Science Bulletin of National Chiao Tung University, Vol. VI, No. 1

MAGNETOACOUSTIC PHENOMENA IN $SrTiO_3$

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(Received May 25, 1972)

Abstract—*The effects of parabolic and nonparabolic band structures on the propagation of ultrasound in $SrTiO_3$ in the presence of a dc magnetic field have been studied by using a quantum treatment which is valid at high frequencies and in strong magnetic fields. The major interaction between the ultrasound and conduction electrons is via deformation coupling. Results are shown that the absorption coefficient and sound velocity depend strongly on the sound frequency and longitudinal dc magnetic field. At the high field region, some oscillations in the absorption coefficient and sound velocity with the dc magnetic field occur at the high ultrasonic frequencies. These oscillations are interpreted as the so-called "giant quantum oscillations" which can be observed when the separation between Landau levels is greater than their thermal broadenings in the region of high frequencies. It is also shown that some discontinuities in the absorption coefficient and change in sound velocity occur in the region of strong magnetic fields.*

I. INTRODUCTION

Some recent experiments in *n*-type InSb on the absorption coefficient and the change in sound velocity for the ultrasound propagating parallel to a dc magnetic field have revealed a magnetic field dependence over a wide range of field strengths.[1,2] It has been shown that the nonparabolicity of the band structure in a nondegenerate semiconductor like *n*-type InSb can lead to a change in the longitudinal magnetoacoustic absorption and the sound velocity in strong magnetic fields.[3] However, there are some oscillations in the absorption coefficient at the region of high frequencies in highly doped *n*-type InSb.[4] These oscillations have been interpreted as the so-called "giant quantum oscillations" which can be observed when the separation between Landau levels is greater than their thermal broadenings in the region of high frequencies. There are also exist discontinuities in these oscillations of the absorption coefficient at high frequencies. This kind of phenomenon is quite different from those of geometric resonances and cyclotron resonances which are in

semiclassical in origin. These geometric and cyclotron resonances in attenuation enter the theory through the Bessel functions appearing in the ac conductivity tensor.[5,6] The geometric resonances can occur when the sound wavelength is of the order of the classical orbit radius of electrons. When the sound frequency is of the order of the cyclotron frequency, we can have cyclotron resonances.[5] If the sound wavelength is greater than the Larmor radius of the conduction electrons but significantly less than their mean free path, then the absorption coefficient and change in sound velocity will oscillate as a function of *dc* magnetic field in high frequencies.[7] However, the oscillations can under conditions be gigantic, i. e., they are not a small correction to a nonoscillatory part of the acoustic attenuation. In this paper we investigate the effects of the longitudinal dc magnetic field and sound frequency on the ultrasonic attenuation for parabolic and nonparabolic band structures in $SrTiO_3$. We study this problem using a quantum treatment which is valid at high frequencies and in strong magnetic fields. We also make the following assumptions:

(1) For a degenerate semiconductor the distribution function of electrons is represented by the Fermi-Dirac statistics and the interesting temperature is very near absolute zero.

(2) We are interested in the frequencies above the microwave region such that $|\vec{q}|l \gg 1$, where $\vec{q}$ is the wavevector of the ultrasound and l is the electron mean free path. Consequently, the effect of collisions can be neglected.[8]

(3) Because of the assumption in (2), the major interaction between the ultrasound and conduction electrons in a degenerate semiconductor is via deformation coupling.[8,9]

In Section II, we follow the previous paper[3] to find the longitudinal ac conductivity tensor for the parabolic and nonparabolic band structures. In Section III, the results of numerical calculations for the absorption coefficient and sound velocity are given by taking $SrTiO_3$ as an example. Then we discuss our results in Section IV.

II. LONGITUDINAL AC CONDUCTIVITY TENSORS FOR THE PARABOLIC AND NONPARABOLIC BAND STRUCTURES

For a parabolic model, the energy eigenvalue equation for electrons in a uniform dc magnetic field $\vec{B}$ directed along the z-direction may be written as

$$H_0\psi_{\vec{k}n} \equiv \frac{1}{2m^*}\left[p_x^2+(p_y-\frac{eBx}{c})^2+p_z^2\right]\psi_{\vec{k}n}=E_{\vec{k}n}\psi_{\vec{k}n}, \qquad (1)$$

