

## THE STUDY OF REFLEX KLYSTRON

by

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Although the mathematical analysis of reflex klystron is somewhat complete, there are many assumptions for simplifying the analysis. It is interesting to compare the experimental results and the results predicted from theories.

It is known that the transit time of electrons in the space between the resonator and the repeller may be written as

$$T = \frac{2S}{V_a + V_r} \sqrt{\frac{2MV_a}{q}} \sqrt{1 + K \frac{V_g}{V_a} \sin \omega t_1} \quad (1)$$

where  $S$ : distance between resonator and repeller.

$V_a$ : beam voltage.

$V_r$ : repeller voltage.

$K$ : beam coupling coefficient.

$V_g$ : r f signal voltage between the gaps of resonator.

$t_1$ : the time when the electrons leaving the resonator.

let

$$t_0 = \frac{2S}{V_a + V_r} \sqrt{\frac{2MV_a}{q}} \quad (2)$$

$$N = \omega t_0 = \frac{\omega t_0}{2\pi} \quad (3)$$

The bunching parameter

$$X = \pi N K \frac{V_g}{V_a}$$

If  $V_g \ll V_a$

$$T = t_0 \left( 1 + \frac{X}{2\pi N} \sin \omega t_1 \right) = \frac{2\pi N}{\omega} + \frac{X}{\omega} \sin \omega t_1$$

$$t_2 = t_1 + T = t_1 + \frac{2\pi N}{\omega} + \frac{X}{\omega} \sin \omega t_1$$

The fundamental component of current induced from the reentrance beam is

$$I_{b1} = 2KI_o J_1(X) e^{j(\frac{\pi}{2} - wt_o)} \quad (4)$$

The equivalent circuit of reflex klystron oscillator may be considered as shown in Appendix.

The power delivered to the load

$$\begin{aligned} P_L &= \frac{-G_L}{G_s + G_L} V_g \text{Re} I_{b1} \\ &= \frac{-G_L}{G_s + G_L} V_a I_o X J_1(X) \frac{\sin 2\pi N}{\pi N} \end{aligned} \quad (5)$$

$$\eta_L = \frac{P_L}{V_a I_o} = - \frac{G_L}{G_s + G_L} X J_1(X) \frac{\sin 2\pi N}{\pi N} \quad (6)$$

For max. efficiency  $-\sin 2\pi N_m = 1$

$$N_m = m - \frac{1}{4}$$

max. value of  $X J_1(X)$  is 1.25 when  $X = 2.4$

$$\text{max. } \eta_L = \frac{0.4}{N_m} \cdot \frac{G_K}{G_s + G_L} \quad (7)$$

The beam admittance

$$Y_1 = \frac{I_{b1}}{V_g} = \frac{2KI_o J_1(X)}{V_g} (\sin wt_o + j \cos wt_o) \quad (8)$$

let  $wt_o = 2\pi N = 2\pi N_m + \phi$

$$Y_1 = \frac{2KI_o J_1(X)}{V_g} (-\cos \phi + j \sin \phi) \quad (9)$$

when oscillation just start  $V_g = 0$   $X = 0$

the real part

$$\begin{aligned} G_s + G_L - \frac{2KI_o J_1(X)}{V_g} \cos \phi &= 0 \\ \cos \phi &= \frac{(G_s + G_L) \frac{V_a}{I_o}}{K^2 \pi N} \end{aligned} \quad (10)$$

the imaginary part

$$wC - \frac{1}{wL} + \frac{2KI_o J_1(X)}{V_g} \sin \phi = 0$$

$$\frac{\Delta f}{f_0} = \frac{1}{2Q_L} \tan \phi$$

Where

$$Q_L = \frac{w_0 c}{G_s + G_L}$$

At first it is necessary to identify the mode. It was shown that when  $t_0 = (m - \frac{1}{4}) T_0$  the output power is maximum. Where  $T_0$  is the period of the signal. Compare two neighboring modes

$$\frac{t_{01}}{t_{02}} = \frac{m_1 - \frac{1}{4}}{m_2 - \frac{1}{4}} = \frac{V_{a2} + V_{r2}}{V_{a1} + V_{r1}} \sqrt{\frac{V_{a1}}{V_{a2}}} \quad (12)$$

From this eq. two methods may be used to identify the modes.

