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STUDIES OF THE BLOCKING EFFECT OF A LANGMUIR PROBE IN MEDIUM PRESSURE PLASMA MEASUREMENT

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Abstract—*The blocking effect of a probe in plasma measurement has been studied theoretically for the cylindrical type probe of finite length and the disk type probe with finite thickness. For a disk probe, the blocking effect increases with increasing probe thickness. In obtaining the electron density N_e, if the probe thickness is not taken into account, the determined N_e will be smaller than its true value. A maximum percentage error of 85% may result with a disk probe if zero thickness is assumed. For a cylindrical probe, the blocking effect increases with increasing probe length. Zakharova's theory assuming an infinite length is not valid at $d(=t/r_p)\lesssim 3$, where $2t=L$, L is the probe length and r_p is the probe radius. This theory is also applicable for obtaining N_e using a large cylinder type sampling probe. Some experiments are presented and discussed.*

1. INTRODUCTION

The classical theory of the Langmuir probe cannot be used for the electron density determinations in medium pressure plasmas in which the probe radius r_p is larger than the electron mean free path λ_e. It is well known that when $r_p \gtrsim \lambda_e$, the electron saturation current I_e collected by a probe is smaller than the usual electron current of random thermal motion because the motion is disturbed by the existence of a large probe. This is called the "Blocking Effect" of a probe and must be taken into account to obtain an exact value of electron density N_e. The blocking effect of a spherical probe was first calculated by Bohm *et. al*[1] using a simple analogy of the electrostatic Gauss' Law in a continuous medium. Their result was confirmed later by the continuum theory of Su and Lam[2] developed using a set of hydrodynamic equations for a mixture and a set of diffusion flux equations deduced from the principle of linear irreversible thermodynamics. Su and Kiel[3], starting from an ellipsoidal shape model, also deduced the blocking effect of a cylindrical type probe with infinite length and of a planar type probe with no thickness. The result of Su and Kiel for a cylindrical probe coincides with the previous work of Zakharova *et. al*[4], and also with the result of Talbot *et. al*[5] starting from fluid dynamic equations.

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In this paper, the blocking effects of a cylindrical probe of finite length and a planar probe with finite thickness are considered using Bohm's analogy as mentioned above from the viewpoint of practical use. Since in practice, there is no cylindrical probe of infinite length and no planar probe with zero thickness can be made. The calculated results are compared with those obtained by assuming zero thickness and infinite length. Some experiments checked by the microwave technique are also presented and discussed.

In some cases, plasma parameters are analyzed by sampling a small part of them through an orifice into a high vacuum chamber; a large cylinder with a small orifice on its surface must be immersed in the plasma space[6,7]. In this paper, the blocking effect of this sampling type probe is also estimated and discussed.

2. THEORETICAL CONSIDERATIONS

Assuming a closed space whose surface is one electron mean free path away from the probe, in which Poisson's equation is valid and the motions of charged particles are collisionless, the diffusion and mobility limited electron current I_e collected by the probe at the plasma space potential through the closed surface from the plasma can be written as[1]

$$I_e = I_o\left(K + \frac{\bar{v}A_p}{16\pi CD}\right)^{-1}, \qquad (1)$$

where I_o is the usual electron random thermal motion current collected by a probe without blocking effect, A_p is the probe surface area, C is the capacitance of the probe (effectively capacitance of the closed surface one mean free path away from the probe), $\bar{v}$ and D are the mean thermal velocity and the diffusion coefficient of electrons, respectively. K is a coefficient varying from 1 to 0.5 depending on λ_e/r_p, where λ_e is the electron mean free path and r_p is the probe radius. The blocking effect of various type probe can be obtained from Eq. (1) if the value of C, the effective capacitance of each probe is known. Results for the blocking effect calculated using two different models will be shown.