where we have used the Landau gauge for the vector potential $\vec{A}_0=(0, Bx, 0)$ and m^* is the effective mass of the electron. For the case of a nonparabolic model, the energy eigenvalue equation in the magnetic field directed along the z-direction takes the form

$$H_0\left(1+\frac{H_0}{E_g}\right)\psi_{\vec{k}n} \equiv \frac{1}{2m^*}\left[p_x^2+(p_y-\frac{eBx}{c})^2+p_z^2\right]\psi_{\vec{k}n}$$

$$=E_{\vec{k}n}\left(1+\frac{E_{\vec{k}n}}{E_g}\right)\psi_{\vec{k}n}, \qquad (2)$$

where E_g is the energy gap between the conduction and valence bands. The eigenfunctions for Eqs. (1) and (2) will have the same representation which can be expressed by the Landau representation,

$$\psi_{\vec{k}n}=\exp(ik_y y+ik_z z)\phi_n\left(x-\frac{\hbar c}{eB}k_y\right), \tag{3}$$

where $\phi_n(x)$ is the wave function for a simple harmonic oscillator. But the eigenvalues of Eq. (1) for a parabolic model are

$$E_{\vec{k}n}=\hbar\omega_c\left(n+\tfrac{1}{2}\right)+\frac{\hbar^2k_z^2}{2m^*}, \tag{4}$$

and the eigenvalues of Eq. (2) for a nonparabolic model are

$$E_{\vec{k}n}=-\tfrac{1}{2}E_g\left\{1-\left\langle 1+(4/E_g)\left[\left(n+\tfrac{1}{2}\right)\hbar\omega_c+\frac{\hbar^2k_z^2}{2m^*}\right]\right\rangle^{\frac{1}{2}}\right\}, \tag{5}$$

where $\omega_c=|e|B/m^*c$ is the cyclotron frequency.

The expression in Eq. (4) is the eigenvalues for a one-band model and Eq. (5) is the eigenvalues of the conduction band for a two-band model. When $(n+\frac{1}{2})\hbar\omega_c+\hbar^2k_z^2/2m^*\ll E_g$, the expression of Eq. (5) can be reduced to those obtained using the parabolic model for the band structure as shown in Eq. (4).

The interaction of the conduction electrons with the ultrasonic wave can be taken into account via the vector potential $\vec{A}_1=\vec{A}_{10}\exp(i\vec{q}\cdot\vec{r}-i\omega t)$ which arises from the the self-consistent field accompanying the ultrasonic wave. To first order in $\vec{A}_1$ the Hamiltonian is

$$H=H_0+H_1, \tag{6}$$

where H_0 is defined as shown in Eq. (1) for the parabolic model and Eq. (2) for the nonparabolic model.

$$H_1=-\left(\frac{e}{2c}\right)(\vec{v}\cdot\vec{A}_1+\vec{A}_1\cdot\vec{v}) \tag{7}$$

is the interaction Hamiltonian. Here $\vec{v}$ is the velocity operator for the unperturbed Hamiltonian, $\vec{v}=-(i/\hbar)[\vec{r}, H_0]$.

The electron current density induced by the self-consistent field is obtained by taking the trace of the product of the current density operator and the single-particle density matrix. The density matrix is calculated to first order in $\vec{A}_1$ as follows. The density operator must satisfy the equation of motion[8]

$$i\hbar\frac{\partial\rho}{\partial t}=[H, \rho]. \tag{8}$$

In the absence of the perturbation, H_1, ρ reduces to its equilibrium value ρ_0. We now set $\rho=\rho_0+\rho_1$, where ρ_1 is a small change in ρ from its equilibrium value ρ_0 caused by the self-consistent field. Furthermore, we assume that ρ_1 varies as the factor $\exp(-i\omega t)$. Taking the matrix elements of the linearlized equation of motion in the representation of Eqs. (3) and (4) for the parabolic model or (3) and (5) for the nonparabolic model, we find that

$$<\vec{k}'n'|\rho_1|\vec{k}n>=\frac{(f_{\vec{k}'n'}-f_{\vec{k}n})<\vec{k}'n'|H_1|\vec{k}n>}{E_{\vec{k}'n'}-E_{\vec{k}n}-\hbar\omega} \tag{9}$$

where $f_{\vec{k}n}=<\vec{k}n|\rho_0|\vec{k}n>$ is the Fermi-Dirac distribution for electrons in degenerate semiconductors.

