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雷射全像術在秘密照相上之應用

On the Laser Holography for the Confidential Picture-Taking Technology

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ABSTRACT — A rigorous theorem and gedanken experiment for the highly reliable optical modulated method for the confidential picture-taking technology is developed. Its reliability and possibility for realization is also presented in this paper.

I. Introduction

For the secret reason, some military and industrial informations should be preserved and transfered secretly.

Now a highly reliable method is given by the author to modulate the image of documents or photos in an optical manner, which possesses many benefits as follows:

(1) The modulated image should be demodulated with the original "Optical key". Even the demodulated process is known by the others, the original image could not be recovered without this "Optical key",

(2) The demodulated process is very easy to excute.

(3) The so called "Optical key" is a picture with random intensity distribution just like a "White noise", therefore it possesses high reliability.

(4) For a specified information, a specified "Optical key" is used. The optical key has great varieties in nature.

(5) The modulated image can be stores in a very limited size. And several kinds of information can be recorded in the same film.

(6) Holograph picture could also be modulated and be reproduced in holographic sense.

II. Theory

This method is performed by three steps:

(1) Manufacture of the "Optical key",

(2) Information modulation, and

(3) Information demodulation.

These three steps will be described in detail respectively as follows:

1. Manufacture of the "Optical key"

The so called "Optical key" is known as the "VANDER LUGT FILTER"(1). It can be illustrated in Fig. 1,

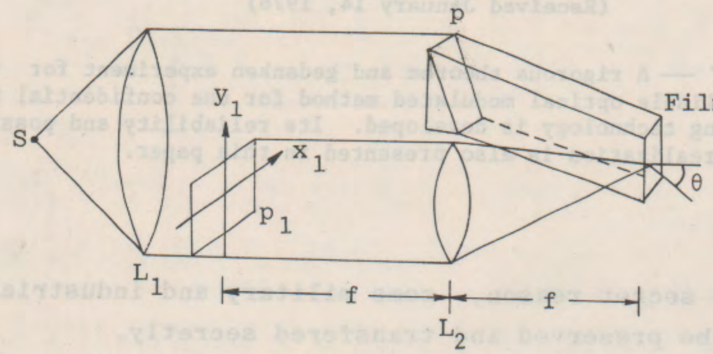

Fig. 1 Manufacture of the "optical key"

In Fig. 1, the point source is a laser light the lens L_1 collimates the light from the point source S. A portion of this light strikes the mask P_1, which has an amplitude transmittance that is equal to the desired random distribution in order to get high reliability. The amplitude distribution of this mask P_1 is called $h(x_1, y_1)$. The lens L_2 Fourier transforms the amplitude distribution h, and yields an amplitude distribution $(1/\lambda f)H(x_2/\lambda f, y_2/\lambda f)$ incident on the film. In addition, a second portion of the collimated light passes above the mask P_1, strikes the prism P, and is finally incident upon the film at an angle θ as shown.

The total intensity incident at each point on the film is determined by the interference of the two amplitude distribution present. The plane wave incident from the prism produces a field distribution,

$$U_r(x_2, y_2) = r_o \exp(-j2\pi\alpha y_2)$$

where the spatial frequency α is given by

$$\alpha = \sin\theta/\lambda \tag{1}$$

The total intensity distribution is given by

$$I_o(x_2,y_2) = |r_o\exp(-j2\pi\alpha y_2)+(1/\lambda f)H(x_2/\lambda f,y_2/\lambda f)|^2 \tag{2}$$

$$= r_o^2+(1/\lambda^2 f^2)|H(x_2/\lambda f,y_2/\lambda f)|^2$$

$$+(r_o/\lambda f)H(x_2/\lambda f,y_2/\lambda f)\exp(j2\pi\alpha y_2)$$

$$+(r_o/\lambda f)H^*(x_2/\lambda f,y_2/\lambda f)\exp(-j2\pi\alpha y_2)$$

The film P_2 is the so called "Optical key".