(a) Charged right circular cylinder model

According to the calculations of Smyth[8] and Miles[9], the capacitance of a charged right circular cylinder as shown in Fig. 1(a) can be written as

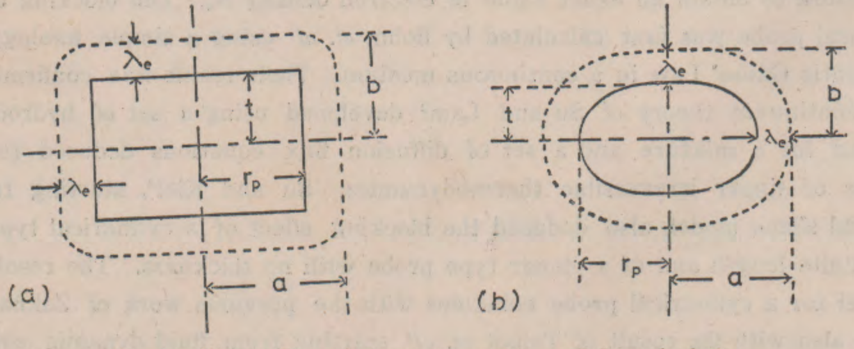

Fig. 1. Illustrations of the probe models used for calculation.
(a) Charged right circular cylinder model.
(b) Ellipsoidal shape model.

$$C=\frac{a}{4\pi}[8+6.95(b/a)^{0.76}], \qquad (0\leq b/a\leq 8), \tag{2}$$

$$C=b[\ln(4b/a)-1+a/(2b)], \qquad (8\leq b/a), \tag{3}$$

where $a=r_p+\lambda_e$, $2b=2(t+\lambda_e)$ ard the radius and the length of the right circular cylinder, respectively. Substituting Eqs. (2), (3) and the classical value of $D=(\lambda_e\,\bar{v})/3$ into Eq. (1), and define the dimensionless shape parameter $d=t/r_p$ and dimensionless pressure parameter $g=\lambda_e/r_p$ for convenience, then it follows from Eq. (1)

$$I_e=I_o\left[K+\frac{3\pi(2d+1)}{2g(1+g)}\left\{8+6.95\left(\frac{d+g}{1+g}\right)^{0.76}\right\}^{-1}\right]^{-1}, \quad (0\leq\frac{d+g}{1+g}\leq 8), \tag{4}$$

$$I_e=I_o\left[K+\frac{3(2d+1)}{8(d+g)g}\left\{\ln\frac{4(d+g)}{1+g}-\frac{2d+g-1}{2(d+g)}\right\}^{-1}\right]^{-1}, \quad (8\leq\frac{d+g}{1+g}). \tag{5}$$

The values inside the brackets of Eqs. (4) and (5) represent I_e/I_o, that is, the reduction rate of electron current collected by a right circular cylinder probe in practical measurement due to the blocking effect.

(b) Ellipsoidal shape model

The capacitance of an ellipsoid as shown in Fig. 1(b) has been obtained by Landau and Lifshitz[10]. For the case of an oblate spheroid ($a>b$),

$$C=(a^2-b^2)^{\frac{1}{2}}/\cos^{-1}(b/a), \tag{6}$$

where a and b are the axes of the spheroid representing $a=r_p+\lambda_e$ and $b=t+\lambda_e$, respectively. Substituting Eq. (6) and A_p, the effective surface area of an oblate spheroid, into Eq. (1) and using the dimensionless parameters d and g, we obtain

$$I_e=I_o\left[K+\frac{3}{8g\sqrt{(1-d)(1+d+2g)}}\left(\cos^{-1}\frac{d+g}{1+g}\right)\left(1+\frac{d^2}{2\sqrt{1-d^2}}\ln\frac{1+\sqrt{1-d^2}}{1-\sqrt{1-d^2}}\right)\right]^{-1}, \tag{7}$$

the reduction rate of electron saturation current collected by an oblate spheroid probe due to the blocking effect. When d and g are much smaller than unity (*i.e.* $d, g\ll 1$), Eqs. (4) and (7) all reduce to

$$I_e\simeq I_o\left(\frac{3\pi}{16g}\right)^{-1} \tag{8}$$

which coincides with the result of Su and Kiel for a planar probe without thickness. On the other hand, for the case of a prolate spheroid ($a<b$),

$$C=\sqrt{b^2-a^2}\left(\ln\frac{b+\sqrt{b^2-a^2}}{a}\right)^{-1}, \tag{9}$$

and thus from Eq. (1), we obtain

$$I_e=I_o\left[K+\frac{3}{8g\sqrt{(d-1)(2g+d+1)}}\left(\ln\frac{d+g+\sqrt{(d-1)(2g+d+1)}}{1+g}\right)\right.$$
$$\left.\times\left(1+\frac{d}{\sqrt{1-1/d^2}}\sin^{-1}(1-1/d^2)\right)\right]^{-1}. \tag{10}$$

When $d\gg 1\gg g$ (*i.e.* $t\gg r_p\gg\lambda_e$), Eqs. (5) and (10) all reduce to

$$I_e\simeq I_o\left[K+\frac{3}{4g}\ln(2d)\right]^{-1} \tag{11}$$

in coincidence with the result of Zakharova *et. al* for the case of a cylindrical probe with infinite length.