The induced current density is

$$\vec{J}=Tr(\rho\vec{J}_{0\rho})=\sum_{k'kn'n}<\vec{k}'n'|\rho|\vec{k}n><\vec{k}n|\vec{J}_{0\rho}|\vec{k}'n'>, \tag{10}$$

where the current density operator to first order in $\vec{A}_1$ at a point $\vec{r}_0$ is

$$\vec{J}_{0\rho}=-\left(\frac{e}{2}\right)[\vec{v}-\left(\frac{e\vec{A}_1}{m^*c}\right),\ \delta(\vec{r}-\vec{r}_0)]_+ \tag{11}$$

for the parabolic model, and

$$\vec{J}_{0\rho}=-\left(\frac{e}{2}\right)[\vec{v}-(1+\frac{2H_0}{E_g})^{-1}\left(\frac{e\vec{A}_1}{m^*c}\right),\ \delta(\vec{r}-\vec{r}_0)]_+ \tag{12}$$

for the nonparabolic model. $[\ ]_+$ denotes the anticommutator. Using the gauge where the scalar potential is zero, we find the relation between the self-consistent field and the vector potential $\vec{A}_1$,

$$\vec{E}=\left(\frac{i\omega}{c}\right)\vec{A}_1. \tag{13}$$

Now we are interested in an ultrasonic wave propagating parallel to the dc magnetic field, then the only component of the conductivity tensor of interest is σ_{zz}. Using Eqs. (3)–(13) we find that

$$\sigma_{zz}(q,\omega)=\frac{i\omega_p^2\omega(m^*)^{3/2}}{8\sqrt{2}\,\pi q^3\hbar\sum_n[E_F-(n+\frac{1}{2})\hbar\omega_c]^{\frac{1}{2}}}$$
$$\cdot\sum_n\left\{\ln\left(\frac{[E_F-(n+\frac{1}{2})\hbar\omega_c]^{\frac{1}{2}}+\frac{q\hbar}{2\sqrt{2}\,(m^*)^{\frac{1}{2}}}-\frac{\omega(m^*)^{\frac{1}{2}}}{\sqrt{2}\,q}}{-[E_F-(n+\frac{1}{2})\hbar\omega_c]^{\frac{1}{2}}+\frac{q\hbar}{2\sqrt{2}\,(m^*)^{\frac{1}{2}}}-\frac{\omega(m^*)^{\frac{1}{2}}}{\sqrt{2}\,q}}\right)\right.$$
$$\left.-\ln\left(\frac{[E_F-(n+\frac{1}{2})\hbar\omega_c]^{\frac{1}{2}}-\frac{q\hbar}{2\sqrt{2}\,(m^*)^{\frac{1}{2}}}-\frac{\omega(m^*)^{\frac{1}{2}}}{\sqrt{2}\,q}}{-[E_F-(n+\frac{1}{2})\hbar\omega_c]^{\frac{1}{2}}-\frac{q\hbar}{2\sqrt{2}\,(m^*)^{\frac{1}{2}}}-\frac{\omega(m^*)^{\frac{1}{2}}}{\sqrt{2}\,q}}\right)\right\} \tag{14}$$

with $E_F\geqslant(n+\frac{1}{2})\hbar\omega_c$ for the parabolic model, and

$$\sigma_{zz}(q,\omega)=\frac{i\omega_p^2\omega(m^*)^{3/2}}{8\sqrt{2}\,\pi q^3\hbar\sum_n[E_F(1+E_F/E_g)-(n+\frac{1}{2})\hbar\omega_c]^{\frac{1}{2}}}$$
$$\sum_n a_n\left\{\ln\left(\frac{[E_F(1+E_F/E_g)-(n+\frac{1}{2})\hbar\omega_c]^{\frac{1}{2}}+\frac{q\hbar}{2\sqrt{2}\,(m^*)^{\frac{1}{2}}}-\frac{\omega(m^*)^{\frac{1}{2}}a_n}{\sqrt{2}\,q}}{-[E_F(1+E_F/E_g)-(n+\frac{1}{2})\hbar\omega_c]^{\frac{1}{2}}+\frac{q\hbar}{2\sqrt{2}\,(m^*)^{\frac{1}{2}}}-\frac{\omega(m^*)^{\frac{1}{2}}a_n}{\sqrt{2}\,q}}\right)\right.$$