2. Information Modulation

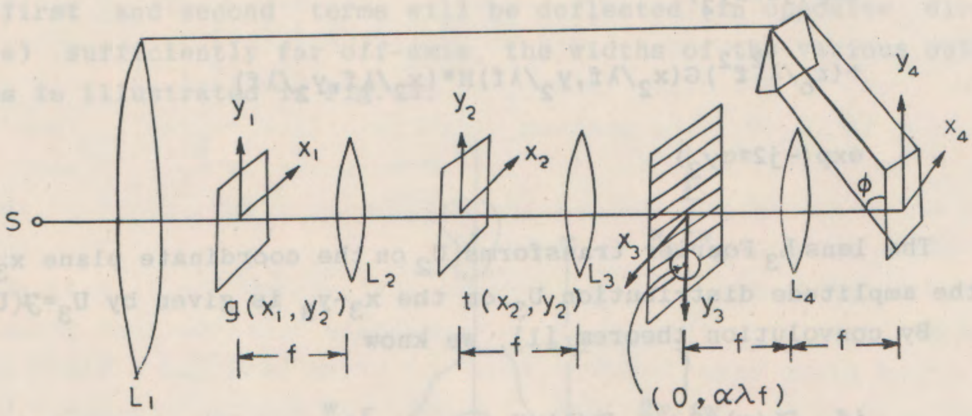

Fig. 2 System to modulate information

As shown in Fig.2, the system is used to modulate the desired information, S is the laser point source. The lens L_1 collimates the light from the source S. On the x_1-y_1 plane, we place our desired information, whose amplitude distribution is $g(x_1,y_1)$. The les L_2 Fourier transforms the amplitude distribution $g(x_1,y_1)$, and yields an amplitude distribution $(1/\lambda f)G(x_2/\lambda f, y_2/\lambda f)$on the x_2-y_2 plane, but at x_2-y_2 plane, we placed the "optical key", whose amplitude transmittance is $t(x_2,y_2)$. We realize that $t(x_2,y_2)$ is proportional to its intensity distribution. If the "optical key" is placed accurately on the x_2-y_2 coordinate, then

$$t(x_2,y_2) \propto r_o^2 + (1/\lambda^2 f^2)|H(x_2/\lambda f, y_2/\lambda f)|^2$$

$$+(r_o/\lambda f) H(x_2/\lambda f, y_2/\lambda f) \exp(j2\pi\alpha y_2)$$

$$+(r_o/\lambda f) H^*(x_2/\lambda f, y_2/\lambda f) \text{ex-}(-j2\pi\alpha y_2)$$

So the field strength transmitted by the mask then obeys the proportionaligy

$$U_2 \propto (1/\lambda f) G(x_2,y_2) . t(x_2,y_2) = (r_o^2/\lambda f) G(x_2/\lambda f,$$

$$y_2/\lambda f) + (1/\lambda^3 f^3)|H(x_2/\lambda f, y_2/\lambda f)|^2 \; G(x_2/\lambda f, y_2/\lambda f)$$

$$+r_o/\lambda^2 f^2 \; G(x_2/\lambda f, y_2/\lambda f) H(x_2/\lambda f, y_2/\lambda f)$$

$$\exp(j2\pi\alpha y_2)$$

$$+(r_o/\lambda^2 f^2) G(x_2/\lambda f, y_2/\lambda f) H^*(x_2/\lambda f, y_2/\lambda f)$$

$$\exp(-j2\pi\alpha y_2)$$

The lens L_3 Fourier transforms U_2 on the coordinate plane x_3-y_3, so the amplitude distribution U_3 on the x_3-y_3 is given by $U_3=\mathcal{F}(U_2)$.

By convolution theorem [1], we know

$$\text{if} \quad \mathcal{F}(g)=G, \quad \mathcal{F}(h)=H \tag{3}$$

$$\text{then} \quad \mathcal{F}(g,h)=G_* H$$

$$U_3(x_3,y_3) \propto r_o^2 \; g(x_3,y_3)$$

$$+1/\lambda^2 f^2 \; h(x_3,y_3) * h^*(-x_3,-y_3) * g(x_3,y_3)$$

$$+r_o/\lambda f \; g(x_3,y_3) * h(x_3,y_3) * \delta(x_3, y_3+\alpha\lambda f)$$

$$+r_o/\lambda f \; g(x_3,y_3) * h^*(-x_3,-y_3) * \delta(x_3, y_3-\alpha\lambda f)$$

Noting that $g(x_3,y_3) * h(x_3,y_3) * \delta(x_3, y_3+\alpha\lambda f)$

$$= \iint_{-\infty}^{\infty} h(x_3-\xi, y_3+\alpha\lambda f-\eta)\, g(\xi,\eta)\, d\xi d\eta$$

This portion of the output is seen to yield the convolution of g and h centered at coordinated $(0, -\alpha\lambda f)$ in the $x_3 y_3$ plane.

