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On the Surface Wave Diffraction by a Step

by

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Abstract-A modified procedure of the Wiener-Hopf technique is developed, and used to determine the transmitted and reflected fields of a surface wave which is normally incident upon the junction of two semi-infinite planes joined together by a step of height a. Both planes have a normalized reactance X, and the step surface has X_0. Reduction of the transmission loss at the step by increasing X_0 is shown to be possible. The relation between edge condition and uniqueness of solution along with field behaviours at the corners of the step are fully investigated. A comparison between the present procedure and the residue cancellation is made.

I. Introduction

A great deal of research has been done on the two-dimensional problems of surface wave diffraction by discontinuity in surface reactance, but a little has been done on those by discontinuity in guide plane. A main difference between the two is that the former is connected with line integral of scattering sources, whereas the latter with surface integral of scattering sources. This paper presents an extension of Johansen's work [1], and moreover gives a correct problem formulation.

The discontinuity studied in this paper is a reactive step of height a as shown in Fig. 1. Two semi-infinite planes ($x>0$, $y=0$, and $x<0$, $y=a$) with a normalized reactance X are joined together by the step ($x=0$, $0<y<a$) with a normalized reactance X_0. The incident field is a TM surface wave travelling in the negative x-direction. The step produces reflected and transmitted surface waves and a radiated field.

It would appear that the Wiener-Hopf technique is the only method available at the moment for dealing with the problem. The solution is thus carried out by a method based on the Wiener-Hopf

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technique. In Section 2 the problem is physically and mathematically formulated into a boundary-value problem, in which a field behaviour near the upper edge of the step is specified. In Section 3 application of the Laplace transform formally reduces the formulation to an equivalent problem, that of solving a transform equation which relates four unknown functions of one complex variable, two of the functions are independent.

It is impractical to tackle the transform equation by the usual Wiener-Hopf procedures, since the equation possesses four related unknown functions in addition to two branch points which occur in a kernel function of the equation. A modification of the procedures is thus developed, which is presented in Section 4. In Section 5 the transmission and reflection coefficients together with some numerical result are given. In Section 6 the relation between edge condition and uniqueness of solution is discussed in some detail. Following the solution, some physical interpretations of the results are given in Section 7. In the final section it is shown that, for numerical computation, formulation by the present procedure is superior to that by the residue cancellation.

2. Problem Formulation

In order to formulate the boundary-value problem, divide the space of concern into three regions: (1) $x>0$, $0<y<a$; (2) $x>0$, $y>a$; (3) $x<0$, $y>a$. Since the incident direction of the TM surface wave is perpendicular to the step surface, the magnetic intensities of the incident as well as scattered waves contain the z-component only, and are functions of x and y. With reference to the time factor $\exp(i\omega t)$, denote the magnetic intensity of the incident wave as

$$H_0(x,y)=\exp(i\beta x-\alpha y) \tag{2-1}$$

where $\beta=k_0\sqrt{1+x^2}$, $\alpha=k_0 X$ and $k_0=\omega\sqrt{u_0\varepsilon_0}$.

The branch of k_0 is taken that Im $k_0<0$. Define $H_n(x,y)$, $n=1,2$ as the magnetic intensity of a scattered field in the regions $x>0$ and $H_3(x,y)$ as that of a scattered field minus the incident field in the region $x<0$. Then, $H_n(x,y)$ satisfy the two-dimensional Helmholz's equations:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+k_0^2\right)H_n(x,y)=0,\ (x,y)\text{ in Region}(n),\ n=1,2,3 \tag{2-2a,b,c}$$

According to definition of the surface reactance, $H_n(n=1,3)$ are subject to the boundary condition:

$$\frac{\partial H_1}{\partial y}(x, 0_+) = -\alpha H_1(x, 0_+), x>0 ; \tag{B1}$$

$$\frac{\partial H_3}{\partial y}(x, a_+) = -\alpha H_3(x, a_+), x<0 ; \tag{B2}$$

$$\frac{\partial H_1}{\partial x}(0_+, y) = -\alpha_0 H_1(0_+, y) - (\alpha_0 + i\beta)e^{-\alpha y}, 0<y<a \tag{B3}$$

where $\alpha_O = k_O X_O$.

By virtue of homogeneity of the medium, all field components are continuous throughout the interior space. This continuity condition is ensured by

$$H_1(x, a-) = H_2(x, a+), \frac{\partial H_1}{\partial y}(x, a-) = \frac{\partial H_2}{\partial y}(x, a+), x>0 ; \tag{C1}$$

$$H_2(0+, y) = H_3(0-, y), \frac{\partial H_2}{\partial x}(0+, y) = \frac{\partial H_3}{\partial x}(0-, y), y>a ; \tag{C2}$$

Field behaviour at the corners of the step is similar to that at the corner of dielectric wedge: Field components remain finite as observation point approaches the edges along any ray. From this physical viewpoint we expect naturally an edge condition given by

$$H_2(0+, a_+) = H_3(0-, a_+) = O(x) \text{ as } x \to \pm 0. \tag{E}$$

We have, in sum, the boundary-value problem: the equations (2-2) are to be solved subject to the conditions (B), (C), (E) and the radiation condition.

3. A Transform Equation

We proceed to show that the boundary-value problem described in the previous section is, under the Laplace transform, equivalent to a transform equation in one complex variable.