(1) If beam voltages keep const. and  $m_2 = m_1 + 1$

$$\text{then } \frac{m_1 - \frac{1}{4}}{m_1 + 1 - \frac{1}{4}} = \frac{V_a + V_{r2}}{V_a + V_{r1}} \quad (13)$$

From Fig. (1) When  $V_a = 300$  volts the power output is maximum when

$$V_r = 35.5 \quad 64 \quad 110 \quad \text{volts}$$

Compare the modes of  $V_r = 64$  and 110

$$\frac{m_1 - \frac{1}{4}}{m_1 + 1 - \frac{1}{4}} = \frac{300 + 64}{300 + 110} = 0.888$$

$$m_1 = 8.18 = 8$$

Compare the modes of  $V_r = 35.5$  and 64

$$\frac{m_2 - \frac{1}{4}}{m_2 + 1 - \frac{1}{4}} = \frac{300 + 35.5}{300 + 64} = 0.92$$

$$m_2 = 11.75 = 12$$

But the difference between  $m_1$  and  $m_2$  should be 1

We see that this method is difficult to identify the mode.

(2) If we let  $V_a = CV_r$

$$\text{then } \frac{m_1 - \frac{1}{4}}{m_1 + 1 - \frac{1}{4}} = \sqrt{\frac{V_{a2}}{V_{a1}}} \quad (14)$$

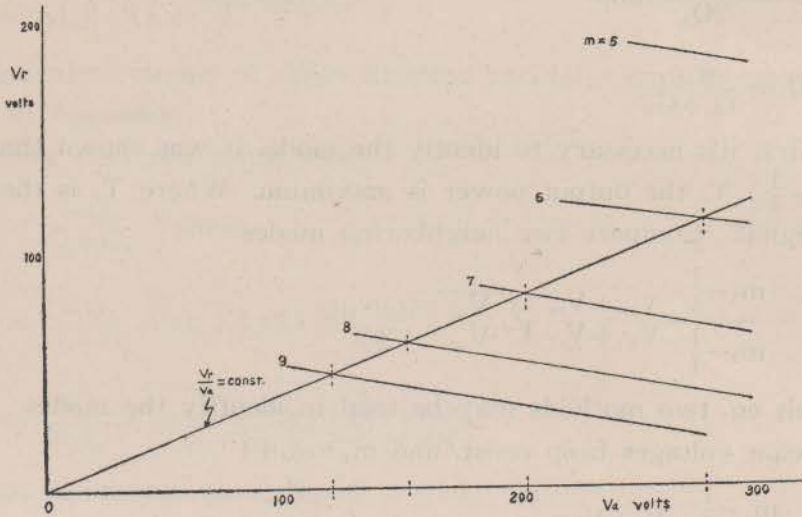


Fig.1 Vr Vs Va for max. power

Here  $V_{a1}, V_{a2}, \dots$  is the beam voltages of each mode on the line  $V_a = CV_r$ . By measuring the repeller voltage when the power output is max. for several beam voltage the curves in Fig. (1) are obtained. On the line  $V_a/V_r = \text{Const.}$  for each mode

$V_a =$	280	204	154	122
$m =$	$m_1$	$m_2$	$m_3$	$m_4$
$\sqrt{\frac{280}{V_a}} =$	1	1.17	1.35	1.514