The fourth term of Eq. (3) may be rewritten as

$$h^*(-x_3, -y_3) * g(x_3, y_3) * \delta(x_3, y_3 - \alpha\lambda f)$$

$$= \iint_{-\infty}^{\infty} g(\xi,\eta)\, h^*(\xi - x_3, \eta - y_3 + \alpha\lambda f)\, d\xi d\eta$$

which is centered at coordinates $(0, \alpha\lambda f)$ in the x_3-y_3 plane.

Note that the first and second terms of Eq. (3) are centered at the origin of the x_3-y_3 plane. Thus it becomes clear that if the "carrier frequency" is chosen sufficiently high, or equivalently, if the reference wave is introduced at a sufficiently steep angle, the first and second terms will be deflected (in opposite directions) sufficiently far off-axis, the widths of the various output terms is illustrated in Fig. 3.

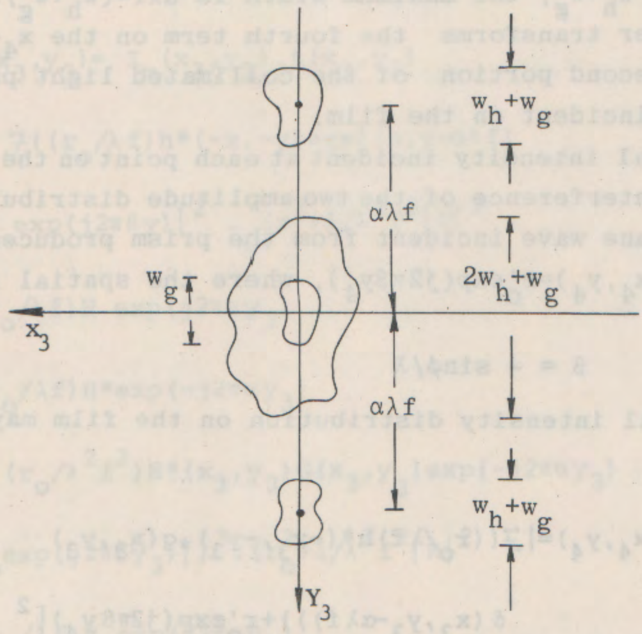

Fig. 3 Location of the various terms of Eq. (3).

If the maximum width of h in the y direction is w_h, and that of g is w_g, then the widths of the various output terms are as follows:

1. $r_o^2 g(x_3,y_3) \rightarrow w_g$

2. $1/\lambda^2 f^2[h(x_3,y_3)*h^*(-x_3,-y_3)*g(x_3,y_3)] \rightarrow 2w_h+w_g$

3. $r_o/\lambda f[h(x_3,y_3)*g(x_3,y_3)*\sigma(x_3,y_3)+f)] \rightarrow w_h+w_g$

4. $r_o/\lambda f[h^*(x_3,y_3)*g(x_3,y_3)*\sigma(x_3,y_3-f)] \rightarrow w_h+w_g$

From this figure, it is clear that the complete separation will be achieved if

$$\alpha > 1/\lambda f(2w_h+w_g)/2+(w_h+w_h+w_g)/2)=1/\lambda f(3/2w_h+w_g)$$

or equivalently, if

$$\sin\theta > 1/f\cdot((3/2)\cdot w_h+w_g) \qquad (4)$$

If we place a screen at x_3-y_3 plane, and open a hole at position $(0,\alpha\lambda f)$, its dimension is such to pass the fourth term completely and to cut off all other terms (the minimum width of the hole in y direction is w_h+w_g, the maximum width is $\alpha\lambda f-(w_h+w_g/2)$). Then the lens U_4 Fourier transforms the fourth term on the x_4-y_4 plane. In addition, a second portion of the collimated light passes prism P with angle ϕ incident on the film.