$$-\ln\left(\frac{[E_F(1+E_F/E_g)-(n+\frac{1}{2})\hbar\omega_c]^{\frac{1}{2}}-\frac{q\hbar}{2\sqrt{2}\,(m^*)^{\frac{1}{2}}}-\frac{\omega(m^*)^{\frac{1}{2}}a_n}{\sqrt{2}\,q}}{-[E_F(1+E_F/E_g)-(n+\frac{1}{2})\hbar\omega_c]^{\frac{1}{2}}-\frac{q\hbar}{2\sqrt{2}\,(m^*)^{\frac{1}{2}}}-\frac{\omega(m^*)^{\frac{1}{2}}a_n}{\sqrt{2}\,q}}\right)\Bigg\} \tag{15}$$

with $E_F(1+E_F/E_g)\geqslant(n+\frac{1}{2})\hbar\omega_c$ for the nonparabolic model. Here $\omega_p=(4\pi n_0e^2/m^*)^{\frac{1}{2}}$ is the plasma frequency, E_F is the Fermi energy and $a_n=[1+4\hbar\omega_c(n+\frac{1}{2})/E_g]^{\frac{1}{2}}$. To obtain Eq. (15) we have used the expansion of Eq. (5),

$$E_{kn}=-\tfrac{1}{2}E_g+\tfrac{1}{2}E_g a_n+\hbar^2k_z^2/2m^*a_n. \tag{16}$$

III. NUMERICAL RESULTS

It has been assumed that the major interaction between the conduction electrons and the ultrasound is via deformation potential coupling. Then the relation between the absorption coefficient α_{11} and the longitudinal ac conductivity σ_{zz} is given by[10]

$$\alpha_{11}=-\frac{\varepsilon}{4\pi\rho v_s^2}(\omega/v_s)^3(C/e)^2\,Im\left(1-\frac{4\pi\sigma_{zz}}{i\omega\varepsilon}\right)^{-1}, \tag{17}$$

where ρ is the density of the material, ε is the static dielectric constant, v_s is the sound velocity, and C is the deformation potential. The change in the sound velocity due to the interaction between the ultrasound and conduction electrons is also related to the longitudinal ac conductivity[10]

$$\frac{\Delta v_s}{v_s}=\frac{\varepsilon}{8\pi\rho v_s^2}(\omega/v_s)^2(C/e)^2\,Re\left(1-\frac{4\pi\sigma_{zz}}{i\omega\varepsilon}\right)^{-1}. \tag{18}$$

It can be seen that α_{11} and $\Delta v_s/v_s$ will depend upon the dc magnetic field when we use the parabolic and nonparabolic models for the energy bands of the degenerate semiconductors. These results are quite different from those of our previous work[3] where the dependence on the dc magnetic field arises solely from the nonparabolicity of the energy bands due to the parameter $a_n=[1+(4\hbar\omega_c/E_g)(n+\frac{1}{2})]^{\frac{1}{2}}$. It can readily be seen that Eq. (15) will reduce to Eq. (14) if $\hbar\omega_c\ll E_g$, so that at low magnetic fields the result using a parabolic model for energy bands should be valid. On the other hand, in strong magnetic fields such that $\hbar\omega_c\simeq E_g$, the nonparabolicity of the energy bands leads to some different results.

In this paper we investigate the absorption coefficient of ultrasound and the change in sound velocity in $SrTiO_3$ at very low temperature limit. The numerical values of physical parameters for this material are[11] $n_0=10^{20}\text{cm}^{-3}$, $m^*=2.5\,m_0$, $\varepsilon=5$, C=15 ev, $E_g=3.2$ ev, $\rho=5.11\,\text{gm/cm}^3$, $E_F=0.015$ ev, and $v_s=6\times10^5\,\text{cm/sec}$. From Figs. 1 and 2, one can see that the absorption coefficient and absolute value of the change in sound velocity increase with increasing of sound frequency. We plot the absorption coefficient and change in sound velocity as a function of dc magnetic field in $SrTiO_3$ as shown in Figs. 3–8. From these numerical results one can see that at low magnetic fields the changes of both α_{11} and $-\Delta v_s/v_s$ with the dc magnetic