3. NUMERICAL RESULTS AND DISCUSSIONS

Typical numerical results of Eqs. (4) and (7) for $g=0.05$ are shown in Fig. 2 by the curves (A) and (B) respectively. The variation of I_o/I_e is plotted with the variation of d. The result of Eq. (8) for the case of $d=0$ is also shown by the dotted line (C). Point (D) represents the blocking effect of a spherical probe ($d=1$) calculated by Bohm *et. al*[1] which is the form $I_o/I_e=\left[K+\frac{3}{4g(1+g)}\right]$. Curves (A) and (B) deviate

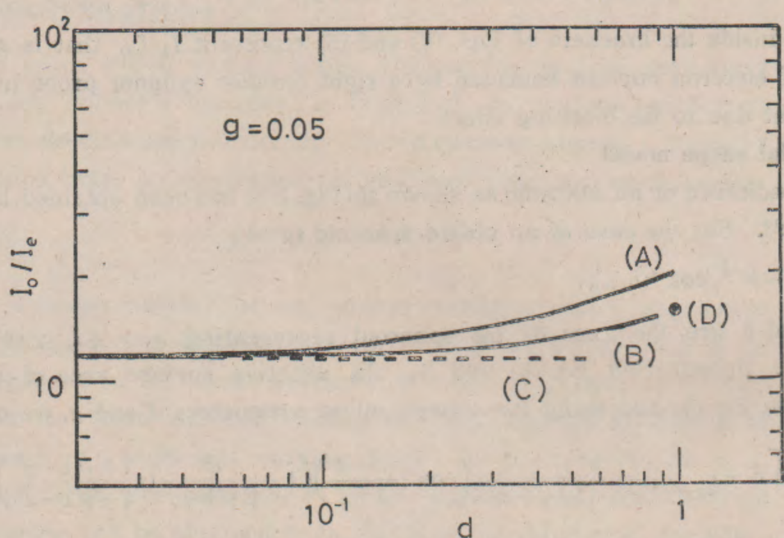

Fig. 2. The variation of I_o/I_e with d at $g=0.05$ for a disk probe. Curves A, B and dotted line C represent Eqs.(4), (7) and (8), respectively. The point D represents the blocking effect of a spherical probe calculated by Bohm *et. al*.

from dotted line (C) approximately at $d=0.05$ and the deviations increase with increasing d. It means that in practical measurement, the blocking effect of a planar probe increases with increasing probe thickness. If the probe thickness is not taken into account and only Eq. (8) is used for obtaining electron density N_e, the measured value will be smaller than its true value. The maximum error (at $d=1$) will be approximately 40% for the oblate spheroid type probe and 85% for the right circular cylinder type probe, respectively. Also the typical results of Eqs. (4), (5), (10) and (11) for $g=0.05$ are shown by curves (A), (B), (C), (D) in Fig. 3 respectively. The blocking effect of a cylindrical probe increases with increasing d (the probe length). Zakharova's theory assuming an infinite length probe (curve D) breaks down when $d\lesssim 3$ because of the edge effect. It means that for a cylindrical probe whose length is whose not sufficiently larger than the probe radius, the measured N_e using curve (D) will be smaller than its true value. The maximum error of curve (D) with respect to the curves (A) and (B) is approximately 75%. On the other hand, the blocking effect of a right circular cylinder type probe (curves A and B) is always approximately 40% larger than that of a prolate spheroid type probe (curve C).

