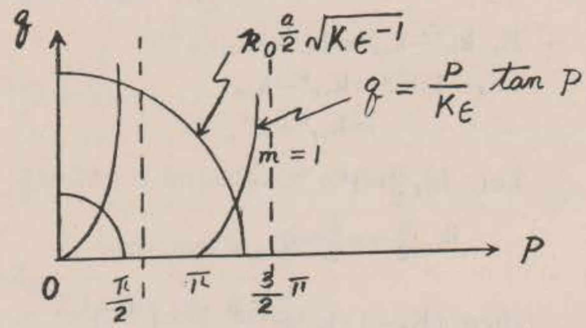
Let: $q = \frac{\alpha a}{2}$

$p = k_{y\epsilon}\frac{a}{2}$

Then: $q = \frac{p}{K_\epsilon} \tan p$

(2) Relation between k_{yo} and $k_{y\epsilon}$ expressed as circles

$(K_\epsilon - 1)(\frac{a}{2})^2 k_o^2 = p^2 + q^2$



(3) Value of k_{yo} and $k_{y\epsilon}$ for a given frequency

For a given frequency, the value of k_o will be a definite quantity and the radius of the circle fixed. The intersection point of the circle and the tangent curve will give the values of P and q, and then k_{yo} and $k_{y\epsilon}$

(b) Cut-off condition

As frequency decreases

↓

Radius of the circle decreases

↓

Intersection point going down
along the tangent curve.
when $q=0$, the intersection will occur on the p-axis, and

$$k_0 \frac{a}{2}\sqrt{K\epsilon-1}=p$$
$$=m\pi$$

where: $m=0, 1, 2, \cdots\cdots$

$$k_o=\frac{m\pi}{\frac{a}{2}\sqrt{K\epsilon-1}}$$

$$2\pi f_c \sqrt{\mu_o \epsilon_o}=\frac{m\pi}{\frac{a}{2}\sqrt{K\epsilon-1}}$$

f_c=Cut-off frequency

$$=\frac{mv}{a\sqrt{K\epsilon-1}}$$

λ_c=Cut-off wavelength

$$=\frac{a\sqrt{K\epsilon-1}}{m}$$

It is interesting to note that the lowest TM-mode does not have a cut-off frequency but can propagate at an arbitrarily low frequency.

Ⅲ. Basic equations for waves along a uniform guiding system in z-direction:

1. Maxwell's equations in component form:

(a) $\nabla \times \overline{E}=-jw\mu\overline{H}$

$$\overline{x_o}\left(\frac{\delta E_z}{\delta y}-\frac{\delta E_y}{\delta z}\right)+\overline{y_o}\left(\frac{\delta E_x}{\delta z}-\frac{\delta E_z}{\delta x}\right)+\overline{z_o}\left(\frac{\delta E_y}{\delta x}-\frac{\delta E_x}{\delta y}\right)$$
$$=-j\omega\mu(\overline{x_o}H_x+\overline{y_o}H_y+\overline{z_o}H_z)$$

x-component:

$$\frac{\delta E_z}{\delta y}-\frac{\delta E_y}{\delta z}=-j\omega\mu E_x$$

$$\frac{\delta E_z}{\delta y}-\frac{\delta}{\delta z}E_{yo}\epsilon^{-jk_zz}=-j\omega\mu H_x$$

$$\frac{\delta E_z}{\delta y}+jk_zE_y=-j\omega\mu H_x$$

y-component:

$$\frac{\delta E_x}{\delta z}-\frac{\delta E_z}{\delta x}=-j\omega\mu H_y$$

$$jk_z E_x+\frac{\delta E_z}{\delta x}=j\omega\mu H_y$$

z-component:

$$\frac{\delta E_y}{\delta x}-\frac{\delta E_x}{\delta y}=-j\omega\mu H_z$$

$$\nabla X\overline{H}=j\omega\epsilon\overline{E}$$

(b) $$\overline{x_o}\left(\frac{\delta H_z}{\delta y}-\frac{\delta H_y}{\delta z}\right)+\overline{y_o}\left(\frac{\delta H_x}{\delta z}-\frac{\delta H_z}{\delta y}\right)+\overline{z_o}\left(\frac{\delta H_y}{\delta y}-\frac{\delta H_x}{\delta y}\right)$$

$$=j\omega\epsilon\,(\overline{x_o}\ E_x+\overline{y_o}\ E_y+\overline{z_o}\ E_z)$$

x-component:

$$\frac{\delta H_z}{\delta y}+jk_z\ H_y=j\omega\epsilon\, E_x$$

y-component

$$-jk_z\ H_x-\frac{\delta H_z}{\delta y}=j\omega\epsilon\, E_y$$

z-component:

$$\frac{\delta H_y}{\delta x}-\frac{\delta H_x}{\delta y}=j\omega\epsilon\, E_z$$

2. Wave equations:

(a) Electric field:

$$\nabla^2\overline{E}=\mu\epsilon\frac{\delta^2\overline{E}}{\delta t^2}=\mu\epsilon\overline{E}_o\frac{\delta^2}{\delta t^2}e^{j(\omega t-K_z z)}$$

$$=-\omega^2\mu\epsilon\,\overline{E}_o e^{j(\omega t-k_z z)}$$

$$=-k^2\overline{E}$$

$$\left(\frac{\delta^2}{\delta x^2}+\frac{\delta^2}{\partial y^2}+\frac{\delta^2}{\partial z^2}\right)\left(\overline{x_o}E_x-\overline{y_o}E_y+\overline{z_o}\,E_z\right)=-k^2\left(\overline{x_o}E_x+\overline{y_o}E_y+\overline{z}\,E_z\right)$$

x-component:

$$\left(\frac{\delta^2}{\partial x^2}+\frac{\delta^2}{\partial y^2}\right)E_x+\frac{\delta^2}{\delta z^2}E_x=-k^2E_x$$

$$\therefore\frac{\delta^2}{\delta z^2}E_x=\frac{\delta^2}{\delta z^2}E_{x_o}e^{j(\omega t-K_z z)}$$

$$=-k_z^2 E_x$$

$$\therefore \left(\frac{\delta^2}{\delta x^2}+\frac{\delta^2}{\delta y^2}\right)E_x-k_z^2\ E_x=-k^2E_x$$

$$\left(\frac{\delta^2}{\delta z^2}+\frac{\delta^2}{\delta y^2}\right)E_x+\left(k^2-k_z^2\right)E_x=0$$

y-component:

$$\left(\frac{\delta^2}{\delta x^2}+\frac{\delta^2}{\delta y^2}\right)E_y+\left(k^2-k_z^2\right)E_y=0$$

z-component:

$$\left(\frac{\delta^2}{\delta x^2}+\frac{\delta^2}{\delta_y{}^2}\right)E_z+\left(k^2-k_z^2\right)E_z=0$$

(b) Magnetic field

$$\nabla^2 H=\mu\epsilon\frac{\delta^2 H}{\delta t^2}=-k^2H$$

x-component:

$$\left(\frac{\delta^2}{\delta x^2}+\frac{\delta^2}{\delta y^2}\right)H_x+\left(k^2-k^2_z\right)H_x=0$$