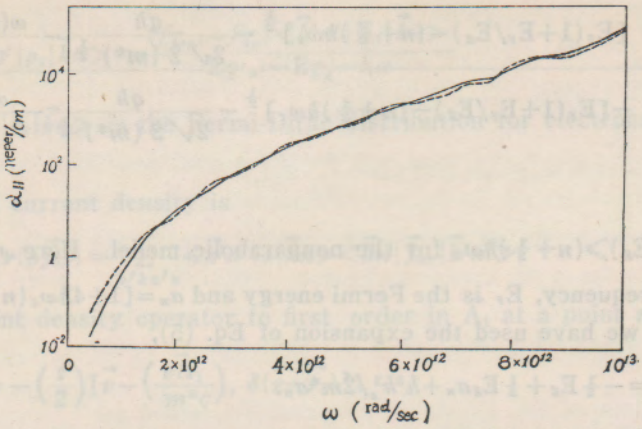

Fig. 1. Absorption coefficient α_{11} as a function of sound frequency ω for parabolic band structure (solid curve) and nonparabolic band structure (dashed curve) in $SrTiO_3$ at B=20 *k*G.

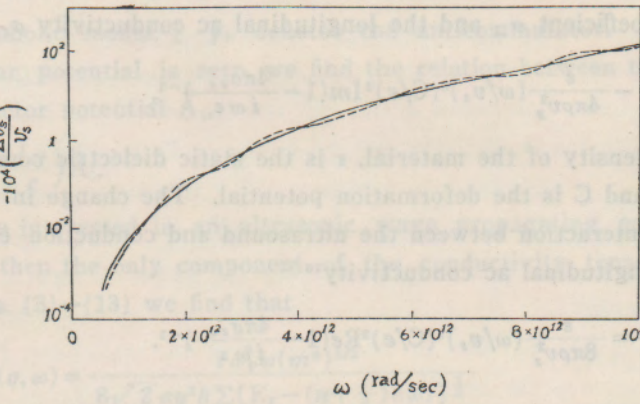

Fig. 2. The change in the sound velocity as a function of sound frequency ω for parabolic band structure (solid curve) and nonparabolic band structure (dashed curve) in $SrTiO_3$ at B=20 *k*G.

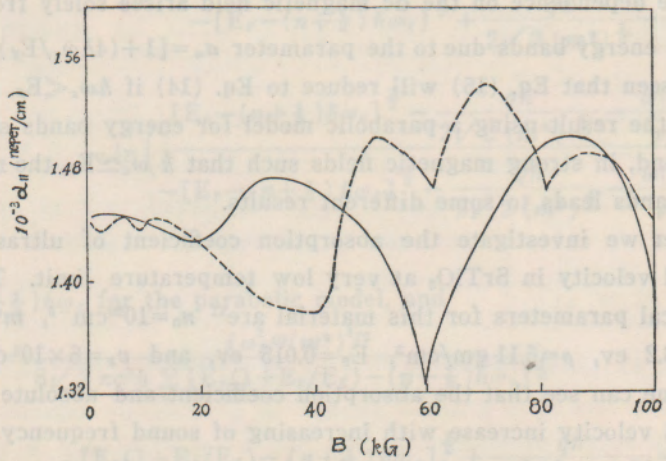

Fig. 3. Absorption coefficient α_{11} as a function of dc magnetic field B for parabolic band structure (solid curve) and nonparabolic band structure (dashed curve) at $\omega=5\times10^{12}$ rad/sec.

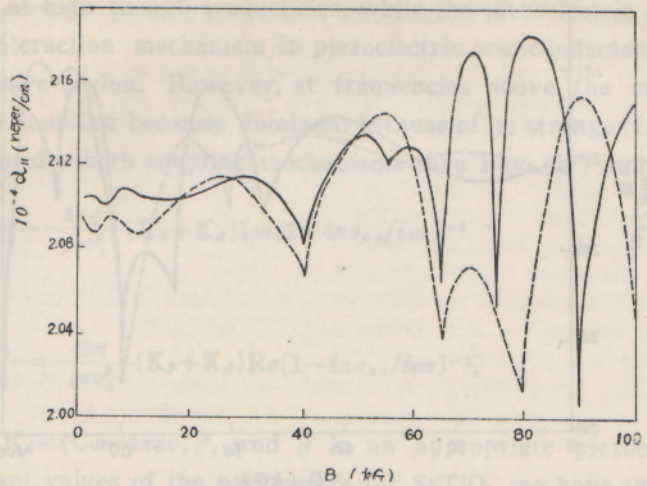

Fig. 4 Absorption coefficient α_{11} as a function of dc magnetic field B for parabolic band structure (solid curve) and nonparabolic band structure (dashed curve) at $\omega = 8 \times 10^{12}$ rad/sec.