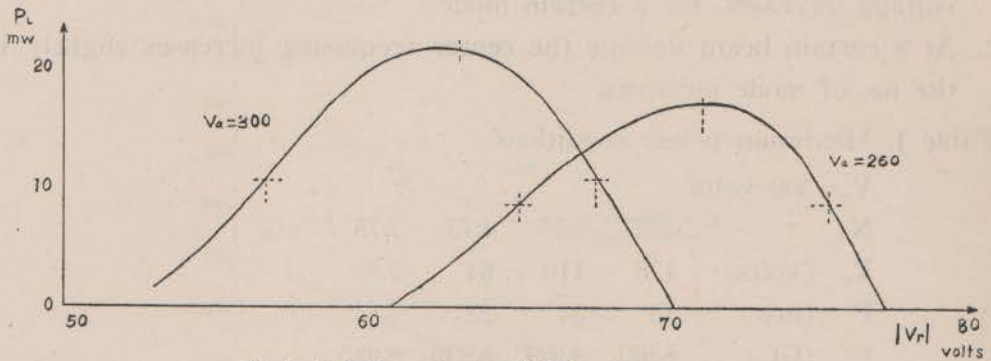
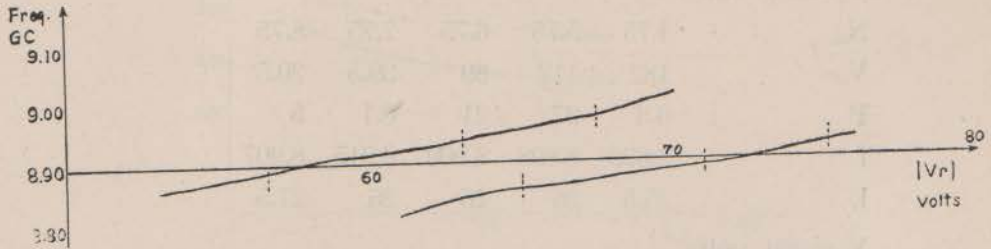
If  $m_1=5$  then  $m_2=5.85$   $m_3=6.75$   $m_4=7.58$

$m_1=6$   $m_2=7.02$   $m_3=8.1$   $m_4=9.1$

$m_1=7$   $m_2=8.2$   $m_3=9.45$   $m_4=10.06$

Compare the results obviously we see that  $m_1=6$

The reason why method I is fail is because  $V_r \ll V_a$  and  $V_r$  is difficult to be measured very accurate

Fig. 2  $P_L$  &  $F_{req.}$  vs  $V_r$  of 7th mode

Now consider the 7th mode.

Measure the power output and frequency as function of repeller voltage and is shown in Fig. 2

The max. power from measurement

$$P_L = 21.5 \text{ mw}$$

D. C. power  $300 \times 30 = 9000 \text{ mw}$

$$\eta_L = \frac{21.5}{9000} = 0.24\%$$

From eq. (7)

$$\eta_L = \frac{0.4}{7-0.25} \cdot \frac{G_L}{G_s + G_L} = 6.4 \frac{G_L}{G_s + G_L} \%$$

Usually  $G_L \gg G_s$   $\therefore \eta_L \doteq 6.4\%$

The efficiency of the reflex klystron is greatly different from the theoretical value.

From Table 1 and Fig. 3

1. The center frequency (when output power is max.) decreases if beam

voltage increases. for a certain mode

- At a certain beam voltage the center frequency increases slightly if the no. of mode increases.

Table 1. Maximum power conditions

$V_a=300$  volts

$N_m$	4.75	5.75	6.75	7.75
$V_r$ (volts)	178	110	64	7.8
P (mw)	13	34	22	7.8
f (GC)	8.921	8.927	8.936	8.945
$I_c$ (mA)	29	30	30	30

$V_a=280$  volts

$N_m$	4.75	5.75	6.75	7.75	8.75
$V_r=$	182	112	69	39.5	20.5
P	6.3	27	21	8.1	5
f	8.893	8.898	8.909	8.915	8.907
$I_c$	25.5	26	26	26	27.5