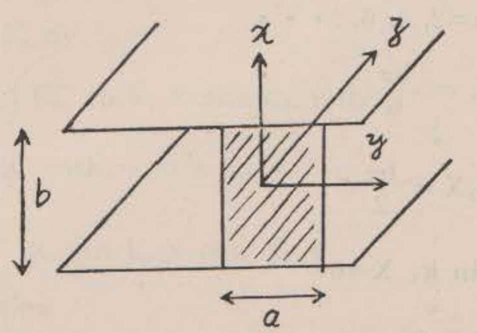
y-component:

$$\left(\frac{\delta^2}{\delta x^2}+\frac{\delta^2}{\delta y^2}\right)H_y+\left(k^2-k_z^2\right)H_y=0$$

z-component:

$$\left(\frac{\delta^2}{\delta x^2}+\frac{\delta^2}{\delta y^2}\right)H_z+\left(k^2-k_z^2\right)H_z=0$$

Ⅳ. Field of TM-mode in y-direction of the H-guide



1. Solution of the wave equation for E_z in dielectric region

$$\frac{\delta^2 E_z}{\delta x^2}+\frac{\delta^2 E_z}{\delta y^2}=-(k-k_z^2)\ E_z$$

$$=-(k_x^2+k_y^2)\ E_z$$

By method or separation of variables, the general solution of the wave equation will be as follows:

$E_z=(A \sin k_x x+B \cos k_x x)(C \sin k_{y\epsilon} y+D \cos k_{y\epsilon} y)$

(a) Determination of constants A, B, C and D for the wave mode having a short circuit plane at $y=0$

(i) when: $y = 0$

$$E_z = 0 \qquad \sin k_{y\epsilon} y = 0$$

then: $D = 0$

(ii) At the upper and conducting surface:

$$E_z=0,\ x=\pm\frac{b}{2}$$

when $n = 1, 3, 5, \cdots$

then:

$$k_x = \frac{h\pi}{b}$$
$$\downarrow$$
$$k_x X=\frac{h\pi}{2}$$
$$\downarrow$$
$$\cos k_x X=0$$
$$\downarrow$$
$$A = 0$$

when $n=2, 4, 6, \cdots$

then $k_x = \frac{h\pi}{b}$

$$\downarrow$$
$$k_x X = \frac{h\pi}{2}$$
$$\downarrow$$
$$\sin k_x X=0$$
$$\downarrow$$
$$B = 0$$

(b) Equation of E_z

when n=odd number

then: $E_z = B\ C \cos k_x X \sin k_{y\epsilon}\ y$

when n=Even number

then: $E_z = A\ C \sin k_x X \sin k_{y\epsilon}\ y$

2. Equations of the field components is the dielectric:

(a) General form with n=odd number:

$H_y = 0$

$E_z = BC \cos k_x X \sin k_{y\epsilon} y$

$E_x = -j\frac{k_x}{k_z} BC \sin k_x X \sin k_{y\epsilon} y$

Basic equation

↓

$$\frac{\delta E_z}{\delta x} + jk_z E_x = j\omega\mu H_y$$

↓

$$E_x = -\frac{1}{jk_z}\frac{\delta E_z}{\delta x} \qquad (\because H_y = 0)$$

$$= -\frac{1}{jk_z}\frac{\delta}{\delta x}(BC \cos k_x x \sin k_{\epsilon y}\ y)$$

$$= -j\frac{k_x}{k_z} BC \sin k_x x \sin k_{y\epsilon}\ y$$

$$H_x = \frac{j\omega\epsilon}{k_{y\epsilon}} BC \cos k_x\ x \cos k_{y\epsilon}\ y$$

Basic Equation

↓

$$\frac{\delta H_y}{\delta x} - \frac{\delta H_x}{\delta y} = j\omega_\epsilon\ E_z$$

↓

$$H_x = -j\omega\epsilon \int E_z\ dy$$

$$= -j\omega\epsilon \int BC \cos k_x\ x \sin k_{y\epsilon}\ y\ dy$$

$$= \frac{j\omega\epsilon}{k_{y\epsilon}} BC \cos k_x\ x \cos k_{y\epsilon}\ y$$

$$H_z = -\frac{k_x\ \omega\epsilon}{k_z\ k_{y\epsilon}} BC \sin k_x\ x \cos_\epsilon k_y\ y$$

Basic equation

↓

$$\frac{\delta H_z}{\delta y} + jk_z\ H_y = j\omega\epsilon E_x$$

$$H_z = j\omega\epsilon \int E_x dy$$

$$= j\omega\epsilon \int \left(-j\frac{k_x}{k_z} BC \sin k_x\ x \sin k_{y\epsilon}\ y\right) dy$$

$$= \frac{\omega\epsilon\ k_x}{k_z} BC \sin k_x\ x \int (\sin k_{y\epsilon}\ y) dy$$

$$= -\frac{\omega\epsilon\ k_x}{k_z\ k_m^{x}} BC \sin k_x\ x \cos k_{y\epsilon}\ y$$

$$E_y = -\frac{jBC}{k_z\ k_{y\epsilon}} (k^2 - k_y{}^2_\epsilon) \cos k_x\ x \cos k_{y\epsilon}\ y$$

Basic equation

$$\frac{\delta E_z}{\delta y} + jk_z\ E_y = -j\omega\mu\ H_x$$

$$E_y = -\frac{1}{jk_z}\left[j\omega\mu\ H_x + \frac{\delta E_z}{\delta y}\right]$$

$$= -\frac{1}{jk_z}\left[j\omega\mu \frac{j\omega\epsilon}{k_{y\epsilon}} BC \cos k_x\ x \cos k_{y\epsilon}\ y + BC\ k_{y\epsilon} \cos k_x\ x \cos k_{y\epsilon}\ y\right]$$

$$= -\frac{jBC}{k_z\ k_{z\epsilon}} (k^2 - k_{y\epsilon}) \cos k_x\ x \cos k_{y\epsilon}\ y$$

(b) Field components expressed in terms of voltage at the center of the dielectric strip.

Let: $\tau = \frac{1}{k^2}\left(k_z^2 + k_x^2\right) = \left(\frac{\lambda_0}{2\pi}\right)^2 \left(k_z^2 + k_x^2\right)$

$$= \left(\frac{\lambda_0}{2\pi}\right)^2 \left(k^2 - k_{y\epsilon}^2\right)$$

$$= \left(\frac{\lambda_0}{2\pi}\right)^2 \left[\omega^2 \mu\epsilon_0 K\epsilon - k_{y\epsilon}^2\right]$$