The total intensity incident at each point on the film is determined by the interference of the two amplitude distributions present. The titled plane wave incident from the prism produces a field distribution $U_r(x_4,y_4)=r_o' \exp(j2\pi\beta y_4)$, where the spatial frequency β is given by

$$\beta = -\sin\phi/\lambda \qquad (5)$$

The total intensity distribution on the film may therefore be written

$$I_m(x_4,y_4)=|\mathcal{F}((r_o/\lambda f)h^*(-x_3,-y_3)*g(x_3,y_3) \qquad (6)$$

$$\delta(x_3,y_3-\alpha\lambda f))+r_o' \exp(j2\pi\beta y_4)|^2$$

The film is the desired confidential picture.

3. Information Demodulation

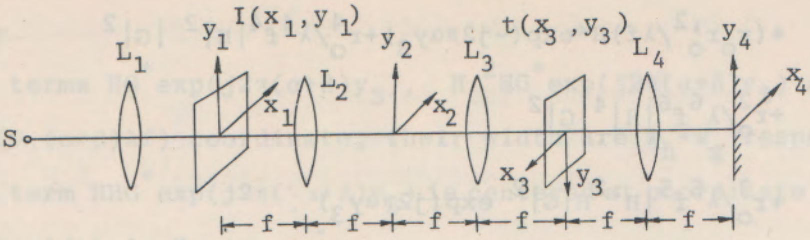

Fig. 4 System to demodulate information.

As shown in Fig.4, the system is used to demodulate the desired information. The lens L_1 collimates the point source S on the plane x_1-y_1, where we placed the confidential picutre. On the plane x_3-y_3, we placed the "optical key". The lens L_2 then Fourier transforms the confidential picture on plane x_2-y_2, the lens L_3 once again Fourier transforms to x_3-y_3 plane, (Because $\mathcal{F}(\mathcal{F}(g(x,y)))=g(-x,-y)$[2]. So the direction of the plane x_3-y_3 is inverse), then the amplitude distribution incident on the x_3-y_3 plane is proportional to $I_m(x_3, y_3)$. The field strength transmitted by the "optical key" mask then obeys the proportionality

$$U_3(x_3,y_3) \propto I_m(x_3,y_3) \cdot t(x_3,y_3) \tag{7}$$

$$=(|\mathcal{F}((r_o/\lambda f) h^*(-x,-y) * g * \delta(x, y-\alpha\lambda f)$$

$$+r_o' \exp(j2\pi\beta y)|^2 \cdot (r_o^2+1/\lambda^2 f^2 |H|^2$$

$$+(r_o/\lambda f) H \exp(j2\pi\alpha y_3)$$

$$+(r_o/\lambda f) H^* \exp(-j2\pi\alpha y_3)$$

$$=(|(r_o/\lambda^2 f^2) H^*(x_3,y_3) G(x_3,y_3) \exp(-j2\pi\alpha y_3)$$

$$+r_o' \exp(j2\pi\beta y_3)|)^2 \cdot (r_o^2+1/\lambda^2 f^2 |H|^2$$

$$+(r_o/\lambda f) H \exp(j2\pi\alpha y_3)$$

$$+(r_o/\lambda f) H^* \exp(-j2\pi\alpha y_3)$$

$$=r_o'^2 r_o^2 + r_o'^2/\lambda^2 f^2 |H|^2 + (r_o r_o'^2/\lambda f) H \exp(j2\pi\alpha y_3)$$

$$+(r_o r_o'^2/\lambda f)H^*\exp(-j2\pi\alpha y_3)+r_o^4/\lambda^4 f^4|H|^2\ |G|^2$$