We introduce, first, the 'unilateral' Laplace transforms defined by

$$F_+(s) = L_+ f(x) = \int_{+0}^{\infty} f(x) e^{-sx} dx, \tag{3-1a}$$

$$F_-(s) = L_- f(x) = \int_{-\infty}^{-0} f(x) e^{-sx} dx \tag{3-1b}$$

where the subscripts +, - for F denote analyticity in the right and left halves of the s-plane, respectively

To obtain the key to the diffraction we require the following steps: (1) apply L_+ to (2-2a); (2) make use of (B3) to eliminate $\frac{\partial H_1}{\partial x}$ (0+,y); (3) change the sign of s in the equation obtained in step(2) and multiply the resultant equation by $(s-\alpha_0)$; (4) multiply the result of step (2) by $(s+\alpha_0)$; (5) add the equations obtained in the last two steps. The result which contains mainly a knowledge of the scattering sources on the step surface is

$$\left(\frac{\partial^2}{\partial y^2}+s^2+k_0^2\right)\{(s+\alpha_0)\hat{F}_+(s,y)+(s-\alpha_0)\hat{F}_+(-s,y)\}= -2s(\alpha_0+i\beta)e^{-\alpha y} \tag{3-2}$$

where $\hat{F}_+(s,y) = L_+H_1(x,y)$. A general solution of this inhomogeneous equation is

$$(s+\alpha_0)\hat{F}_+(s,y)+(s-\alpha_0)\hat{F}_+(-s,y)=B(s)e^{iW(y-a)}+C(s)e^{-iWy} - \frac{2s(\alpha_0+i\beta)}{s^2+\beta^2}e^{-\alpha y} \tag{3-3}$$

where $W=\sqrt{s^2+k_0^2}$. The branch of W must be chosen that W reduces to k_0 if $s = 0$.

Subjecting to (B1), B and C are in fact dependent:
(1) Set $y \doteq 0$ in (3-3) and multiply the result by α; (2) differentiate (3-3) with respect to y and then set $y = 0$ in the resultant equation; (3) summing the equations obtained in steps (1) and (2), we obtain

$$C(s) = -\frac{\alpha+iW}{\alpha-iW}e^{-Wa}B(s) \tag{3-4}$$

In favor of functions whose regions of regularity are known, eliminate B and C among (3-4), (3-3) and the derivative of (3-3). This gives, $0<y<a$,

$$J(s,y)\{(s+\alpha_0)\hat{F}_+(s,y)+(s-\alpha_0)\hat{F}_+(-s,y)\}$$
$$=(s+\alpha_0)\frac{\partial\hat{F}_+}{\partial y}(s,y)+(s-\alpha_0)\frac{\partial\hat{F}_+}{\partial y}(-s,y)$$
$$-(\alpha_0+i\beta)e^{-\alpha y}\{J(s,y)+\alpha\}\frac{2s}{s^2+\beta^2}$$

where $$J(s,y) = \frac{W(\alpha \cos Wy + W \sin Wy)}{\alpha \sin Wy - W \cos Wy} \tag{3-5}$$

Next, set up a functional relation between L_+H_2 and L_-H_3 which will be donoted by $F_+(s,y)$ and $F_-(s,y)$ respectively. Apply L_+ L_- to (2-2b), (2-2c) respectively, add the resultant equations and simplify the sum by (C2) to find

$$\left(\frac{\partial^2}{\partial y^2} + s^2 + k_0^2\right)\{F_+(s,y) + F_-(s,y)\} = 0, \; y > a. \tag{3-6}$$

This equation has an appropriate solution

$$F_+(s,y) + F_-(s,y) = A(s)e^{-iW(y-a)}, \; y > a. \tag{3-7}$$

From this equation we obtain

$$F_+(s,a_+) + F_-(s,a_+) = A(s) \tag{3-8a}$$

$$\frac{\partial F_+}{\partial y}(s,a_+) + \frac{\partial F_-}{\partial y}(s,a_+) = -iWA(s) \tag{3-8b}$$

Use (3-7) and the Laplace inversion formula to find

$$\int_{c_0 - i\infty}^{c_0 + i\infty} A(s)e^{-iW(y-a)+sx}\,ds = \begin{cases} H_2(x,y), \; x > 0 & \text{(3-9a)} \\ H_3(x,y), \; x < 0. & \text{(3-9b)} \end{cases}$$

Insert (3-9b) in (B2), and reverse the order of integration and differentiation to obtain

$$\int_{c_0 - i\infty}^{c_0 + i\infty} (\alpha - iW)A(s)e^{sx}\,ds = 0, \; x < 0. \tag{3-10}$$

On account of a theorem due to Jordan [4] the deformation of the path of integration Re $s = c_0$, such that for $x<0$ the new contour encloses the positive real s-axis, is certainly permissible if $(\alpha - iW)$ $A(s)$ tends to zero, uniformly with respect to arg s, for $|s| \to \infty$ and s in the right half-plane. The edge condition (E) implies that $A(s) = o(|s|^{-1})$ as $|s| \to \infty$ and therefore $(\alpha - iW)A(s) = o(1)$ as $|s| \to \infty$. From the arguments mentioned above and Cauchy's theorem, (3-10) gives

$$(\alpha - iW)A(s) = R_+(s). \tag{3-11}$$

It follows that (3-8) can be rewritten as

$$F_+(s, a_+) = \frac{1}{\alpha - iW} R_+(s) - F_-(s, a_+), \tag{3-12a}$$

$$\frac{\partial F_+}{\partial y}(s, a_+) = -\frac{iW}{\alpha - iW} R_+(s) + \alpha F_-(s, a_+) \tag{3-12b}$$