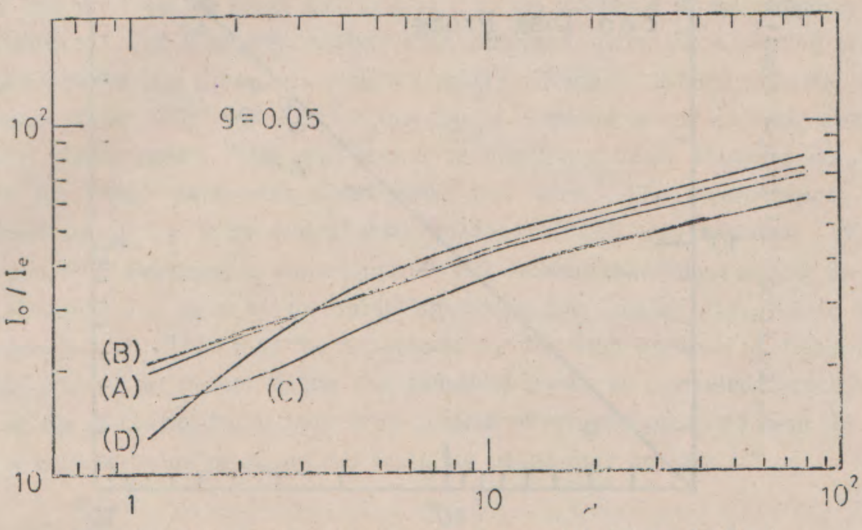

Fig. 3. The variation of I_o/I_e with d at g=0.05 for a cylindrical probe. Curve A, B, C and D represent Eqs. (4), (5), (10) and (11), respectively.

4. EXPERIMENTAL RESULTS AND DISCUSSIONS

Experiments have been done in the argon plasmas produced with the same discharge system as reported in Ref. 11. The electron random thermal motion current I_o can be obtained from the measured electron saturation current of the probe by taking the blocking effect into account. The electron density N_e can be obtained from the relation $I_o = A_p N_e e (kT_e/2\pi m_e)^{1/2}$ where A_p is the probe surface area, k is the Boltzmann constant, T_e is the electron temperature and e, m_e are the charge and the mass of an electron, respectively. The measured N_e using a cylindrical probe (r_p=0.25 *mm*, t=6 *mm*), a disk probe (r_p=3 *mm*, t=1 *mm*) and a cylindrical sampling probe (r_p=5 *mm*, t=4 *mm*) are compared with those determined from a spherical probe (r_p=0.4 *mm*). The reliability of N_e determination using a spherical probe taking the blocking effect calculated by Bohm *et. al*[1] into account had been checked previously by the microwave free space method; the results of the two methods are in agreement within a factor of two[11]. In Fig. 4, typical results for N_e determined from the disk probe using Eq. (4) and N_e from the cylindrical probe using Eq. (5) are shown compared with those determined from a spherical probe. The pressure parameter $g(=\lambda_e/r_p)$ was varied from 0.05 to 0.25. The results of the three probes almost agree with each other however N_e from the spherical probe is approximately 40% smaller than those from other probes. This difference may be caused by the existence of the supporter of each probe. In practice, if the dimension of the supporter is large

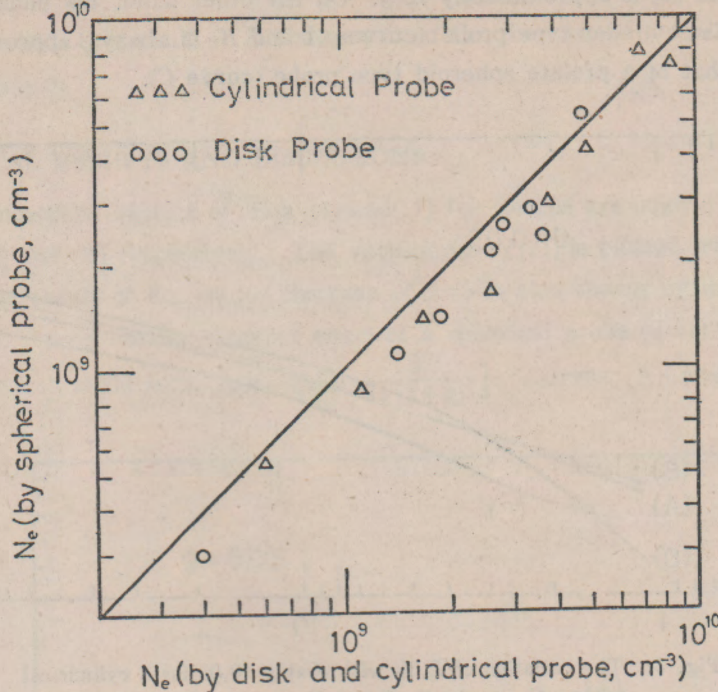

Fig. 4. Comparison of N_e obtained from the saturation electron current I_e of a disk and a cylindrical probe with those obtained from a spherical probe. Argon gas at g=0.05~0.25 was used.