$V_a=260$  volts

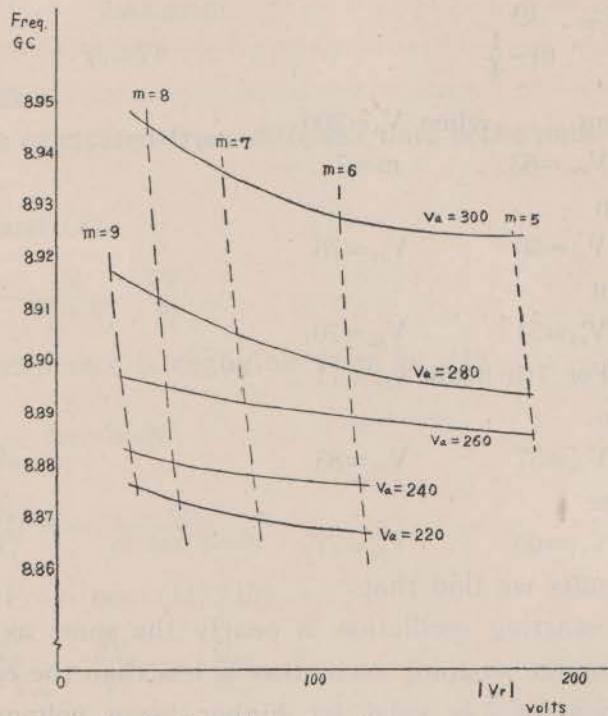
$N_m$	4.75	5.75	6.75	7.75	8.75
$V_r$	185	117	72	42.5	24.2
P	16	21	17	18.2	2
f	8.885	8.891	8.892	8.895	8.897
$I_c$	27	24	24	24	24

$V_a=240$  volts

$N_m$	5.75	6.75	7.75	8.75
$V_r$	118	75	46	27.2
P	16	15	7.9	1.1
f	8.876	8.878	8.879	8.884
$I_c$	21	21	21	22

$V_a=220$  volts

$N_m$	5.75	6.75	7.75	8.75
$V_r$	120	7.9	80	30.5
P	12	12	7.2	2.6
f	8.866	8.87	8.868	8.877
$I_c$	18.5	18.5	18.5	19

Fig. 3 Center Freq. vs.  $V_r$  &  $V_a$ 

we may estimate the range of repeller voltages in which the oscillation may occur.

From eq. (10) we see that the oscillation starts when

$$\phi = \cos^{-1} \frac{(G_s + G_L) \frac{V_a}{I_o}}{K^2 \pi N}$$

For simplicity assume  $\phi = \frac{\pi}{2}$

let  $t_{om}$  represent the transit time for max. output

then

$$\frac{t_o}{t_{om}} = \frac{V_a + V_r}{V_a + V_{rm}} = \frac{MT_o}{(m - \frac{1}{4})T_o}$$

$M = m - \frac{1}{2}$  when oscillation start

$M = m$  when oscillation stop

$$\frac{V_a + V_{rl}}{V_a + V_{rm}} = \frac{m - \frac{1}{2}}{m - \frac{1}{4}}$$

$$\frac{V_a + V_{r2}}{V_a + V_{rm}} = \frac{m}{m - \frac{1}{4}}$$

In the experiment when  $V_a = 300$

For 7th mode  $V_{rm} = 63$   $m = 7$

From calculation

$$V_{r1} = 50 \quad V_{r2} = 76$$

From experiment

$$V_{r1} = 51 \quad V_{r2} = 70$$

When  $V_a = 260$  For 7th mode  $v_{rm} = 71$

From calculation

$$V_{r1} = 57 \quad V_{r2} = 85$$

From experiment

$$V_{r1} = 60 \quad V_{r2} = 77$$

Compare the results we find that

1. The voltage starting oscillation is nearly the same as the estimate value. The voltage stopping oscillation is less than the estimate value.
2. The assumption  $\phi = \frac{\pi}{2}$  is valid for higher beam voltage than lower beam voltage, this may be seen from

$$\cos\phi = \frac{(G_{rs} + G_L)}{K^2 \pi N} \cdot \frac{V_a}{I_0}$$

we see from Table 1 that  $\frac{V_a}{I_0}$  increase if  $V_a$  decrease

and  $K^2$  decrease if  $V_a$  decrease

thus when  $V_a$  increase  $\frac{1}{K^2} \cdot \frac{V_a}{I_0}$  decrease and the assumption

$\phi = \frac{\pi}{2}$  is approached.