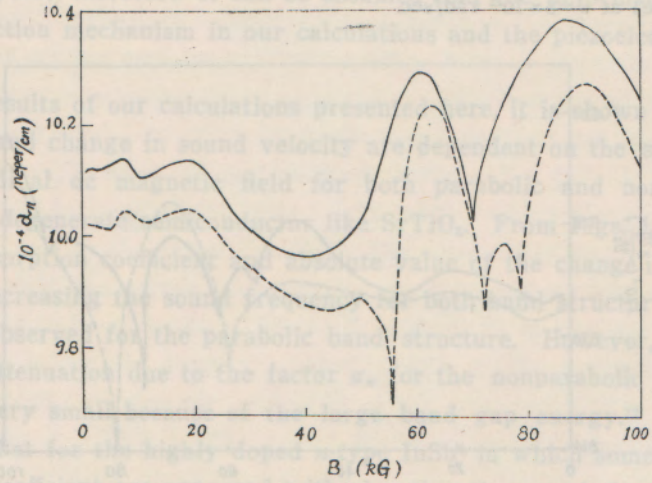

Fig. 5. Absorption coefficient α_{11} as a function of dc magnetic field B for parabolic band structure (solid curve) and nonparabolic band structure (dashed curve) at $\omega = 10^{13}$ rad/sec.

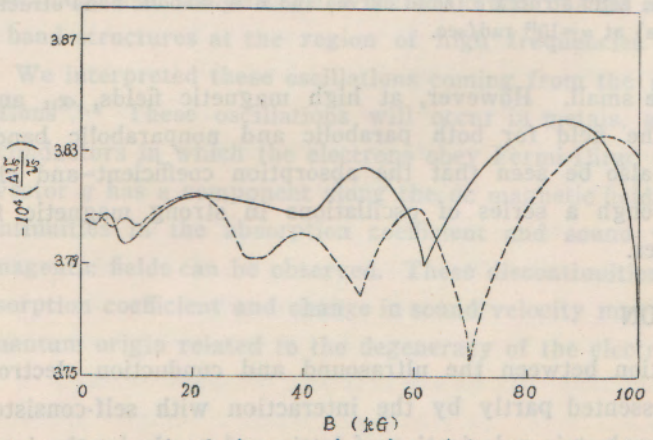

Fig. 6. The change in sound velocity as a function of dc magnetic field B for parabolic band structure (solid curve) and nonparabolic band structure (dashed curve) at $\omega = 5 \times 10^{12}$ rad/sec.

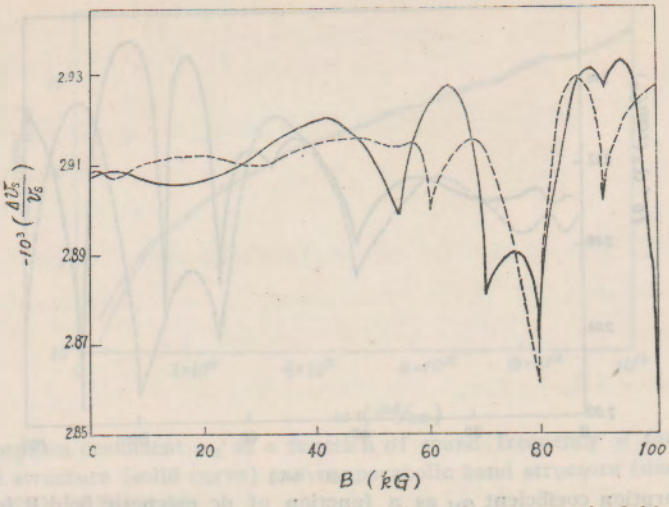

Fig. 7. The change in sound velocity as a function of dc magnetic field B for parabolic band structure (solid curve) and nonparabolic band structure (dashed curve) at $\omega = 8 \times 10^{12}$ rad/sec.