Finally, to complete the derivation of our transform equation replace $\hat{F}_+, \frac{\partial \hat{F}_+}{\partial y}$ in (3-5) by the right sides of (3-12a), (3-12b) respectively on the ground of the condition (C1). Rearrange to obtain

$$\frac{1}{(s^2+\beta^2)K(s)} R_+(s) - F_-(s) + \frac{s-\alpha_0}{s+\alpha_0} \frac{1}{(s^2+\beta^2)K(s)} R_+(-s)$$

$$-\frac{s-\alpha_0}{s+\alpha_0} F_-(-s) = -2(\alpha_0 + i\beta)e^{-\alpha a} \frac{s}{s+\alpha_0} \frac{1}{s^2+\beta^2}$$

where $K(s) = e^{-iWa} \dfrac{\sin Wa}{W}$, $F_-(s) \triangleq F_-(s, a_+)$. (3-13)

4. The Procedure of Spectral Representations

The practical details of the analysis carried out later tend to obscure the essential simplicity of the present procedure, which is therefore summarized in this paragraph. Decompose the transform euuation into two equations: one is analytic in the right half-plane, the other in the left half-plane. Express each unknown function as a sum of a discrete set and a continuous set of expansion functions, and then insert these spectral representations in the decomposed equations. Since the expansion functions are linearly independent, the solution of the transform equation is reduced to that of a mixed system of simultaneous linear algebraic and integral equations.

First decompose K(s) in the form $K_+(s) \cdot K_-(s)$[5] with

$$K_{\pm}(s)=\sqrt{\frac{\sin k_0 a}{k_0}}\ \exp\{\mp S(s)-\frac{iWa}{\pi}\cos^{-1}(\frac{s}{\pm ik_0})\}\prod_{n=1}^{\infty}(1\pm s/c_n)$$

(4-1)

$\exp(\mp s/c_n)$

where

$$S(s)=\frac{sa}{\pi}\{\ln\frac{2\pi}{ik_0a}+0.4228+\sum_{n=1}^{\infty}(\frac{1}{n}-\frac{\pi}{c_n a})\}$$

$$c_n\simeq\begin{cases}\sqrt{(\frac{n\pi}{a})^2-k_0^2}\ , & \mathrm{Re}\ k_0<\frac{n\pi}{a}\\ i\sqrt{k_0^2-(\frac{n\pi}{a})^2}\ , & \mathrm{Re}\ k_0>\frac{n\pi}{a}\end{cases}$$

Note that $K_-(-s)=K(s)$; $K_+, K_- \sim |s|^{-\frac{1}{2}}$ as $|s|\to\infty$ in appropriate half-planes; and the branch of $\cos^{-1}s$ is chosen such that $\cos^{-1}0=\pi/2$.

Introduce the notation for representing a unique additive decomposition of a function as

$$F(s)=[F(s)]_+ + [F(s)]_-$$

Then, by the Wiener-Hopf procedure we may express (3-13) in the form

$$P(s)=\frac{1}{(s+i\beta)K_+(s)}R_+(s)+\left[\frac{s-\alpha_0}{s+\alpha_0}\cdot\frac{R_+(s)}{(s+i\beta)K_+(s)}\right]_+$$
$$-\left[\frac{s-\alpha_0}{s+\alpha_0}(s-i\beta)K_-(s)F_-(-s)\right]_+ +2(\alpha_0+i\beta)e^{-\alpha a}\left[\frac{s}{s+\alpha_0}\frac{K_-(s)}{s+i\beta}\right]_+$$
$$=(s-i\beta)K_-(s)F_-(s)-\left[\frac{s-\alpha_0}{s+\alpha_0}\frac{R_+(s)}{(s+i\beta)K_+(s)}\right]_-$$
$$+\left[\frac{s-\alpha_0}{s+\alpha_0}(s-i\beta)K_-(s)F_-(-s)\right]_- -2(\alpha_0+i\beta)e^{-\alpha a}\left[\frac{s}{s+\alpha_0}\frac{K_-(s)}{s+i\beta}\right]_-$$

(4.2)

From the edge condition (E) we can readily show that $P(s) \equiv 0$. Thus, (4.2) is actually separable into two equations: the second part is equal to zero, and so is the third part,

As shown in Fig. 2 the complex s-plane is cut from ik to infinity, -ik to infinity in the right half-plane, the left half-plane, respectively. Then, by pole removal we find

$$\left[\frac{s}{s+\alpha_0}\frac{K_-(s)}{s+i\beta}\right]_+ = \frac{\alpha_0}{\alpha_0 - i\beta}\frac{K_-(-\alpha_0)}{s+\alpha_0} - \frac{i\beta}{\alpha_0 - i\beta}\frac{K_-(-i\beta)}{s+i\beta}, \tag{4-3a}$$

$$\left[\frac{s}{s+\alpha_0}\frac{K_-(s)}{s+i\beta}\right]_- = \frac{\alpha_0}{\alpha_0 - i\beta}\frac{K_-(s)-K_-(-\alpha_0)}{s+\alpha_0} - \frac{i\beta}{\alpha_0 - i\beta}\frac{K_-(s)-K_-(-i\beta)}{s+i\beta} \tag{4-3b}$$