Then: $k_{y\epsilon}^2 = \omega^2 \mu\epsilon_0 K\epsilon - \left(\frac{2\pi}{\lambda_0}\right)^2 \tau$

$$k_{y}\epsilon = \frac{2\pi}{\lambda_0} \sqrt{K\epsilon - \tau}$$

Let: $E_{0\epsilon}$ = Electric field at the center of the dielectric strip

E_z and E_x at the center of the dielectric are equal to zero

$E_{o\epsilon}$ = y-component of the electric at that center point

$$= -\frac{jBC}{k_z k_{y\epsilon}}(k^2 - k^2_{y\epsilon})$$

$$= -\frac{jBC}{k_z}\ \frac{1}{\frac{2\pi}{\lambda_o}\sqrt{K\epsilon-\tau}}\ \tau\left(\frac{2\pi}{\lambda_o}\right)^2$$

$$= -\frac{jBC}{\sqrt{K\epsilon-\tau}}\ \tau\ \frac{\frac{2\pi}{\lambda_o}}{\frac{2\pi}{\lambda_g}}$$

$$E_{o\epsilon} = \frac{jBC}{\sqrt{K\epsilon-\tau}}\ \frac{\lambda_g}{\lambda_o}$$

$$BC = jE_{o\epsilon}\frac{\sqrt{K\epsilon-\tau}}{\tau}\ \frac{\lambda_o}{\lambda_g}$$

then: Field equations will be as follows:

$$H_y = 0$$

$$E_y = E_{o\epsilon}\cos k_x\, x \cos k_y\, y\, e^{-jk_z z}$$

$$E_z = BC \cos k_x\, x \sin k_{y\epsilon}\, y\, e^{-jk_z z}$$

$$= j\frac{E_{o\epsilon}}{\tau}\sqrt{K\epsilon-\tau}\ \frac{\lambda_o}{\lambda_g}\cos k_x\, x \sin k_{y\epsilon}\, y\, e^{-jk_z z}$$

$$E_x = -j\frac{k_x}{k_z}BC \sin k_x\, x \sin k_{y\epsilon}\, y\, e^{-jk_z z}$$

$$= \frac{E_{o\epsilon}}{\tau}\sqrt{K\epsilon-\tau}\ \frac{\lambda_o}{\lambda_g}\ \frac{k_x}{\frac{2\pi}{\lambda_g}}\sin k_x\, x \sin k_{y\epsilon}\, y\, e^{-jk_z y}$$

$$= \frac{E_{o\epsilon}}{\tau}\sqrt{K\epsilon-\tau}\ \frac{\lambda_o}{2\pi}\ k_x \sin k_x\, x \sin k_{y\epsilon}\, y\, e^{-jk_z z}$$

$$H_x = \frac{j\omega\epsilon}{k_{y\epsilon}}BC \cos k_x\, x \cos k_{y\epsilon}\, y\, e^{-jk_z z}$$

$$= \frac{j\omega\epsilon}{k_{y\epsilon}}\left(j\ \frac{E_{o\epsilon}}{\tau}\ \frac{\lambda_o}{\lambda_g}\sqrt{K\epsilon-\tau}\right)\cos k_x\, x \cos k_{y\epsilon}\, y\, e^{-jk_z z}$$

$$= -\frac{\omega\epsilon\ E_{o\epsilon}}{\frac{2\pi}{\lambda_o}\sqrt{K\epsilon-\tau}}\ \frac{\sqrt{K\epsilon-\tau}}{\tau}\ \frac{\lambda_o}{\lambda_g}\cos k_x\, x \cos k_{y\epsilon}\, y\, e^{-jk_z z}$$

$$= -\frac{\omega\epsilon E_{o\epsilon}}{\omega\sqrt{\mu_o \epsilon_o}} \frac{1}{\tau} \frac{\lambda_o}{\lambda_g} \cos k_x x \cos k_{y\epsilon} y \, e^{-jK_z z}$$

$$= -\frac{E_o \epsilon}{\eta_o} K\epsilon \frac{\lambda_o}{\lambda_g} \frac{1}{\tau} \cos k_x x \cos k_{y\epsilon} y \, e^{-jk_z z}$$

$$H_z = -\frac{\omega\epsilon k_x}{k_z k_{y\epsilon}} BC \sin k_x x \cos k_{y\epsilon} y \, e^{-jk_z z}$$

$$= -\frac{k_x}{k_z} \frac{\omega\epsilon}{k_{y\epsilon}} BC \sin k_x x \cos k_{y\epsilon} y \, e^{-jk_z z}$$

$$= -\left(\frac{k_x}{\frac{2\pi}{\lambda_g}}\right)\left(j \frac{E_o \epsilon}{\eta_o} \frac{\lambda_o}{\lambda_g} \frac{1}{\tau}\right) \sin k_x x \text{ con } k_{y\epsilon} y \, e^{-jk_z z}$$

$$= -\frac{jE_{o\epsilon}}{\eta_o} \frac{K\epsilon}{\tau} \frac{\lambda_o k_x}{2\pi} \sin k_x x \cos k_{y\epsilon} y e^{-jk_z z}$$

3. Field components in air region:
   (a) Relation between the wave numbers in the air region and that in dielectric:

   k_x and k_z:

   To satisfy the conditions on the air-dielectric interface, the waves on both sides wi llhave equal components in x-direction and also that in z-directions, i.e. k_x and k_z will not be changed.

   k_{yo}:

   To allow the wave decaying in y-direction in the air region, k_{yo} will be an imaginary quantity.

   $k_{yo} = -j^{\alpha}$

   where α is an absolute quantity

   (b) Electric field at the center of the interface between the dielectric and air region:

   Continuity of the normal component of electric displacement.

$$\downarrow$$

$$D_{n1} = D_{n2}$$

$$\downarrow$$

$$\epsilon_o k_\epsilon E_{y\epsilon} = \epsilon_o E_o$$

$$\downarrow$$

$$k\epsilon E_{o\epsilon} \text{ coo } k_x x \cos k_{y\epsilon} y = E_o$$

$$\downarrow$$

$$K\epsilon \ E_{o\epsilon} \cos k_{y\epsilon} \frac{a}{2} = E_o$$

where: E_o = y-component the electric field at the center of interface

(c) Equations of the field components:

$H_y = 0$

$$E_z = \frac{jE_o}{\tau}\sqrt{\tau-1}\left(\frac{\lambda_o}{\lambda_g}\right) \text{ocs } k_x \, x e^{-(y-\frac{a}{2})\alpha} e^{-jk_z z}$$

$$E_x = \frac{E_o}{\tau}\sqrt{\tau-1}\left(\frac{\lambda_o k_x}{2\pi}\right)\sin k_x \, x e^{-(y-\frac{a}{2})\alpha} e^{-jk_z z}$$

$$H_x = \frac{E_o}{\eta_o} \frac{1}{\tau} \frac{\lambda_o}{\lambda_g} \cos k_x \, x e^{-(y-\frac{a}{2})\alpha} e^{-jk_z z}$$

$$H_z = -\frac{jE_o}{\eta_o} \frac{1}{\tau}\left(\frac{\lambda_o k_x}{2\pi}\right)\sin k_x \, x e^{-(y-\frac{a}{2})\alpha} e^{-jk_z z}$$

$$E_y = E_o \text{ csok}_x \, x e^{-(y-\frac{a}{2})\alpha} e^{-jk_z z}$$

V. Power distribution:

1. power flow in the dielectric:

P_d = power in dielectric

$$= -\frac{1}{2}\int_{y=-\frac{a}{2}}^{\frac{a}{2}} \int_{x=-\frac{b}{2}}^{\frac{b}{2}} E_y H_x \, dx \, dy$$

$$E_y = E_{o\epsilon} \cos k_x \, x \cos k_{y\epsilon} \, y$$

$$H_x = -\frac{E_{o\epsilon}}{\eta_o} \frac{\lambda_o}{\lambda_g} \frac{k_\epsilon}{\tau} \cos k_x \, x \cos k_{y\epsilon} \, y$$

$$= H_x^*$$

$$E_y H_x^* = -\frac{E_{o\epsilon}^2}{\eta_o} \frac{\lambda_o}{\lambda_g} \frac{k_\epsilon}{\tau} \cos^2 k_x \, x \cos^2 k_{y\epsilon} \, y$$

$$= -\frac{E_o{}^2{}_\epsilon}{\eta_o} \frac{\lambda_o}{\lambda_g} \frac{k_\epsilon}{\tau} (1+\cos 2 k_x \, x)(1+\cos 2 k_{y\epsilon} \, y)$$

$$P_d = \frac{1}{8}\left(\frac{E_{o\epsilon}^2}{\eta_o} \frac{\lambda_o}{\lambda_g} \frac{k_\epsilon}{\tau}\right)\int_{y=-\frac{a}{2}}^{\frac{a}{2}} \int_{x=-\frac{b}{2}}^{\frac{b}{2}} (1+\cos 2k_x x)$$