$$+r_o^2/\lambda^6 f^6|H|^4|G|^2$$

$$+r_o^3/\lambda^5 f^5|H^{2}{}^{2}H|G|^2\ \exp(j2\pi\alpha y_3)$$

$$+(r_o^3/\lambda^5 f^5)H^*|H|^2\ |G|^2\ \exp(-j2\pi\alpha y_3)$$

$$+(r_o^3 r_o'/\lambda f)H^*G\ \exp(-j2\pi(\alpha+\beta)y_3$$

$$+r_o r_o'/\lambda^3 f^3|H|^2 H^*G\ \exp(-j2\pi(\alpha+\beta)y_3)$$

$$+(r_o^2 r_o'/\lambda^2 f^2)H.H^*.G\ \exp(-j2\pi\beta y_3)$$

$$+(r_o' r_o^2/\lambda^2 f^2)H^*H^*G\ \exp(-j2\pi(2\alpha+\beta)y_3)$$

$$+(r_o^3 r_o'/\lambda^2 f^2)HG^*\exp(j2\pi(\alpha+\beta)y_3)$$

$$+(r_o r_o'/\lambda^4 f^4)|H|^2 HG^*\exp(j2\pi(\alpha+\beta)y_3)$$

$$+(r_o^2 r_o'/\lambda^3 f^3)H.H.G^*\exp(j2\pi(2\alpha+\beta)y_3)$$

$$+(r_o^2 r_o'/\lambda^3 f^3)H^*HG^*\exp(j2\pi\beta y_3)$$

Then lens L_4 Fourier transforms U_3 to x_4-y_4 plane. The constant term, $|H|^2$, $|H|^2|G|^2$, $|H|^4|G|^2$ are centered at the origin of the x_4-y_4 plane, their width in y direction are 0, $2w_h+2w_g$, $4w_h+2w_g$ respectively.

The terms $|H|^2|G|^2H\exp(j2\pi\alpha y_3)$, $H\exp(j2\pi\alpha y_3)$ are centered at $(0,\alpha\lambda f)$ coordinate, their width in y direction are $3w_h+2w_g$, w_h respectively.

The terms $|H|^2|G|^2H^*\exp(-j2\pi\alpha y_3)$, $H^*\exp(-j2\pi\alpha y_3)$ are centered at $(0, -\alpha\lambda f)$ coordinate, their width are $3w_h+2w_g$, w_h respectively.

The terms $H^*G\exp(-j2\pi(\alpha+\beta)y_3$, $H^{\,2}H^*G\exp(-j2\pi(\alpha+\beta)y_3)$ are centered at $(0,-(\alpha+\beta)\lambda f)$ coordinate, their width are w_h+w_g, $3w_h+w_g$ respectively.

The terms $HG^*\exp(j2\pi(\alpha+\beta)y_3)$, $H^2HG^*\exp(j2\pi(\alpha+\beta)y_3)$ are centered at $(0,(\alpha+\beta)\lambda f)$ coordinate, their width are w_h+w_g respectively.

The term $HHG^*\exp(j2\pi(2\alpha+\beta)y_3)$ is centered at coordinate $(0,(2\alpha+\beta)\lambda f)$, its width is $2w_h+w_g$.

The term $H^*H^*G\exp(-j2\pi(2\alpha+\beta)y_3)$ is centered at coordinate $(0,-(2\alpha+\beta)\lambda f)$, its width is $2w_h+w_g$.

The term $HH^*G\exp(-j2\pi\beta y_3)$ is centered at coordinate $(0,\beta\lambda f)$, its width is $2w_h+w_g$.

The term $H^*HG^*\exp(j2\pi\beta y_3)$ is centered at coordinate $(0,-\beta\lambda f)$, its width is $2w_h+w_g$.

The widths of the various output terms is illustrated in Fig.5.

We are interested in last two terms, from the fiture it is clear that the last two terms will be completely separated from other terms

if $(\beta-\alpha)\lambda f > (2w_h+w_g)/2+(3w_h+2w_g)/2$

$=(5/2)w_h+(3/2)w_g$

and $(\alpha+\beta)\lambda f-\beta\lambda f > (3w_h+w_g)/2+(2w_h+w_g)/2$

$=(5/2)w_h+w_g$

or equivalently, if

$$\alpha\lambda f > (5/2)w_h+w_g \tag{8}$$

$$\beta\lambda f > 5w_h+(5/2)w_g \tag{9}$$

is satisfied.

We see that Eqs. (4),(8) and (9) are the least limitation for angle θ and ϕ.

If we take θ and ϕ satisfy Eqs. (4),(8),(9), then at x_4-y_4 plane we wi-l get two separable terms, that is

$\mathcal{F}(HH^{*}G\exp(-2\pi\beta y_3))$ and $\mathcal{F}(HH^{*}G^{*}\exp(j2\pi\beta y_3))$

If we take the function h be a random intensity distribution as like a "white noise", that is in the interested frequency (the cut-off frequency of desired confidential picture)[2], $H\cdot H^{*}$ is a constant.