In addition, by taking into account the contributions from the simple poles $-i\beta$, $-\alpha_0$, $-c_n$, n=1,2,..... and the branch point -ik, deformations of the contour to the left half-plane gives

$$\left[\frac{s-\alpha_0}{s+\alpha_0}\frac{R_+(-s)}{(s+i\beta)K_+(s)}\right]_+ = \frac{\alpha_0+i\beta}{\alpha_0-i\beta}\frac{R_+(i\beta)}{K_+(-i\beta)}\frac{1}{s+i\beta} + \frac{2\alpha_0}{\alpha_0-i\beta}\frac{R_+(\alpha_0)}{K_+(-\alpha_0)}\frac{1}{s+\alpha_0} - \sum_{n=1}^{\infty}\frac{\alpha_0+c_n}{\alpha_0-c_n}\cdot\frac{R_+(c_n)}{(c_n-i\beta)K_+'(-c_n)}\frac{1}{s+c_n}$$
$$-\frac{1}{2\pi i}\int_C \frac{z-\alpha_0}{z+\alpha_0}\frac{R_+(-z)}{(z+i\beta)K_+(z)}\frac{dz}{z-s}. \tag{4-4}$$

A path C is taken that no singularities except -ik fall inside C. In a similar way we find

$$\left[\frac{s-\alpha_0}{s+\alpha_0}(s-i\beta)K_-(s)F_-(-s)\right]_- = \frac{1}{2\pi i}\int_C \frac{z-\alpha_0}{z+\alpha_0}(z-i\beta)F_-(-z)\frac{dz}{z+s}. \tag{4-5}$$

Facility in finding out suitable spectral representations of $R_+(s)$ and $F_-(s)$ comes only with observation. Observing (4-2) by the help of (4-3) through (4-5), we learn

$$R_+(s) = M_0\cdot(s+i\beta)K_+(s)\left\{\frac{d_1}{s+i\beta} + \frac{d_2}{s+\alpha_0} + \sum_{n=3}^{\infty}\frac{d_n}{s+c_{n-2}} + \int_C \frac{u(s)}{z-s}dz\right\}, \tag{4.6a}$$

$$F_-(s) = \frac{M_0}{(s-i\beta)K_-(s)}\left\{d_1'\frac{K_-(s)-K_-(-i\beta)}{s+i\beta} + d_2'\frac{K_-(s)-K_-(-\alpha_0)}{s+\alpha_0}\right. \tag{4-6b}$$

$$\left. + \int_C \frac{v(z)}{z+s}\,dz\right\}$$

where $M_0 = \frac{2\pi}{\sqrt{a}} \cdot \frac{\alpha_0+i\beta}{\alpha_0-i\beta} \cdot e^{-\alpha a}$.

The given constant M_0 is inserted for simplifying numerical computation. Now, a mixed system of simultaneous algebraic and integral equations for the unknown constants d_1', d_2', d_n, $n=1,2\ldots\ldots$ and the unknown functions u, v can be obtained by the following steps: (1) substitute the spectral representations of R $(\pm s)$, $F_-(\pm s)$ into (4-2); (2) deform the contour of each $[\ ]_+$ to the two sides of the lower branch-cut by counting contributions from simple poles enclosed on shifting the contour, and so deform the contour of each $[\ ]_-$ to the upper branch-cut; (3) in vertue of linear independence of all the expansion functions, equate the coefficient of each expansion function in each equation to zero.
We observe in passing that the system has a unique solution.

Consideration of the contributions from $\pm ik$ (the branch-cut integrals) which mainly give rise to the radiation field, in fact, involves the complicated integral equations. Thus, these integrals are hereupon omitted in further development. It follows that the system is reduced to:

$$d_1' = -\frac{i\beta\sqrt{a}}{\pi}, \quad d_2' = \frac{\alpha_0\sqrt{a}}{\pi};$$

and d_1, d_2, ... satisfy an infinite system of simultaneous linear algebraic equations.

$$\sum_{n=1}^{\infty} a_{mn}\, d_n = g_m, \quad m=1,2,\cdots\cdots \tag{4-7}$$

Introduce simplifying notations

$p_1 = i\beta,\ p_2 = \alpha_0,\ p_n = c_{n-2},\ n=3,4,\cdots\cdots;$

$$M_n = \begin{cases} (-1)^n \dfrac{(p_2+p_1)K_+(p_n)}{(p_2-p_1)K_+(-p_n)}, & (n=1,2), \\[2ex] \dfrac{(p_n+p_1)(p_n+p_2)(n-2)^2}{(p_n-p_1)(p_n-p_2)2(p_n a/\pi)^2} \cdot \dfrac{K_+^2(p_n)}{a}, & (n=3,4,\cdots\cdots); \end{cases}$$

$$K_{n1} = \frac{1}{\sqrt{a}}\left\{K_+(-p_n) - K_+(p_1)\right\}, K_{n2} = \frac{1}{\sqrt{a}}\left\{K_+(-p_n) - K_+(p_2)\right\}, n=1, 2, \cdots\cdots;$$

$$G_n = M_n \cdot \frac{p_n a}{\pi}\left\{-\frac{2p_1}{p_n+p_1} K_{n1} + \frac{2p_2}{p_n+p_2} K_{n2}\right\}, n=1, 2, \cdots\cdots \tag{4-8}$$

