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CONDUCTIVITY OF AN ANISOTHERMAL MAGNETOPLASMA FROM A QUANTUM-MECHANICAL CONVERGENT KINETIC EQUATION

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Abstract—*The quantum-mechanical convergent kinetic equation is used to calculate the collisional part of the ac electrical conductivity tensor of a multispecies, anisothermal magnetoplasma in the long wave-length limit. When the temperature ratio of electrons to ions is of the order or higher than 10^3 the usual Coulomb logarithmic contribution is no longer dominant.*

I. INTRODUCTION

Following a general method recently proposed[1], the collisional (or correlational) part of the ac electrical conductivity tensor of an isothermal magnetoplasma has been calculated using various collisional models, namely, the Landau's model[1,2], the Beliaev-Budker's model[3], the convergent classical (or the Kihara-Aono's) model[4], and the convergent quantum-mechanical model.[5] With the assumption of weak collisions, detailed calculations have been carried out to second order in the wave vector and the motions of ions have been fully accounted for in Refs. 2, 4 and 5.

In an earlier paper[6], the general method given in Ref. 1 was extended to cover the case of an anisothermal magnetoplasma in which the particles of α kind are in a Maxwellian distribution $f_0^{(\alpha)}$ of temperature T_α. The only assumption made above is the temperature relaxation (or equilibration) time being much longer than the collisional damping time of the wave. Detailed calculations of the conductivity was then carried out in the long wavelength limit using the convergent classical (or the Kihara-Aono's) model.[6]

As a sequel to earlier works, the present paper extends the result of Ref. 5 to the anisothermal case on the one hand and the work of Ref. 6 to higher temperatures on the other. The collisional model used is the convergent quantum-mechanical model in the Baldwin's version[7] which was first discussed by Kihara and Honda[8]. As a result, the starting equation for the collisional conductivity becomes[9] [campare with Eq. (16) of Ref. 1]:

$$\sigma_{hl}^{(c)} = K^{-1} \sum_{\alpha} \sum_{\beta} \int d^3v^{(\alpha)} \tilde{\rho}_h^{(\alpha)} C_{\alpha\beta}(\rho_l^{(\alpha)} f_0^{(\alpha)}/T_\alpha;\ \rho_l^{(\beta)} f_0^{(\beta)}/T_\beta) \tag{1}$$

where $C_{\alpha\beta}$ is the linearized collision term (between particles of α kind and β kind) given by, in Baldwin's form,

$$C=C^{B,\kappa}+C^{BLG}-C^{BLG,\kappa} \qquad (2)$$

with $C^{B,\kappa}$ being the quantum-mechanical Boltzmann collision term with the screened Coulomb potential, exp $(-\kappa r)/r$, C^{BLG} the Balescu-Lenard-Guernsey (hereafter denoted as B−L−G) term and $C^{BLG,\kappa}$ the same B−L−G term except that the correct dynamical dielectric function $\varepsilon(\underset{\sim}{k}, \omega)$ is replaced by the static one, $1+\kappa^2/k^2$. Also in Eq. (1), K is the Boltzmann constant, ρ_l is a vector function given by Eq. (10) of Ref. 1 such that $\underset{\sim}{\rho}^{(\alpha)}\cdot\underset{\sim}{E}f_0^{(\alpha)}/T_\alpha$ is the well-known solution of the linearized Vlasov equation, and $\tilde{\rho}_h$ is equal to ρ_h by a reverse of the magnetic field. From Eqs. (1) and (2), it is obvious that the collisional conductivity can be expressed as a sum of three terms,

$$\sigma^{(C)}=\sigma^{(B,\kappa)}+\sigma^{(BLG)}-\sigma^{(BLG,\kappa)} \qquad (3)$$

The last two terms in Eq. (3) can always be treated classically for a non-degenerate plasma in which the thermal de-Broglie wavelength is much smaller than the average interparticle distance[8], however, the Boltzmann term must be handled quantum-mechanically whenever the temperature is high enough that the classical impact radius of closest approach is no longer larger than the electron thermal de-Broglie wavelength. In the following considerations, we shall restrict to a non-degenerate plasma so that the classical B−L−G result obtained in Ref. 6 can be used. In the next section, the last two terms in Eq. (3) shall be calculated in the long wavelength limit and the Boltzmann term will be considered in Sect. III.