$(1+\cos 2\,k_{y\epsilon}\,y)\;dxdy$

$$=\frac{1}{8}\left(\frac{E_{o\epsilon}^2}{\eta_o}\;\frac{\lambda_o}{\lambda_g}\;\frac{k_\epsilon}{\tau}\right)\left[x+\frac{\sin 2k_x x}{2k_x}\right]_{x=-\frac{b}{2}}^{\frac{b}{2}}\left[y+\frac{\sin 2k_{y\epsilon}y}{2k_{y\epsilon}}\right]_{y=-\frac{a}{2}}^{\frac{a}{2}}$$

$$\because\ \sin\ 2k_x x\int_{-\frac{b}{2}}^{\frac{b}{2}}=2\sin\left(2\frac{2\pi}{\lambda_x}\;\frac{b}{2}\right)$$

$$=2\sin\ \pi=0$$

$$\left[x+\sin 2k_x x\right]_{-\frac{b}{2}}^{\frac{b}{2}}=b$$

$$\sin\ 2k_{y\epsilon}\ y\int_{-\frac{a}{2}}^{\frac{a}{2}}=2\sin^2 k_{y\epsilon}\frac{a}{2}$$

$$=2\sin\ k_o\ a\sqrt{K\epsilon-\tau}$$

$$\left[y+\frac{\sin 2k_{y\epsilon}y}{2k_{y\epsilon}}\right]_{-\frac{a}{2}}^{\frac{a}{2}}=a+\frac{\sin\ k_o\ a\sqrt{K\epsilon-\tau}}{k_o\sqrt{K\epsilon-\tau}}$$

$$\therefore\ Pd=\frac{1}{8}\ \frac{E_{o\epsilon}^2}{\eta_o}\ K\epsilon\ \frac{\lambda_o}{\lambda_g}\ \frac{b}{\tau}\left[a+\frac{\sin k_o\ a\sqrt{K\epsilon-\tau}}{k_o\sqrt{K\epsilon-\tau}}\right]$$

2. Power flow in air region:

P_a=Power in air

$$=2\left[-\frac{1}{2}\int_{x=-\frac{b}{2}}^{\frac{b}{2}}\int_{y=\frac{a}{2}}^{\infty}E_y\,H_x{}^*\,dx\ dy\right]$$

$$\because\ E_y=E_o\ \cos\ k_x\ xe^{-(y-\frac{a}{2})\alpha}$$

$$H_x=-\frac{E_o}{\eta_o}\ \frac{1}{\tau}\ \frac{\lambda_o}{\lambda_g}\ \cos\ k_x\ x\ e^{-(y-\frac{a}{2})\alpha}$$

$$\therefore\ P_a=\frac{E_o^2}{\eta_o}\ \frac{1}{\tau}\ \frac{\lambda_o}{\lambda_g}\int_{x=-\frac{b}{2}}^{\frac{b}{2}}\int_{y=\frac{a}{2}}^{\infty}\cos^2\ k_x x e^{-2(y-\frac{a}{2})\alpha}\ dx\ dy$$

$$=\frac{E_o^2}{\eta_o}\frac{1}{\tau}\frac{\lambda_o}{\lambda_g}\frac{b}{2}\frac{1}{-2\alpha}e^{-2(y-\frac{a}{2})\alpha}\Big|_{y=\frac{a}{2}}^{\infty}$$

$$=\frac{E_o^2}{\eta_o}\frac{1}{\tau}\frac{\lambda_o}{\lambda_g}\frac{b}{2}\frac{1}{-2\alpha}(-1)$$

$$=\frac{1}{4}\frac{E_o^2}{\eta_o}\frac{1}{\tau}\frac{\lambda_o}{\lambda_g}\frac{b}{\alpha}$$

$$\because k_o^2=k_x^2+k_y^2+k_z^2$$

$$=k_x^2+k_z^2-\alpha$$

$$\alpha=\sqrt{(k_x^2+k_z^2)-k_o^2}$$

$$k_o\sqrt{\frac{k_x^2+k_z^2}{k_o^2}-1}=k_o\sqrt{\tau-1}$$

$$\therefore P_a=\frac{E_o^2}{4}\frac{b}{\eta_o}\frac{\lambda_o}{\lambda_g}\frac{1}{k_o\tau\sqrt{\tau-1}}$$

3. Total power flow

$$P_t=P_d+P_a$$

Ⅵ. Attenuation

1. Power loss in metal conducting plates:

Let: W_m = power loss per unit length in the top and bottom conducting plates

$1_{y\epsilon}$ = Current densi ty per unit length in the metal of the dielectric region

T_{yo} = Current density per unit length in the metal of the air region.

R_s = Surface resistivity

Then:

$$W_m=\int_{-\frac{a}{2}}^{\frac{a}{2}}\left|J_{y\epsilon}\right|^2 R_s\,dy+2\int_{\frac{a}{2}}^{\infty}\left|J_{yo}\right|^2 R_s\,dy$$

$$=\int_{-\frac{a}{2}}^{\frac{a}{2}}\left|H_{z\epsilon}\right|^2 R_s\,dy+2\int_{\frac{a}{2}}^{\infty}\left|H_{zo}\right|^2 R_s\,dy$$

$$= \int_{-\frac{a}{2}}^{\frac{a}{2}} \left[\frac{E_o\epsilon}{\eta_o} \frac{K\epsilon}{\tau} \frac{\lambda_o k_x}{2\pi} (-\sin k_x \frac{b}{2})(\cos k_{y\epsilon}\, y) \right]^2 R_s\, dy$$

$$+2 \int_{\frac{a}{2}}^{\infty} \left[\frac{E_o}{\eta_o} \frac{1}{\tau} \frac{\lambda_o k_x}{2\pi} (-\sin k_x \frac{b}{2}) e^{-(y-\frac{a}{2})\alpha} \right]^2 R_s\, dy$$

$$= \left(\frac{\lambda_o k_x}{2\pi} \frac{1}{\tau\eta_o} \sin k_x \frac{b}{2} \right)^2 R_s \left[\int_{-\frac{a}{2}}^{\frac{a}{2}} E_o\epsilon^2 K\epsilon \operatorname{Cos}^2 k_{y\epsilon}\, y\, dy \right.$$

$$\left. + 2 \int_{\frac{a}{2}}^{\infty} E_o^2 e^{-2(y-\frac{a}{2})\alpha} dy \right]$$

$$= \left(\frac{\lambda_o}{2b} \right) \frac{1}{\eta_o^2} \frac{R_s}{\tau^2} \left[\frac{1}{2} E_o^2\epsilon K_\epsilon^2 (y + \frac{\sin 2k_{y\epsilon}\, y}{2k_{y\epsilon}}) \Big|_{-\frac{a}{2}}^{\frac{a}{2}} \right.$$

$$\left. - \frac{2E_o^2}{2\alpha} e^{-2(y-\frac{a}{2})\alpha} \Big|_{y=\frac{a}{2}}^{\infty} \right]$$

$$= \left(\frac{\lambda_o}{2b} \right)^2 \frac{1}{\eta_o} \frac{R_s}{\tau^2} \left[\frac{1}{2} E_o\epsilon^2 K\epsilon^2 (a + \frac{\sin k_o a \sqrt{K\epsilon - \tau}}{k_o \sqrt{K\epsilon - \tau}}) + \frac{E_o^2}{\alpha} \right]$$

$$= \left(\frac{\lambda_o}{2b} \right)^2 \frac{1}{\eta_o^2} \frac{R_s}{2\tau^2} \left[E_o\epsilon^2 K\epsilon^2 (a + \frac{\sin k_o a \sqrt{K\epsilon - \tau}}{k_o \sqrt{K\epsilon - \tau}}) + \frac{2E_o^2}{k_o \sqrt{\tau - 1}} \right]$$