Therefore, we can get the desired two terms,

one is $\mathcal{F}(H\cdot H^{*}G\exp(-j2\pi\beta y_3)=\text{Constant}\cdot g(x_4,y_4-\beta\lambda f)$ (10)

the other is $\mathcal{F}(HH^{*}G^{*}\exp(j2\pi\beta y_3))=\text{Constant}\cdot g(-x_4,-y_4+\beta\lambda f)$ (11)

The Eq. (10) is the original information.

The Eq. (11) is the inversely original information.

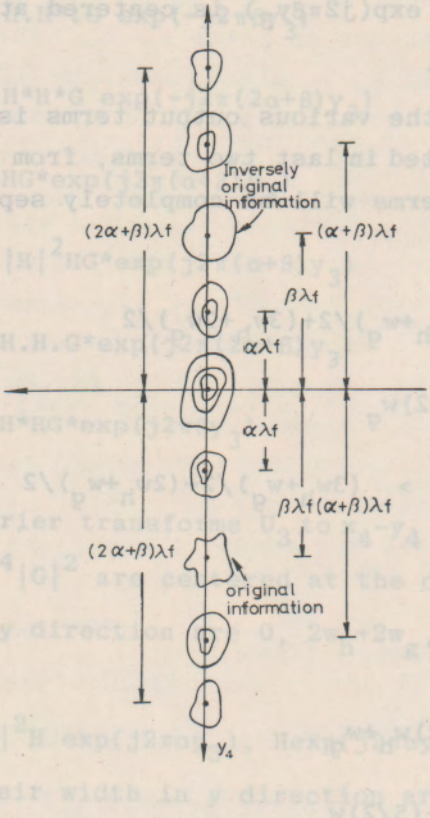

Fig. 5 Location of the various terms on x_4-y_4 plane.

III. Discussion

1. Reliability:

The information in the modulated picture is

$$I(x,y)=|(r_o/\lambda^2 f^2)H^*G\exp(-j2\pi\alpha y) \tag{12}$$

$$+r_o'\exp(j2\pi\beta y)|^2$$

$$=r_o^2/\lambda^4 f^4|H|^2|G|^2+r_o'^2$$

$$+(r_o r_o'/\lambda^2 f^2)H^*G\exp(-j2\pi(\alpha+\beta)y)$$

$$+(r_o r_o'/\lambda^2 f^2)HG^*\exp(j2\pi(\alpha+\beta)y)$$

We know from Eq.(12). Nothing will be get unless take Fourier transform, the results as follows:

The first term: $h*h^* *g*g^*$

The second term: $\delta(x,y)$

The third term: $h^* *g$

The fourth term: $h*g^*$

Because h is a random function, $h^* *g$ or $h*g^*$ will loss the original information completly.

Any others want to get the original information, they must add a new "optical key" and follow the same process for demodulation. Let the amplitude distribution of this new "optical key" be h'(x,y), then he will get the two terms:

One is $\mathcal{F}(H'H^*G\exp(-j2\pi\beta y))$

the other term is $\mathcal{F}(H\,H'^*G^*\exp(j2\pi\beta y))$.

Because $$H=\iint h(x,y)\exp(-j2\pi(f_x x+f_y y))dxdy \tag{13}$$

$$=\iint h(x,y)\cos(2\pi(f_x x+f_y y)dxdy$$

$$-j\iint h(x,y)\sin(2\pi(f_x x+f_y y)dxdy$$

$$=r(f_x,f_y)\exp(-j\theta(f_x,f_y))$$

$\therefore$ $H\cdot H^*=r^2(f_x,f_y)\approx$ Constant (the condition is satisfied, because h is like as white noise).

Compare Eq. (13) and Eq. (14) we get

$$r^2(f_x,f_y)= \left(\iint_{-\infty}^{+\infty} h(x,y)\cos(2\pi(f_x x+f_y y))\,dxdy\right)^2$$

$$+ \left(\iint_{-\infty}^{+\infty} h(x,y)\sin(2\pi(f_x x+f_y y)\,dxdy\right)^2 \qquad (14)$$

$$\tan\theta(f_x,f_y)=-\left(\iint h(x,y)\sin(2\pi(f_x x+f_y y))\,dxdy\right)/$$

$$\left(\iint h(x,y)\cos(2\pi(f_x x+f_y y))\,dxdy\right) \qquad (15)$$

We will prove that $\theta(f_x,f_y)$ is a very random value. In order to state the fact, we consider a simple case.