Then, the given elements of (4-7) becomes

$$a_{mn} = \begin{cases} 1+M_m, & (n=m), \\ M_m \dfrac{2p_m}{p_m+p_n}, & (n \neq m); \end{cases}$$

$$g_n = \begin{cases} \dfrac{i\beta\sqrt{a}}{\pi} K_+(i\beta) + G_1, & (n=1); \\ -\dfrac{\alpha_0\sqrt{a}}{\pi} K_+(\alpha_0) + G_2, & (n=2), \\ G_n, & (n=3, 4, \cdots\cdots). \end{cases} \tag{4-9}$$

Existence of $[a_{mn}]^{-1}$ is ensured by the existence of a unique solution of the problem. As implied by (4-8) and (4-9),we observe that the inhomogeneous elements g_n and the off-diagonal entries a_{mn} converges to zero as n and m increase. Moreover, rapidity of the convergence depends on the height of the step a/λ where λ denotes the wavelength of the incident wave. This means that the number of equations necessary for accurate computing d_n depends on a/λ. For $a/\lambda < 1$ it is sufficient to find an approximate solution of the infinite system (4-7) by solving the first four equations in the first four unknowns d_1 through d_4.

5. The Transmission and Reflection Coefficients

From (3-7), (3-11) and (4-6a) the total magnetic field $H_T(x,y)$ for $y>a$ is approximately given by the inverse transform

$$H_T(x,y) \cong \frac{1}{2\pi i} \int_{c_0 - i\infty}^{c_0 + i\infty} \frac{M_0 \cdot (s+i\beta) K_+(s)}{\alpha - iW} \left\{ \frac{d_1}{s+i\beta} + \frac{d_2}{s+\alpha_0} + \sum_{n=3}^{\infty} \frac{d_n}{s+c_{n-2}} \right\} \exp\{sx - iW(y-a)\} ds + \exp(i\beta x - \alpha y). \tag{5-1}$$

Two fields, transmitted and reflected, are of our primary interest. The pole at $s = -i\beta$ contributes to the reflected field, while the pole at $s = i\beta$ contributes to the transmitted field. This gives the reflection and transmission coefficients as follows:

$$r \simeq -2\pi \frac{\alpha}{i\beta} \cdot \frac{\alpha_0 + i\beta}{\alpha_0 - i\beta} \cdot \frac{K_+(-i\beta)}{\sqrt{a}} \cdot d_1 \tag{5-2a}$$

$$t \simeq e^{-\alpha a}\{1 - 2\pi \cdot \frac{\alpha}{i\beta} \cdot \frac{\alpha_0 + i\beta}{\alpha_0 - i\beta} \cdot \frac{K_+(i\beta)}{\sqrt{a}} \cdot (d_1 + \frac{2i\beta}{\alpha_0 + i\beta} d_2$$

$$+ \sum_{n=3}^{\infty} \frac{2i\beta}{c_{n-2} + i\beta} d_n)\} \tag{5-2b}$$

In addition, from (5-1) we find that there is a peculiar mode of a small percentage of the incident power propagating in the y direction, which is contributed by the pole at $s = -\alpha_0$.

The absolute values of the coefficients r and t are plotted as functions of a/λ, X and X_0. One feature of the numerical result, which deserves particular attention, is presented in Fig. 3. The practical significance of these curves is explained in Section 7.

6. Edge Condition and Uniqueness of Solution

The equations in (4-2) determine $R_+(s)$, $F_-(s)$ to within the arbitrary polynomial P(s). However, an edge condition which ensures $P(s) \equiv 0$ might still give a non-unique solution of the problem, since there is a coupling between the two equations through the second and third terms of each equations. This kind of nonunique solution is investigated in the next paragraph.

We no longer obtain a unique solution of this diffraction problem, if we relax the edge condition (E) to a weaker state:

$$H_2(0_+, a_+) = b, H_3(0_-, a_+) = c, (b \neq c). \tag{E'}$$

Instead of $A(s) = o(|s|^{-1})$ as $|s| \to \infty$ which is given in Section 3, the new condition (E') implies $A(s) = O(|s|^{-1})$ as $|s| \to \infty$ It follows that $(\alpha - iW)A(s)$ does not tend to zero uniformly with respect to arg s, for $|s| \to \infty$ so a suitable solution for (3-10) is not $R_+(s)$. Therefore, the relations (3-12a,b) must be replaced by a single expression

$$\frac{\partial F_+}{\partial y}(s, a_+) = -iWF_+(s, a_+) + (\alpha - iW)F_-(s, a_+) \tag{6-1}$$

Substitution of (6-1) into (3-5) yields a new transform equation

$$\frac{1}{K_o(s)} F_+(s)+F_-(s)+\frac{s-\alpha_0}{s+\alpha_0}\frac{1}{K_o(s)} \cdot F_+(-s)+\frac{s-\alpha_0}{s+\alpha_0}\cdot F_-(-s)$$

$$=-2(\alpha_0+i\beta)e^{-\alpha a}\cdot\frac{s\sin Wa/W}{(s+\alpha_0)(\cos Wa-\alpha\sin Wa/W)} \tag{6-2}$$

where $K_0(s)=e^{-iWa}(\cos Wa - \alpha\sin Wa/W)$.
Cos Wa - α sin Wa/W, an integral function, can be factorized by the infinite product method.