2. Power loss in dielectric:

Let: W_{diel} = power loss per unit length due to finite conductivity of the dielectric material

= Conductivity of the dielectric material

Then:

$$W_{diel} = \frac{\sigma}{2} \int_{x=-\frac{b}{2}}^{\frac{b}{2}} \int_{y=-\frac{a}{2}}^{\frac{a}{2}} \left[\left| E_x \right|^2 + \left| E_y \right|^2 + \left| E_z \right|^2 dx\, dy \right.$$

$$= \frac{\sigma}{4} \left| E_o\epsilon \right|^2 \left\{ \frac{K\epsilon^n}{2\tau} + (1 + \frac{K\epsilon}{2\tau}) \frac{\sin k_o\, a \sqrt{K\epsilon - \tau}}{k_o \sqrt{K\epsilon - \tau}} \right\}$$

3. Attenuation:

a = attenuation

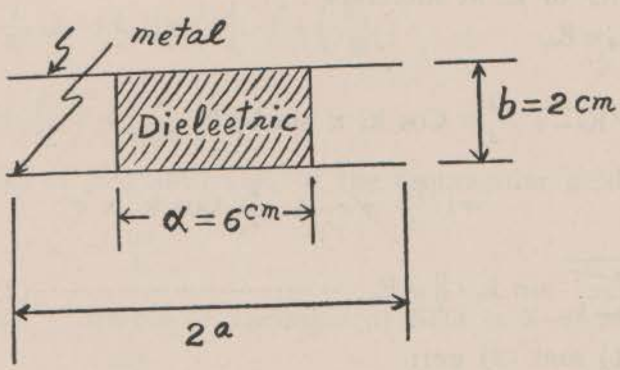
$$= \frac{{}^{1}/_{2}\text{ (heat loss per unit length)}}{\text{total power flowing down the guide}}$$

$$= A_{metal} + A_{diel}$$

$$= \frac{W_m + W_{diel}}{P_t}$$

Ⅶ. Calculation for a hypothetical H-guide.

1. Dimension and Material



(a) Matal plates _______ Silver with resistivity 1.62×10^{-6} ohm-cm.

(b) Dielectric Material _______ Polyfoam with dielectric constant of 1.03 and loss tangenst $\tan\delta = 0.1 \times 10^{-4}$

(c) $\frac{\text{Width of dielectric slab}}{\text{Height of dielectric slab}} = 3$

(d) Height of slab=Separation between plates.

The separation between the plates is so chosen that frequencies above 7.5 kmc/sec would propagate inside the guide. Use of 2 cm for this dimension will make the cut-off frequency nearly identical with that of the smallest rectangular waveguide available in our laboratory.

2. Relative frequency Parameter $K_0 a$.

(a) Characteristic equation of the H-Guide.

Continuity of normal component of electric flux displacement at the air-dielectric interface.

$$D_{n1} = D_{n2}$$

$$\downarrow$$

$$K\epsilon \; E_{y\epsilon} = E_o$$

$$\downarrow$$

$$K\epsilon \; E_{o\epsilon} \; \mathrm{Cos} \; k_x \; x \; \mathrm{Cas} \; k_{y\epsilon} \; y = E_o$$

$$\downarrow$$

$$K\epsilon \; E_{o\epsilon} \; \mathrm{Cos} \; k_{y\epsilon} \frac{a}{2} = E_o \quad \cdots\cdots\cdots (1)$$

$\uparrow$ y-component of the electric field at the center of interface.

Continuity of E_z at interface

$$E_{z\epsilon} = E_{zo}$$

$$\downarrow$$

$$j \frac{E_{o\epsilon}}{\tau} \sqrt{K\epsilon - \tau} \; \frac{\lambda_o}{\lambda_g} \; \mathrm{Cos} \; k_x \; x \; \sin \; k_{y\epsilon} \frac{a}{2}$$

$$= i \frac{E_o}{\tau} \sqrt{\tau - 1} \; \frac{\lambda_o}{\lambda_g} \; \mathrm{Cos} \; k_x \; x \; e^o$$

$$E_o \epsilon \frac{\sqrt{K\epsilon - \tau}}{\sqrt{\tau - 1}} \sin \; k_y \epsilon \frac{a}{2} = E_o \quad \cdots\cdots\cdots (2)$$

From (1) and (2) get:

$$K\epsilon \; \mathrm{Cos} \; k_{y\epsilon} \; \frac{a}{2} = \frac{\sqrt{K\epsilon - \tau}}{\sqrt{\tau - 1}} \sin \; k_{y\epsilon} \; \frac{a}{2}$$

$$\downarrow$$

$$\tan \; k_{y\epsilon} \; \frac{a}{2} = \frac{K\epsilon \sqrt{\tau - 1}}{\sqrt{K\epsilon - \tau}}$$

(b) Relation between $k_{y\epsilon}$ and k_o

$$\tau = \frac{1}{k_o^2} (k_z^2 + k_x^2) = \frac{1}{k_o^2} (k_{o\epsilon}^2 - k_{y\epsilon}^2)$$

$$= \frac{1}{k_o^2} (k_o^2 \; k\epsilon - k_{y\epsilon}^2)$$

$$= K\epsilon - \left(\frac{k_{y\epsilon}^2}{k_o} \right)^2$$

$$(k_{y\epsilon})^2 = k_o^2 \; (k\epsilon - \tau)$$

$$k_{y\epsilon} = k_o \sqrt{K\epsilon - \tau}$$

$$= \frac{2\pi}{\lambda_o} \sqrt{K\epsilon - \tau}$$

(c) Relation between $k_o a$ and τ

$$\tan k_y \epsilon \frac{a}{2} = \frac{K\epsilon\sqrt{\tau-1}}{\sqrt{K\epsilon-\tau}}$$

$$\tan \left(k_o \frac{\sqrt{K\epsilon-\tau}}{2}\right) = K\epsilon \frac{\sqrt{\tau-1}}{\sqrt{K\epsilon-\tau}}$$

3. value of $\frac{\lambda_o}{\lambda_g}$:

(a) τ expressed in $\frac{\lambda_o}{\lambda_g}$ and $\frac{\lambda_o}{2b}$

$$\tau = \frac{1}{k^2}(k_z^2+k_x^2) = \left(\frac{\lambda_o}{\lambda_g}\right)^2 + \left(\frac{k_x}{k}\right)^2$$

$$= \left(\frac{\lambda_o}{\lambda_g}\right)^2 + \left(\frac{\lambda_o}{2\pi}\right)^2 k_x^2$$