II. THE BALESCU-LENARD-GUERNSEY TERMS

For a non-degenerate plasma, $\sigma^{(BLG)}$ in Eq. (3) is just the expression given by Eqs. (19) and (20) of Ref. 6 and $\sigma^{(BLG,\kappa)}$ can be obtained from the same B−L−G term through a replacement of the dynamical dielectric constant by the static dielectric constant, $1+\kappa^2/k^2$. This replacement has been discussed in Sect. IV of the same reference (see also Ref. 5). The desired conductivity may now be written:

$$\sigma_{hl}^{(BLG)}-\sigma_{hl}^{(BLG,\kappa)}=\boldsymbol{\vartheta}\{[\ln(\kappa/k_D)+1/2]\Pi_{hl}^{(\alpha\beta)}+\Theta_{hl}^{(\alpha\beta)}(\Delta_1,\Delta_2,W)\} \qquad (4)$$

$$\boldsymbol{\vartheta}=[4(2\pi)^{1/2}/3K^{3/2}]\sum_{\alpha,\beta}n_\alpha n_\beta(q_\alpha q_\beta)^2$$
$$\times[(m_\alpha+m_\beta)/(m_\alpha T_\beta+m_\beta T_\alpha)]^{3/2}(\mu_{\alpha\beta})^{1/2}, \qquad (5)$$

where Π and Θ are the same matrices as those in Eqs. (19) and (20) of Ref. 6. The other symbols in Eqs. (4) and (5) are the particle density n, the charge q, the mass m, the temperature T, the usual reduced mass μ and the Debye wave vector k_D. Note that $\boldsymbol{\vartheta}$ contains a double summation and thus should be treated as an operator.

III. THE BOLTZMANN TERM

The isothermal expression in Eq. (5) of Ref. 5 can easily be modified to an anisothermal expression

$$\sigma_{hl}^{(B,\kappa)}=K^{-1}\sum_{\alpha,\beta}\int d^3v^{(\alpha)}\int d^3v^{(\beta)}f_0^{(\alpha)}f_0^{(\beta)}(\rho_l^{(\alpha)}/T_\alpha+\rho_l^{(\beta)}/T_\beta)g \times \sum_{n=1}^{\infty}(n!)^{-1}\frac{\partial^n\tilde{\rho}_h^{(\alpha)}}{\partial \underset{\sim}{v}^{(\alpha)n}}\cdot\int d\sigma \underset{\sim}{\Delta}^n \tag{6}$$

where the differential scattering cross section $d\sigma$, the relative velocity $\underset{\sim}{g}=\underset{\sim}{v}^{(\beta)}-\underset{\sim}{v}^{(\alpha)}$ and the change of velocity $\underset{\sim}{\Delta}$ during one scattering are given in Ref. 5. Using the velocity transformations introduced in Eqs. (3)-(7) of Ref. 6, the integrals over $\underset{\sim}{G}$ and the directions of $\underset{\sim}{g}$ can be evaluated without difficulty. The remaining integral is of the form

$$\int_0^\infty dx\exp(-x)x^2\int d\sigma(1-\cos\theta) \tag{7}$$

which can be reduced to a single integral[5,8,10]

$$E(z)=\int_z^\infty dt\exp(-t)/t, \tag{8}$$

with the following limiting behaviors,

$$E(z)\rightarrow\exp(-z)/z \qquad \text{as } z\gg1 \tag{9}$$

$$E(z)\rightarrow-\ln(\gamma z) \qquad \text{as } z\ll1. \tag{10}$$

In Eq. (10), $\ln(\gamma)$ is the familiar Euler constant. With the integral $E(z)$, the quantum-mechanical Boltzmann contribution to the collisional conductivity becomes