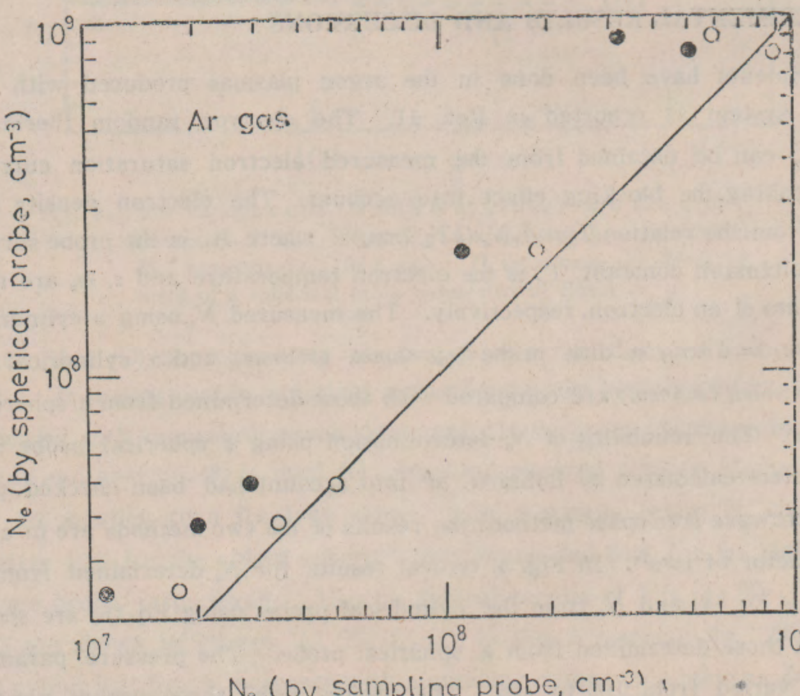

Fig. 5. Comparison of N_e determined from the sampling probe with those from a spherical probe. Argon gas at g=0.05~0.25 was used. Open points are those from Eq. (4) and closed points are from Eq.(8).

compared to the probe itself, the current collected by the probe will be reduced because of the blocking effect of the supporter and a smaller value of N_e will be obtained. In this experiment, the dimension of the spherical probe ($r_p=0.4\ mm$) is much smaller than the other probes and thus the influence of the supporter may not be neglected. Fig. 5 shows another data obtained using the sampling probe. This sampling probe is a glass shield right circular cylinder extended from the wall of the discharge tube; only one side at the top is exposed in the plasma working as a circular planar probe. The open points in Fig. 5 are those obtained using Eq. 4 in which the shape parameter $d=t/r_p=0.8$ was used. The closed points are those obtained using Eq. 8 in which zero thickness ($d=0$) was assumed. N_e obtained assuming zero thickness is approximately 40% smaller than those taking the thickness into account. All data by the sampling probe are smaller than those by a small spherical probe. This may be explained by the non-uniform distribution of the plasma around the probe. Since the sampling probe is extended directly from the wall of the discharge tube, there is no plasma effectively provided from the tube wall, thus a smaller value of N_e by the sampling probe may be resulted.

5. CONCLUSIONS

The blocking effect of a disk probe with finite tkickness and a cylindrical probe of finite length has been studied. The following conclusions are given:

(a) The blocking effect of a disk probe increases with increasing probe thickness. In determining electron density N_e, if the thickness of the probe is not taken into account, N_e obtained will be smaller than its true value. For the case of $d=1$ (*ie.* thickness of the probe is equal to its diameter), the blocking effect of a right circular cylinder type probe is approximately 85% greater than that of $d=0$ (zero thickness). Namely, a maximum percentage error of 85% may be introduced in determining N_e.

(b) The blocking effect of a cylindrical probe increases with increasing probe length. Blocking effect of a right circular cylinder is approximately 40% greater than that of a prolate spheroid. The result of Zakharova *et. al*[4] is not valid at $d \lesssim 3$, since in their calculation, an infinite length of the probe is assumed.

(c) The experimental results agree with the theoretical all within a factor of 2. It is possible to determine N_e using a large cylinder type sampling probe extended from the tube wall if the blocking effect of the probe is taken into account.

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