$\because$ (k_x of H-guide) = (k_x of the rectangnlar guide)

$$\therefore\ k_x = \frac{n\pi}{b}$$

when n=No of variation of field in X-direction
=1

$$\therefore\ k_x = \frac{\pi}{b}$$

$$\therefore\ \tau = \left(\frac{\lambda_o}{\lambda_g}\right) + \left(\frac{\lambda_o}{2\pi}\ \frac{\pi}{b}\right)^2$$

$$= \left(\frac{\lambda_o}{\lambda_g}\right)^2 + \left(\frac{\lambda_o}{2b}\right)^2$$

$$\left(\frac{\lambda_o}{\lambda_g}\right)^2 = \sqrt{\tau - \left(\frac{\lambda_o}{2b}\right)^2}$$

(b) Relation between $\frac{\lambda_o}{2b}$ and $k_o a$

$$\frac{k_x}{k_o} = \frac{\lambda_o}{2\pi}\ \frac{\pi}{b} = \frac{\lambda_o}{2b}$$

$$= \frac{\pi}{b}\ \frac{1}{k_o} = \frac{\pi}{b}\ \frac{1}{k_o}\ \frac{a}{a}$$

$$= \left(\frac{\pi a}{b}\right)\left(\frac{1}{k_o a}\right)$$

4. Attenuation:

P_d = Power flow in dielectric

$$=\frac{1}{8}\frac{|E_{o\epsilon}|^2}{\eta_o}K\epsilon\frac{\lambda_o}{\lambda_g}\frac{b}{\tau}\left[a+\frac{\text{Sin }k_o a\sqrt{K\epsilon-\tau}}{k_o\sqrt{K\epsilon-\tau}}\right]$$

$$=\frac{|E_{o\epsilon}|^2}{8}\frac{ab}{\eta_o}\frac{\lambda^o}{\lambda_g}\frac{K\epsilon}{\tau}\left[1+\frac{2K\epsilon(\tau-1)/(K\epsilon-1)}{k_o a\sqrt{\tau-1}(\tau K\epsilon+\tau-K\epsilon)}\right]$$

P_a = power flow in air region

$$=\frac{|E_o|^2}{4}\frac{b}{\eta_o}\frac{\lambda_o}{\lambda_g}\frac{1}{k_o\tau\sqrt{\tau-1}}$$

$$=\frac{|E_{o\epsilon}|^2}{8}\frac{ab}{\eta_o}\frac{\lambda_o}{\lambda_g}\frac{K\epsilon}{\tau}\left[\frac{2K\epsilon\ (K\epsilon-\tau)/(K\epsilon-1)}{k_o a\sqrt{\tau-1}\ (\tau\ K\epsilon+\tau\ K\epsilon)}\right]$$

W_m = Power loss per nuit length in the top and bottom conducting plates.

$$=\left(\frac{\lambda_o}{2b}\right)\frac{1}{\eta_o^2}\frac{R_s}{2\tau^2}\left[E_{o\epsilon}^2\ k_\epsilon^2\left(a+\frac{\sin k_o a\sqrt{K\epsilon-\tau}}{k_o\sqrt{K\epsilon-\tau}}\right)+\frac{2E_o^2}{k_o\sqrt{\tau-1}}\right]$$

$$=\frac{R_x}{2}\frac{a}{\eta_o^2}\left(\frac{\lambda_o}{2b}\right)^2|E_{o\epsilon}|^2\frac{K\epsilon^2}{\tau^2}\left[1+\frac{2\tau}{k_o a\sqrt{\tau-1}(\tau K\epsilon+\tau-K\epsilon)}\right]$$

W_d = Power loss per unit length inside the dielectric material.

$$=\frac{\sigma}{4}|E_{o\epsilon}|^2\left\{\frac{K\epsilon a}{2\tau}+\left(1-\frac{K\epsilon}{2\tau}\right)\frac{\sin k_o a\sqrt{K\epsilon-\tau}}{k_o\sqrt{K\epsilon-\tau}}\right\}$$

$$=\frac{\omega\epsilon_o K\epsilon\,\text{btan}\,\delta}{4}|E_o\epsilon|^2\left\{\frac{K\epsilon a}{2\tau}+\left(1-\frac{k\epsilon}{2\tau}\right)\frac{\sin k_o a\sqrt{K\epsilon-\tau}}{k_o\sqrt{K\epsilon-\tau}}\right\}$$

$$=\frac{|E_{o\epsilon}|^2}{8}\frac{ab}{\eta_o}\left(\frac{2\pi}{\lambda_o}\right)\frac{K\epsilon^2}{\tau}\tan\delta\left[1+\frac{2(2\tau-K\epsilon)(\tau-1)/(K\epsilon-1)}{k_o a\sqrt{\tau-1}(\tau K\epsilon+\tau-K\epsilon)}\right]$$

A_m = Attenuation due to metal

$$=\frac{Wm}{2(Pa+Pd)}$$

$$=\frac{2R_s}{b\eta_o}\left(\frac{\lambda_o}{2b}\right)^2\left(\frac{\lambda_g}{\lambda_o}\right)\frac{K\epsilon}{\tau}\frac{1+\frac{2\tau}{k_o a\sqrt{\tau-1}(\tau K\epsilon+\tau-K\epsilon)}}{1+\frac{2K\epsilon}{k_o a\sqrt{\tau-1}(\tau K\epsilon+\tau-K\epsilon)}}$$

A_d = Attenuation due to dielectric

$$= \frac{Wd}{2(Pa+Pd)}$$

$$= \frac{\pi \, K\epsilon \, \tan \delta}{\lambda_o} \left(\frac{\lambda_g}{\lambda_o} \right) \frac{1 + \frac{2(\tau-1)\,(2\tau - K\epsilon)/(K\epsilon-1)}{k_o \, a \sqrt{\tau-1}(\tau K\epsilon + \tau - K\epsilon)}}{1 + \frac{2K\epsilon}{k_o \, a \sqrt{\tau-1}(\tau K\epsilon + \tau - K\epsilon)}}$$

5. Calculation of the attenuation:

The three values τ, $\frac{\lambda_o}{2b}$ and $\frac{\lambda_o}{\lambda_g}$ are used to determine attenuation due to the metal, and that due the dielectric. A knowledge of τ and K_o a is sufficient for the determination. As an illustration, calculate the attenuation of the hypothetical H-guide as follows.