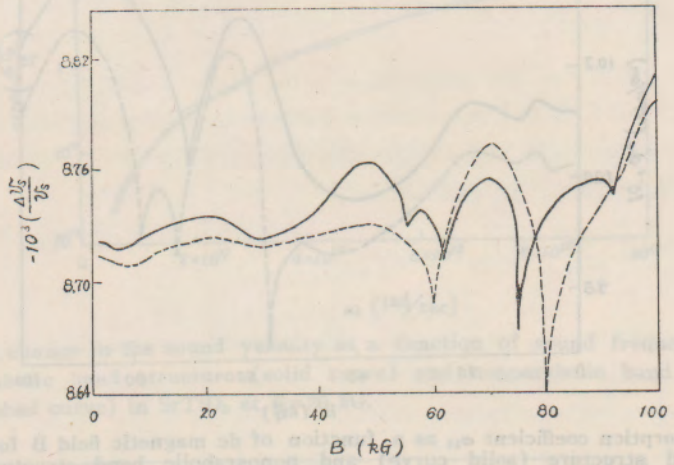

Fig. 8. The change in sound velocity as a function of dc magnetic field B for parabolic band structure (solid curve) and nonparabolic band structure (dashed curve) at $\omega = 10^{13}$ rad/sec.

field B are quite small. However, at high magnetic fields, α_{11} and $-\Delta v_s/v_s$ are dependent on the field for both parabolic and nonparabolic band structures in $SrTiO_3$. It can also be seen that the absorption coefficient and change in sound velocity go through a series of oscillations in strong magnetic fields and high sound frequencies.

IV. DISCUSSION

The interaction between the ultrasound and conduction electrons in semiconductors is represented partly by the interaction with self-consistent field which includes the piezoelectric polarization of lattic and partly by the interaction with a deformation potential.[1] The deformation coupling is dominant in semimetals and

semiconductors at high enough frequencies, while the piezoelectric coupling will be the dominant interaction mechanism in piezoelectric semiconductors for frequencies into the microwave region. However, at frequencies above the microwave region the deformation coupling becomes dominant because of its stronger frequency dependence. If we consider both coupling mechanisms then Eqs. (17) and (18) become as

$$\alpha_{11} = -\frac{4\pi q}{\rho \varepsilon v_s^2}(K_p + K_d) Im(1 - 4\pi\sigma_{zz}/i\omega\varepsilon)^{-1} \tag{19}$$

and

$$\frac{\Delta v_s}{v_s} = \frac{2\pi}{\rho \varepsilon v_s^2}(K_p + K_d) Re(1 - 4\pi\sigma_{zz}/i\omega\varepsilon)^{-1}, \tag{20}$$

where $K_p = \beta^2$, $K_d = (C\omega\varepsilon/4\pi e v_s)^2$, and β is an appropriate piezoelectric constant. Using the relevant values of the parameters for $SrTiO_3$, we have the relation $K_p/K_d = 10^{21}/\omega^2$. Since we are interested in the frequencies above the microwave region, i.e. $\omega > 10^{12}$ rad/sec, therefore it can be assumed that the deformation coupling is only the interaction mechanism in our calculations and the piezoelectric coupling is neglected.

From the results of our calculations presented here, it is shown that the absorption coefficient and change in sound velocity are dependent on the sound frequency and the longitudinal dc magnetic field for both parabolic and nonparabolic band structures in a degenerate semiconductor like $SrTiO_3$. From Figs. 1 and 2, we can see that the absorption coefficient and absolute value of the change in sound velocity increase with increasing the sound frequency for both band structures, but no oscillations can be observed for the parabolic band structure. However, oscillations in the ultrasonic attenuation due to the factor a_n for the nonparabolic band structure in $SrTiO_3$ are very small because of the large band gap energy.[12] This result is different from that for the highly doped *n*-type InSb[4] in which some oscillations in the absorption coeffcient are appeared with changing the sound frequency for both band structures because the smaller band gap energy. From Figs. 3–8, it can be seen that some oscillations in the absorption coefficient and change in sound velocity appear for both band structures at the region of high frequencies and strong dc magnetic fields. We interpreted these oscillations coming from the so-called "giant quantum oscillations".[7,8] These oscillations will occur in metals, semimetals, and degenerate semiconductors in which the electrons obey Fermi-Dirac statistics when the sound wavevector $\vec{q}$ has a component along the dc magnetic field $\vec{B}$. It is shown that some discontinuities in the absorption coefficient and sound velocity at the region of high magentic fields can be observed. These discontinuities are due to the fact that the absorption coefficient and change in sound velocity may have singularities of purely quantum origin related to the degeneracy of the electron gas.[13,14]

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