$$\sigma_{hl}^{(B,\kappa)}=\boldsymbol{\theta}\left\{\ln\frac{4K(m_\alpha T_\beta+m_\beta T_\alpha)}{\gamma^2(m_\alpha+m_\beta)|q_\alpha q_\beta|\kappa}-\frac{1}{2}\exp(z)E(z)-\frac{1}{2}\right\}\Pi_{hl}^{(\alpha\beta)} \tag{11}$$

where $\boldsymbol{\theta}$ is given by Eq. (5) and

$$z=\frac{m_\alpha m_\beta(\gamma q_\alpha q_\beta)^2}{2K(m_\alpha T_\beta+m_\beta T_\alpha)\hbar^2} \tag{12}$$

is a dimensionless parameter whose square root indicates the ratio of the average classical impact radius of closest approach and the thermal de-Broglie wavelength.

IV. THE TOTAL CONDUCTIVITY

The total collisional part of the ac electrical conductivity in the long wavelength limit may now be written by combining Eqs. (4) and (11) which is

$$\sigma_{hl}^{(c)}=\boldsymbol{\theta}\left\{\left[\ln\frac{4K(m_\alpha T_\beta+m_\beta T_\alpha)}{\gamma^2(m_\alpha+m_\beta)|q_\alpha q_\beta|k_D}-\frac{1}{2}\exp(z)E(z)\right]\Pi_{hl}^{(\alpha\beta)} + \Theta_{hl}^{(\alpha\beta)}(\Delta_1,\Delta_2,W)\right\} \tag{13}$$

where $\boldsymbol{\theta}$ is again given by Eq. (5), the matrices Π_{hl} and Θ_{hl} are the same as those in Eq. (19) and (20) of Ref. 6, $E(z)$ is given by Eq. (8) and z by Eq. (12). It should be noted that the parameter κ disappears as expected from a convergent kinetic equation.

The result in Eq. (13) applies to a multi-species magneto-plasma in a wide range of temperatures. In the limit of low temperature ($z\gg1$), the second term in the square bracket of Eq. (13) becomes negligible and the classical result studied in Ref. 6 is recovered. In the other limit, the Born approximation can be obtained according to Eq. (10):

$$\sigma_{hl}^{(\mathrm{Born})}=\mathscr{S}\Big\{\frac{1}{2}\ln[8K\mu_{\alpha\beta}(m_\alpha T_\beta+m_\beta T_\alpha)/(m_\alpha+m_\beta)\gamma\hbar^2k_D^2]\Pi_{hl}^{(\alpha\beta)} + \Theta_{hl}^{(\alpha\beta)}(\Delta_1,\Delta_2,W)\Big\}. \quad (14)$$

In the isothermal limit in which all particles have the same temperature, Eq. (13) reduces to the long-wavelength result in Eq. (37) of Ref. 5.

V. DISCUSSION

Using the quantum-mechanical convergent kinetic equation we have calculated the collisional part of the ac electrical conductivity for a multi-species, anisothermal magneto-plasma in the long wavelength limit. The result is given in Eqs. (13) and (5). The total ac electrical conductivity should, of course, be a sum of the collisional part reported here and the well-known collisonless (or Vlasov) part; and the result can then be used to find out the dispersion relations and thus the collisional damping of various types of waves.

It has been concluded in Ref. 6 that, for a two-component plasma, the logarithmic contribution (due to the static coulomb scattering) is no longer dominant at $t\equiv T_e/T_i\gtrsim10^3$; in fact, the other contribution due to the dynamical dielectric constant in B−L−G equation (through the matrix Θ_{hl}) turns out to be increasingly dominant as t increases. The same conclusion holds here, for the logarithmic contribution (i.e., the coefficient of the matrix Π_{hl} in Eq. (13)) is again of order 10 and the other contribution comes from the same Θ_{hl}. The importance of the dynamical effect in the anisothermal case is independent of the quantum consideration of the close binary scatterings as expected.

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