Let: $\tau = 1.02$ which is within the range $1 < \tau < K\epsilon$

(a) Value of $K_o a$.

$$\mathrm{Tan}\left(k_o \, a \frac{\sqrt{k\epsilon - \tau}}{2} \right) = K\epsilon \frac{\sqrt{\tau - 1}}{\sqrt{K\epsilon - \tau}}$$

$$= 1.03 \frac{\sqrt{1.02-1}}{\sqrt{1.03-1.02}} = 1.456$$

$$k_o \, a \frac{\sqrt{K\epsilon - \tau}}{2} = 55.5^\circ = 1.47 \text{ Radians}$$

$$k_o \, a = \frac{2 \times 1.47}{\sqrt{1.03-1.02}} = 29.4$$

(b) $\frac{\lambda_o}{2b}$ · $\frac{\lambda_o}{\lambda_g}$

$$\left(\frac{\lambda_o}{2b} \right) = \frac{\pi a}{b} \, \frac{1}{k_o a} = \frac{3\pi}{29.4} = 0.32$$

$$\frac{\lambda_o}{\lambda_g} = \sqrt{\tau - \left(\frac{\lambda_o}{2b} \right)^2}$$

$$= \sqrt{1.02 - 0.32^2} = 0.955$$

(c) Attenuation due to metal

$$am = \frac{2ks}{b\eta_o} \left(\frac{\lambda_o}{2b} \right)^2 \frac{\lambda_g}{\lambda_o} \, \frac{K\epsilon}{\tau} \, \frac{1 + \frac{2\tau}{k_o \, a \sqrt{\tau-1}(\,K\epsilon + \tau - K\epsilon)}}{1 + \frac{2K\epsilon}{k_o \, a \sqrt{\tau-1}(\,K\epsilon + \tau - K\epsilon)}}$$

$$\frac{2R_s}{b\eta_o}=\frac{2\times1.62\times10^{-6}}{2\times377}=4.3\times10^{-9}$$

$$\frac{K\epsilon}{\tau}=\frac{1.03}{1.02}=1.01$$

$$\frac{2\tau}{k_o\,a\sqrt{\tau-1}(\ K\epsilon+\ -K\epsilon)}=\frac{2\times1.02}{29.4\sqrt{0.02}\,(1.02\times1.03+1.02-1.03)}$$

$$=\frac{2.04}{29.4\times0.1414\times1.04}=0.481$$

$$\frac{2K_\epsilon}{k_o\,a\sqrt{\tau-1}(\ K\epsilon+\tau-K\epsilon)}=\frac{2.06}{29.4\times0.1414\times1.04}=0.486$$

$$A_m=4.3\times10^{-9}\ (0.32)^2\ \frac{1}{0.955}\times1.01\,\frac{1+0.481}{1+0.486}$$

$=4.38\times10^{-10}$ per cm.

$=1.33\times10^{-8}$ per ft.

(d) Attenuation due to dielectric

$$\lambda_o=\frac{2\pi}{k_o}=\frac{2\pi a}{k_o a}=\frac{2\pi\times6}{29.4}=1.28\text{cm}$$

$$\frac{\pi\ K\epsilon\ \tan\delta}{\lambda_o}\ \left(\frac{\lambda_g}{\lambda_o}\right)=\frac{\pi\,1.03\times0.1\times10^{-4}}{1.28}\ \frac{1}{0.955}$$

$$=2.65\times10^{-5}$$

$$\frac{2(\tau-1)\ (2\tau-K\epsilon)/(K\epsilon-1)}{k_o\,a\sqrt{\ -1}(\ K\epsilon+\ -K\epsilon)}=\frac{2\times0.02\times(2.04-1.03)/0.03}{29.4\times0.1414\times1.04}$$

$$=\frac{1.38}{29.4\times0.1414\times1.04}=0.325$$

$$A_d=2.65\times10^{-5}\ \frac{1+0.325}{1+0.486}$$

$=2.37\times10^{-5}$ per cm.

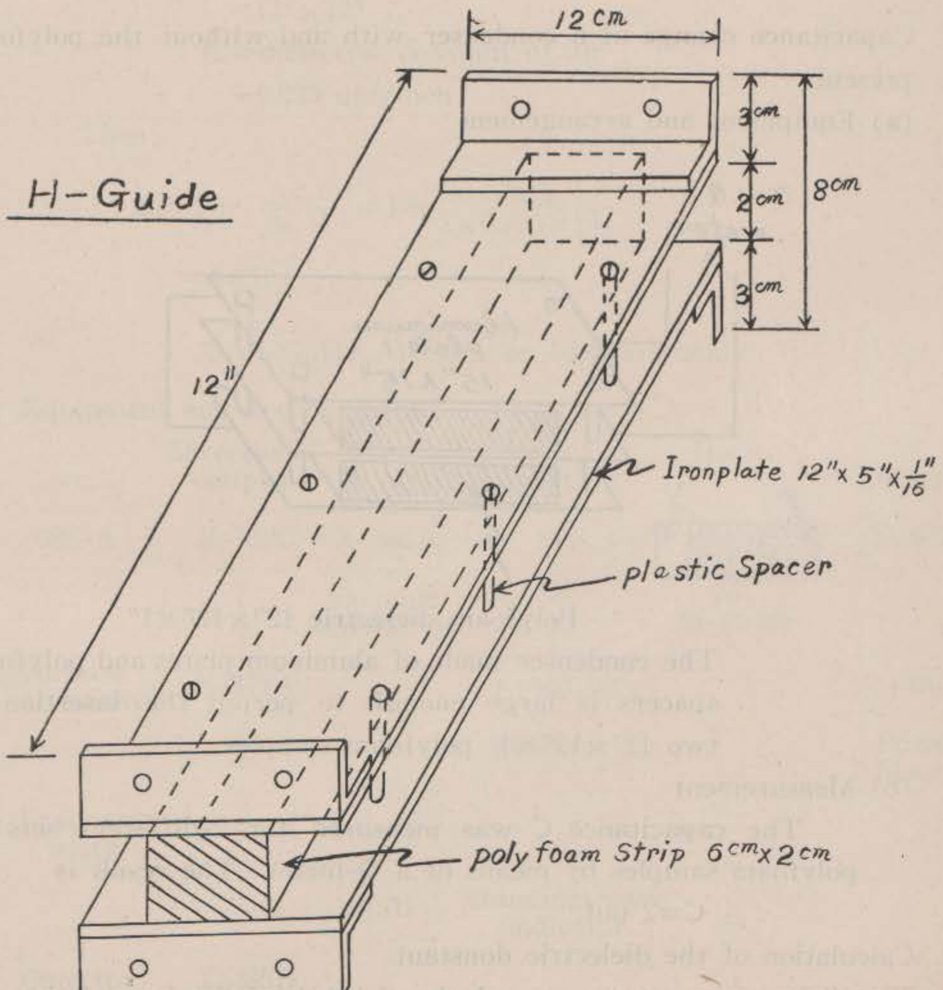
$=7.2\times10^{-4}$ per ft.

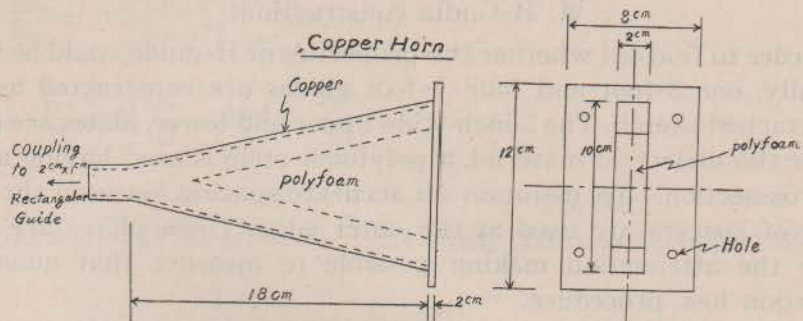
Comparison between A_m and A_d, it is evident that loss factor of the dielectric is much greater than that of the metal strip.

Ⅷ. H-Gudie construction:

In order to find out whether the properties of H-guide could be realized practically, one 3-foot and four 1-foot guides are constructed as shown in the attached sketch. The 5-inch-wide upper and lower plates are made of iron. For the dielectric material, a polyfoam strip is used having a 6 cm x 2-cm crosssection. To maintain an accurate spacing between the plates, a series of spacers are used at the outer edges. Iron plates are used to increase the attenuation making possible to measure that quantity by an insertion loss procedure.