The distance between the cavity and repeller can be estimated

Since

$$N = ft_0 = \frac{2sf}{V_a + V_r} \sqrt{\frac{2mV_a}{q}}$$

$$= 6.74 \cdot 10^{-6} sf \frac{\sqrt{V_a}}{V_a + V_r} \quad \text{s in cm}$$

From experimental value in Fig. 2

(a)  $N_m = 6.75$   $f = 8.935 \times 10^9$

$V_a = 300$   $V_r = 63$

then  $s = 0.235$  cm



$$(b) N_m = 6.75 \quad f = 8.8910^9$$

$$V_a = 260 \quad V_r = 71$$

$$\text{then } s = 0.232$$

The value of  $s$  calculated from different data is the same thus this value maybe correct.

From eq. (2) and (3)

$$\frac{\partial N}{\partial f} = \frac{N}{f} - \frac{N}{V_a + V_r} \frac{\partial V_r}{\partial f} \quad (14)$$

If the load susceptance is negligible from eq. (1)

$$\frac{f}{f_0} = -\frac{1}{2Q_L} \tan 2\pi\Delta N$$

$$\frac{\partial N}{\partial f} = \frac{\partial(\Delta N)}{\partial(\Delta f)} = -\frac{Q_L}{\pi f \sec^2 2\pi\Delta N} \quad (15)$$

When  $N = N_m$  From eqs. (14), (15)

$$\frac{N_m}{f_0} - \frac{N_m}{V_a + V_{rm}} \cdot \frac{\partial V_r}{\partial f} = -\frac{Q_L}{\pi f_0}$$

$$\left(\frac{\partial f}{\partial V_r}\right) N_m = \frac{\pi N_m f_0}{(V_a + V_{rm})(\pi N_m + Q_L)} \quad (16)$$

From Fig. 2

$$(a) V_a = 300 \quad f_0 = 8.953 \quad N_m = 6.75$$

$$V_{rm} = 63 \quad Q_L = \frac{8.953}{8.98 - 8.89} = 99.3$$

$$\text{From eq. (16)} \quad \frac{\partial f}{\partial V_r} = 4.35 \quad \text{MC/volt}$$

$$\text{From Fig. (2)} \quad \frac{\partial f}{\partial V_r} = 9 \quad \text{MC/volt}$$

$$(b) V_a = 260 \quad f_0 = 8.89 \quad N_m = 6.75$$

$$V_{rm} = 71 \quad Q_L = \frac{8.89}{8.925 - 8.856} = 129$$

$$\text{From eq. (16)} \quad \frac{\partial f}{\partial V_r} = 3.8 \quad \text{MC/volt}$$

$$\text{From Fig. (2)} \quad \frac{\partial f}{\partial V_r} = 6 \quad \text{MC/volt}$$

The estimate value is nearly one half of the actual value.

Conclusions

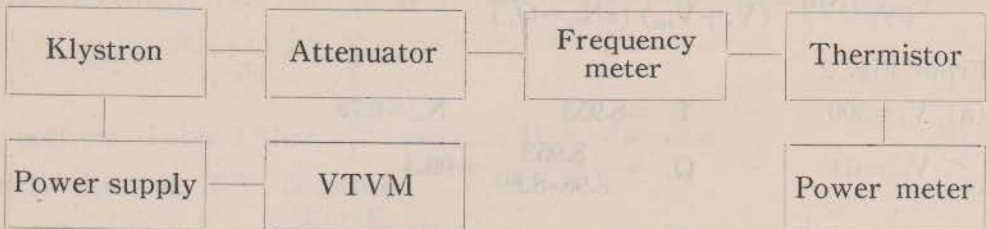
The experimental results are deviated from theoretical discussion to a certain degree. The simplified equations is not good enough to predicate the experimental results satisfactorily, experiment is important in studying the reflex klystron.

Reference:

- |                           |                                   |
|---------------------------|-----------------------------------|
| 1. Spangenberg            | Vacuum Tubes                      |
| 2. Hamillton              | Reflex klystrons                  |
| 3. Hamilton, Knipp, Kuper | Klystron and Microwave Triode     |
| 4. Slater                 | Microwave Electronics             |
| 5. Sucher Fox             | Handbook of Microwave Measurement |
| 6. Ginzton                | Microwave Measurements            |

APPENDIX

1. The tube Raytheon 723 A/B is used in the whole experiment.
2. The block diagram is shown as follow



3. The equivalent circuit of reflex klystron