Following a similar procedure of spectral representations, we now reduce (6-2) to a new system of simultaneous algebraic and integral equations, whose solution can be readily shown to be nonunique.

7. The Implications of the Results

The computed result as shown in Fig. 3 indicates that, for constant X and a, there is a considerable increase of the transmitted power with increasin X_0 . It would appear that the above result is correct in spite of neglecting the branch-cut integrals: A possible physical interpretation is that an increased guiding effect of the step surface brings about an increase in power flow along the step surface.

As the solution of the problem with the given condition (E) exists and is unique, we might conclude that all field components of the total field remain finite as observation point approaches either edge of the step along any ray. Besides, the magnetic intensity of the total field is continuous not only at the upper edge but also at the lower edge of the step. Examining (4-6b), we find

$$\lim_{s\to-\infty} sF_-(s)=0.$$

Therefore, the edge condition (E) practically becomes

$$H_2(0+,a+)=H_3(0_-,a+)=0.$$

This shows a remarkable feature of field behaviors: the magnetic intensity of the total field at the upper edge is equal to that of the unperturbed incident field. In view of definition of the surface reactance and the discontinuities of the boundary condition, we also find that the electric intensity of the total field suffers a discontinuity at either edge.

8. Futher Remarks

The present procedure provides a significant extension of the range of problem that can be solved by the usual procedures of the Wiener-Hopf technique, including the residue cancellation. Furthermore, with regard to accuracy of numerical work the present procedure is superior to the residue cancellation. This is demonstrated by the following example.

$$\frac{R_+(s)}{(s^2+\beta^2)K(s)}+\frac{s+\alpha}{s-\alpha}\cdot F_+(-s)+\frac{s+\alpha}{s-\alpha}\cdot\frac{\cos Wa-\alpha\sin Wa/W}{(s^2+\beta^2)\sin Wa/W}R_+(-s)$$

$$=-\frac{2s(i\beta-\alpha)e^{-\alpha a}}{(s^2+\beta^2)(s-\alpha)} \tag{8-1}$$

By the residue cancellation the equation (8-1) is reduced to that of solving an infinite system of linear algebraic equations for the unknowns $R_+(i\beta)$, $R_+(c_n)$, $(n=1,2,\dots)$,

$$a_{11}R_+(i\beta)+\sum_{n=2}^{\infty}a_{1n}R_+(c_{n-1})=b_1,$$

$$a_{m1}R_+(i\beta)+\sum_{n=2}^{\infty}a_{mn}R_+(c_{n-1})=b_m, m=2,3,4,\cdots\cdots \tag{8-2}$$

where $b_m \neq 0$ for $m=1,2,\dots$. On the other hand, by using

$$R_+(s)=(s+i\beta)K_+(s)\cdot\left\{\frac{d_1}{s+i\beta}+\sum_{n=2}^{\infty}\frac{d_n}{s+c_{n-1}}\right\},$$

the procedure of spectral representations gives

$$a_{11}d_1+\sum_{n=2}^{\infty}a'_{1n}d_n=b'_1 \quad ,$$

$$a'_{m1}d_1+\sum_{n=2}^{\infty}a_{mn}d_n=0, m=2,3,\cdots\cdots \tag{8-3}$$

where d_1, d_2,.... are unknowns. A striking difference between

(8-2) and (8-3) is: the former contains an infinite mumber of non-zero inhomogeneous terms, while the latter contains only a finite number of non-zero inhomogeneous terms.

For complete information of the diffraction, especially the radiation field, contributions from the branch points $\pm ik_0$ to the solution must be taken into consideration. It would seem that a suitable approach to deal with a mixed system of simultaneous algebraic and integral equations is the process of iterations. This will be thoroughly studied in a sequel to this work.