For the purpose of exciting the H-guide, horns are made that typered gradually from the rectangular waveguide to a 5″ aperture opening into the H-guide.

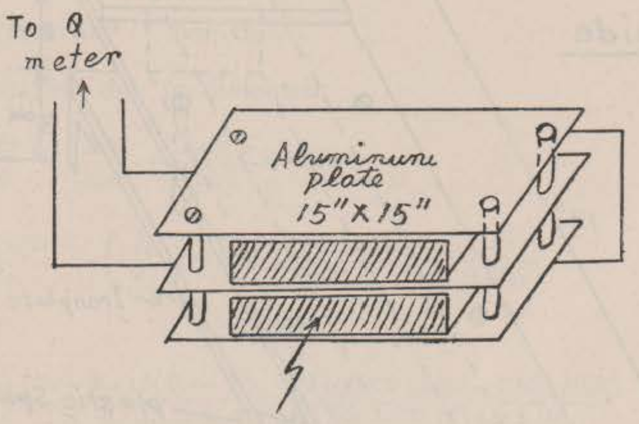




Ⅺ. Measurement of dielectric constant

1. Capacitance change of a condenser with and without the polyfoam present.

   (a) Equipment and arrangement



Polyfoam dielectric 12″×12″×1″

The condenser made of aluminum plates and polyfoam spacers is large enough to permit the insertion of two 12″×12″×1′ polyfoam samples.

   (b) Measurement

   The capacitance C was measured for 2-different sets of polyfoam samples by means of a Q-meter. The result is

   $\triangle$ C=2 uuf.

2. Calculation of the dielectric donstant

   The dielectric constant was calculated by the following relation

$$K\epsilon - 1 = \frac{\frac{\triangle c}{2}\, d}{\frac{\epsilon_0 A}{d}\, d_1 - \frac{\triangle c}{2}\,(d-d_1)}.$$

where: K_E=dielectric constant of the polyfoam

C=Observed capacitance change in uuf.

d=distance between top (or bottom) and center plate =1″

d′=Height of the sample 1″

A=cross-sectional area of the sample parallel to the plates =12″×12″

E_0=dielectric constant of air =0.225 uuf/inch

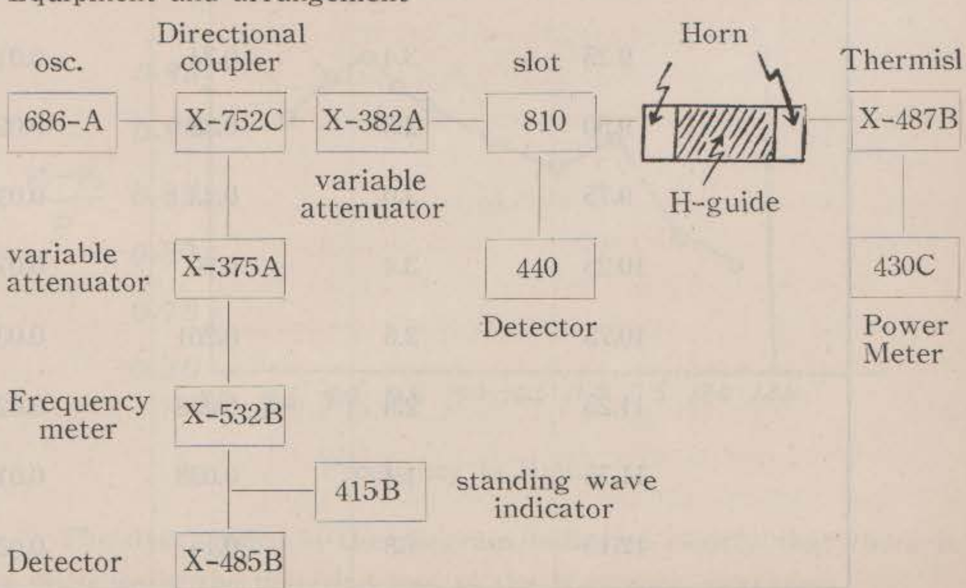
Then:

$$K_E = 1 + \frac{\triangle c\ d}{2\epsilon_0\ A} = 1 + \frac{2\times 1}{2\times 0.225\times 144}$$

$$=1.031$$

X. H-Guide Attenuation Measurements

1. Equipment and arrangement



A waveguide sample 5 feet long was constructed by firmly fastening together two 1-foot lengths and one 3-foot length.

2. Measurement

Measurements were made to determine the magnitude of the attenuation of H-guide, and the manner in which it changed as the frequency was varied. It consisted of inserting the sample between a lanncher and a receiver and noting the reduction in the power to a balometer at the end of the receiver. The launcher and the receiver were identical, each being of a horn exciter having a 10 cm. aperture to which was attached a one-foot length of H-guide.

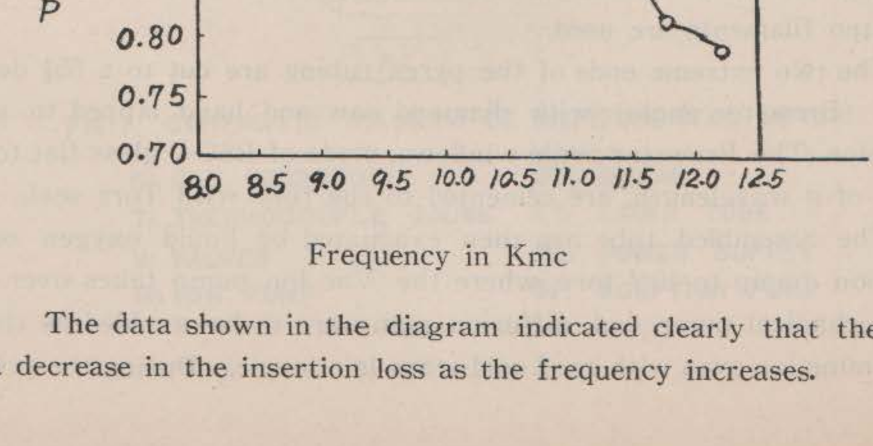
3. Results

(a) Data:

Feequency KMC.	Power measured in M.w.		
	without guide	with 2-ft. guide	with 5-ft guide
8.75	4.8	0.44	0.037
9.00	4.6	0.24	0.014
9.25	3.4	0.34	0.014
9.50	2.3	0.38	0.024
9.75	3.0	0.43	0.033
10.25	3.4	0.70	0.075
10.75	2.5	0.261	0.034
11.25	2.3	0.258	0.026
11.75	1.4	0.088	0.017
12.15	1.8	0.14	0.028

(b) Loss in 3-ft guide

Frequency Kmc	Loss in 3-ft waveguide	
	in m.w. p_1-p_2	$\frac{p_1-p_2}{p_1}$
8.75	0.403	0.916
9.00	0.226	0.940
9.25	0.326	0.958
9.50	0.356	0.936
9.75	0.397	0.924
10.25	0.625	0.891
10.73	0.227	0.870
11.25	0.232	0.898
11.75	0.071	0.810
12.15	0.112	0.793



Frequency in Kmc

The data shown in the diagram indicated clearly that there is a decrease in the insertion loss as the frequency increases.