Let $h(x,y)= \sum_k[1+\sin(2\ (f_{kx}x+f_{ky}y)+\theta_k)]P(x_k,x'_k)P(y_k,y'_k)$, where

$$P(x_k,x'_k)= \begin{cases} 1 & \text{when } x_k \le x \le x'_k \\ 0 & \text{otherwise} \end{cases}$$

$$P(y_k,y'_k)= \begin{cases} 1 & \text{when } y_k \le y \le y'_k \\ 0 & \text{otherwise} \end{cases}$$

The amplitude distribution of the function $h(x,y)$ may be illstrates in Fig. 6.

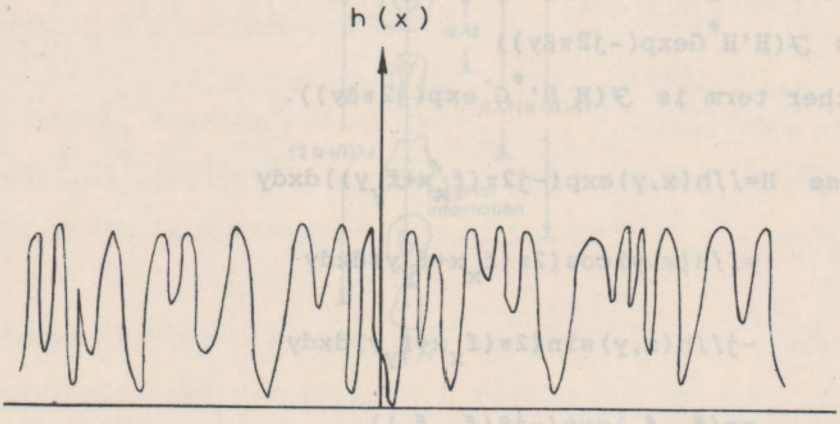

Fig. 6 The amplitude distribution of the simple case $h(x,y)$.

From Eq. (15), we obtain

$$\tan\theta(f_x,f_y)=-\left(\sum_k \int_{y_k}^{y'_k}\int_{x_k}^{x'_k}[1+\sin(2\ (f_{kx}x+f_{ky}y) \qquad (16)$$

$$+\theta_k)]\sin(2\pi(f_x x+f_y y))dxdy)/\sum_k\int_{y_k}^{y'_k}\int_{x_k}^{x'_k}[1$$

$$+\sin(2\pi(f_{kx}x+f_{ky}y)+\theta_k]\cos(2\pi(f_x x+f_y y))dxdy)$$

From Eq. (15), we obtain

If the distribution of f_{kx} or f_{ky} is very closed, then we only consider for $f_x \simeq f_{kx}$, $f_y \simeq f_{ky}$ condition. Then the integration result of Eq. (16) will be

$$\tan\theta(f_{kx},f_{ky})=-((I)+(II)+(III))/((A)+(B)) \qquad (17)$$

The term $(I)=(4\sin(\pi(f_{kx}(x'_k+x_k)+f_{ky}(y'_k+y_k))\sin(\pi f_{kx}(x'_k-x_k))\sin(\pi f_{ky}(y'_k-y_k)))/(4\pi^2 f_{kx}f_{ky})$

The term $(II)=\cos\theta_k\cdot(x'_k-x_k)(y'_k-y_k)$

The term $(III)=-4(\cos(2\pi(f_{kx}(x'_k-x_k)+f_{ky}(y'_k-y_k)+2\theta_k)\sin(2\pi f_{kx}(x'_k-x_k))\sin(2\pi f_{ky}(y'_k-y_k))$

$16\pi^2 f_{kx}^2 f_{ky}^2$

The term $(A)=(4\cos(\pi(f_{kx}(x'_k+x_k)+f_{ky}(y'_k+y_k))\sin(\pi f_{kx}(y'_k-y_k)))/(4\pi^2 f_{kx}f_{ky})$