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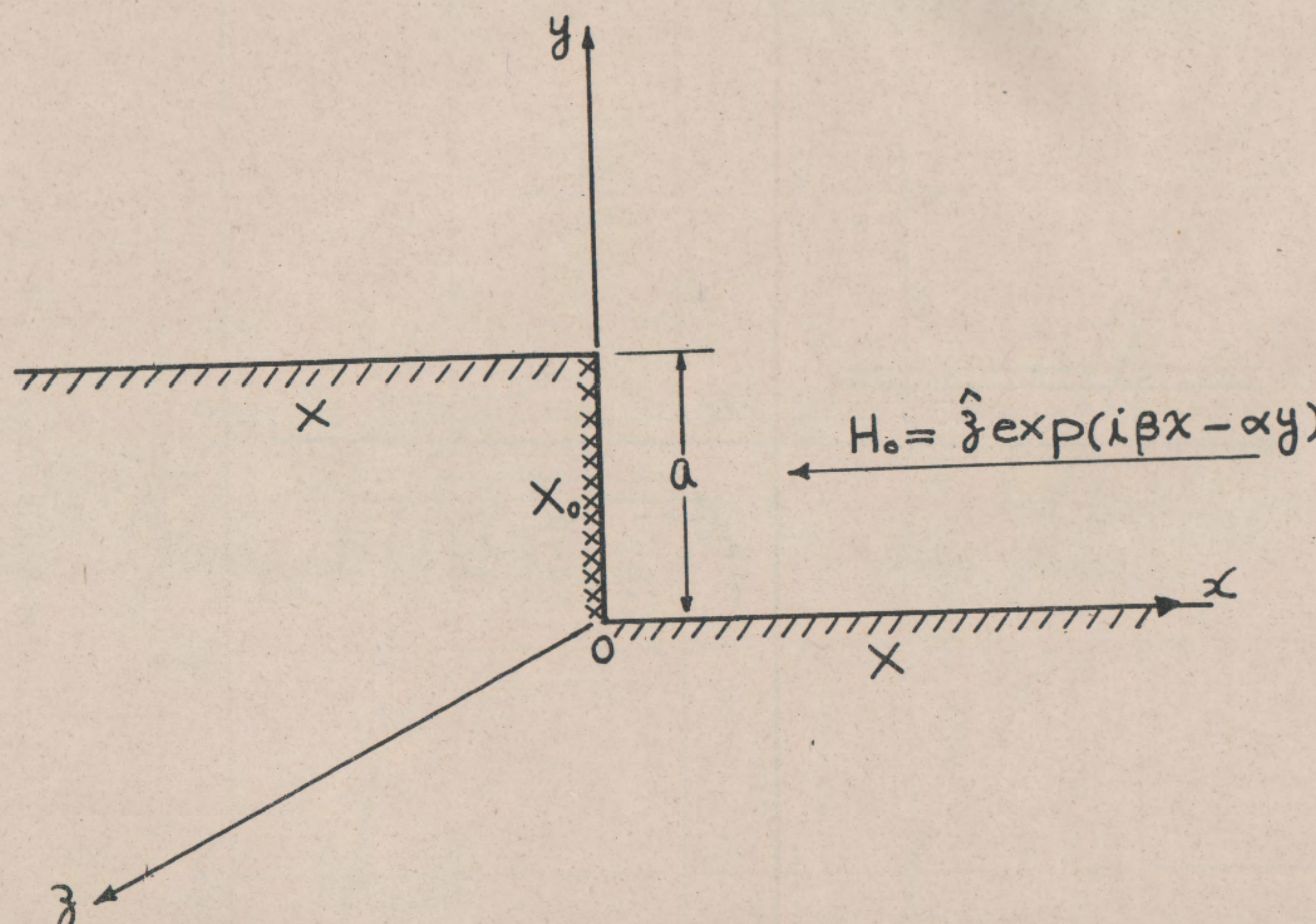

Fig. 1. Geometry of the discontinuity in a guide plane.

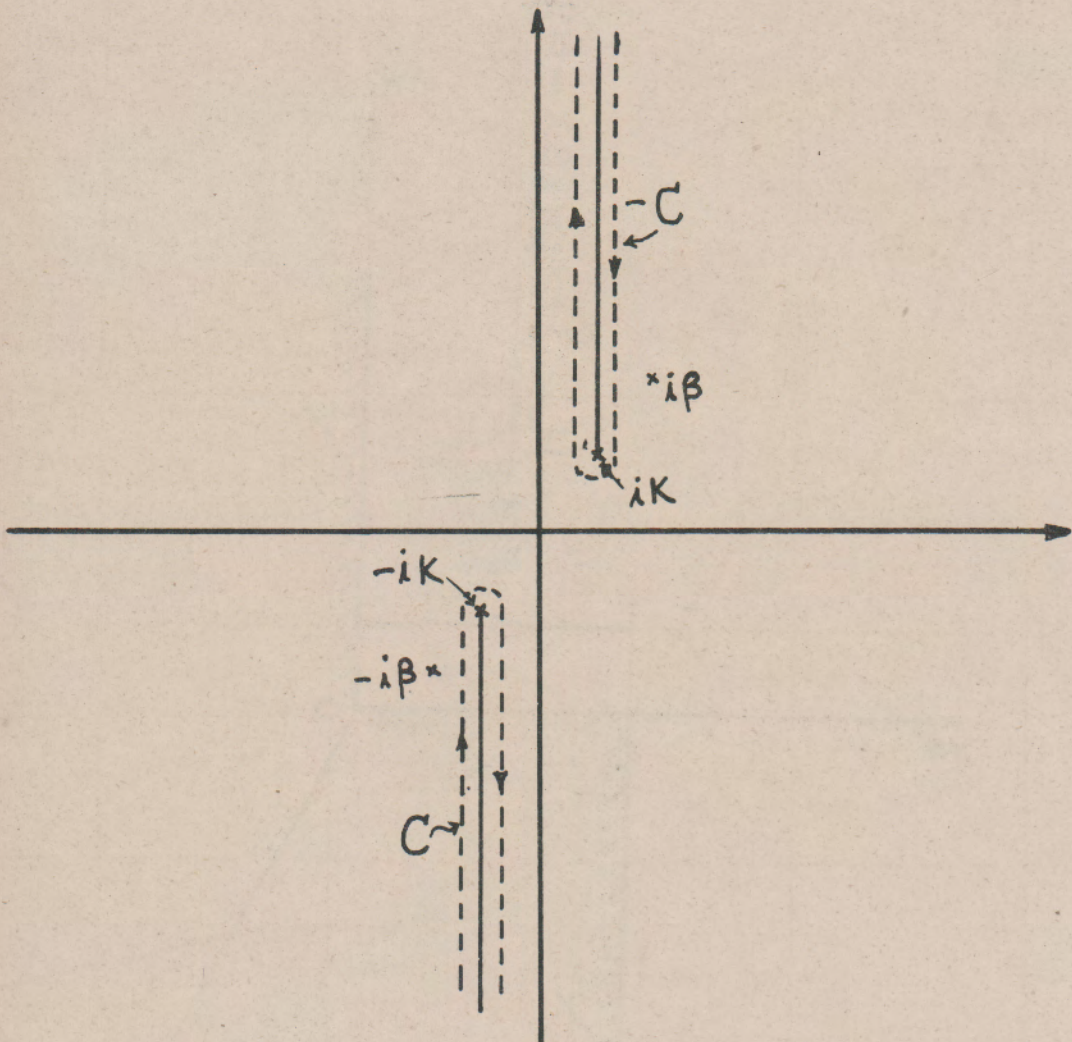

Fig. 2. The complex s-plane.

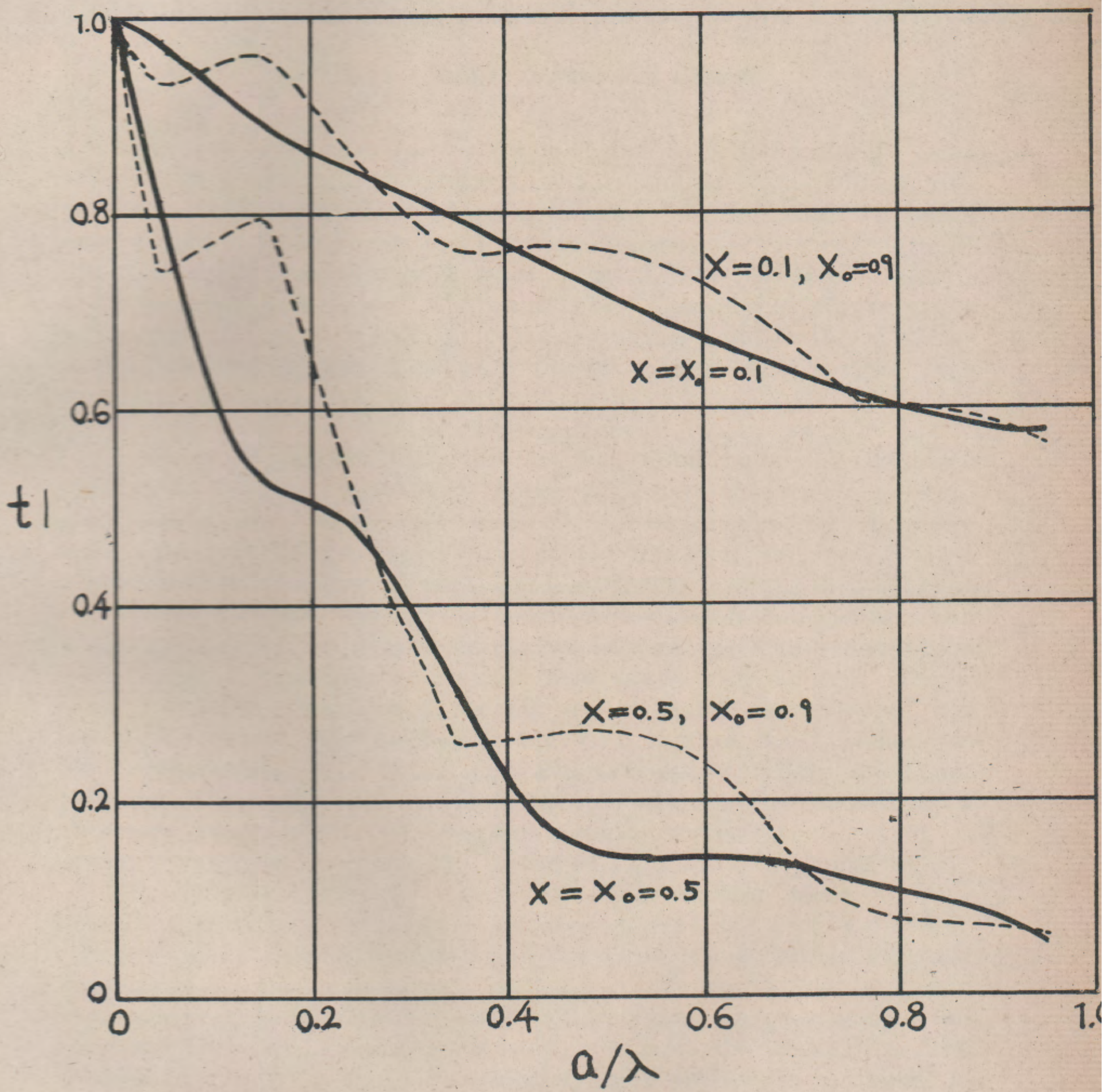

Fig. 3. The transmission coefficients.