The term $(B)=(4\sin(2\pi(f_{kx}(x'_k+x_k)+f_{ky}(y'_k+y_k))+2\theta_k)\sin(2\pi f_{kx}(x'_k-x_k))\sin(2\pi f_{ky}(y'_k-y_k)))/(16\pi^2 f_{kx}^2 f_{ky}^2)$

Because $(x'_k-x_k)^2(y'_k-y_k)^2$ is the spectrum density of the frequency f_{kx}, f_{ky}, therefore $(x'_k-x_k)^2(y'_k-y_k)^2$ is almost a constant. Specially, we take (x'_k-x_k) and (y'_k-y_k) are all constant. But $(x'_k+$

x_k) or ($y_k'+y_k$) is a random value, and θ_k is also a random value. From these condition, we see that the term (I) has function value between 0 and some constant.

The terms (II),(III), (A),(B) also have their functional value between 0 and some constant.

If the value x_k, y_k, θ_k are mutually independent, then the functional value of these terms (I),(II),(III) are mutually independent. The functional value of these terms (A) and (B) are independent. Therefore the function $\theta(f_{kx},f_{ky})$ is a random variable.

Now, we want to see these terms:

(1) $\mathcal{F}(H' \cdot H^* G\exp(-j2\pi\beta y))$

(2) $\mathcal{F}(H \cdot H'^* G^* \exp(j2\pi\beta y))$

Let $H=r\exp(-j\theta)$ where r is a constant

θ is a random variable.

$H'=r'\exp(-j\phi)$ where r' is a constant

ϕ is the other random variable.

If $h'(x,y) \neq h(x,y)$, then θ and ϕ are independent,

$$\therefore \quad \mathcal{F}(H'.H^*)=\iint_{-\infty}^{+\infty} r'r\exp(-j(\phi-\theta))\exp(-j2\pi(f_x x+f_{yy}))df_x df_y$$

$$=rr'\iint_{-\infty}^{+\infty}\exp(-j(\phi-\theta+2\pi(f_x x+f_y y))df_x df_y$$

Because θ and ϕ are independent, therefore $\phi-\theta$ is a random variable of frequency f_x and f_y. Then the value $\phi-\theta+2\ (f_x x+f_y y)$ is a random value.

Let $\phi-\theta+2\ (f_x x+f_y y)=\sigma$, then σ is a random variable,

$$\therefore \quad \mathcal{F}(H'\cdot H^*)=r'r\iint\exp(-j\sigma)df_x df_y$$

$$=r'r(\iint\cos\ df_x df_y-j\iint\sin\ df_x df_y)$$

Because σ is a random variable

$$\therefore \quad \iint\cos\ df_x df_y \simeq 0$$

$$\iint\sin\ df_x df_y \simeq 0$$

$$\therefore \quad \mathcal{F}(H'\cdot H^*) = 0$$

$$\therefore \quad \mathcal{F}(H'\cdot H^*\cdot G) = \mathcal{F}(H'\cdot H^*) * \mathcal{F}(G)$$

$$= 0 * g = 0$$

The same reason, $\mathcal{F}(H \cdot H'^{*} \cdot G^{*})=0$,

Therefore, we conclude that if the using "Optical key" is not the special "optical key", then we cannot obtain the original information. But the "optical key" is a random pattern, therefore the reliability is very high.

2. The Error Influence Due To Alignment

In the preceeding discussion of information modulation. We assume the origion of the "optical key" is accurately aligned on the origin of the x_2-y_2 plane. We can know if the "optical key" is seted far from the accurate position, it will loss the modulation characteristic. Let the error due to alignment in the x direction be Δx, in the y direction be Δy, that is

$$x_2' = x_2 + \Delta x, \quad y_2' = y_2 + \Delta y$$

or

$$x_2 = x_2' - \Delta x, \quad y_2 = y_2' - \Delta y$$

Thus the transmittance of the "optical key" will be replaced by

$$t(x_2',y_2') \alpha r_o^2 + 1/\lambda^2 f^2 |H((x_2'-\Delta x)/\lambda f, (y_2'- y)/\Delta f)|^2$$

$$+(r_o/\lambda f) H((x_2'-\Delta x)/\lambda f, (y_2'-\Delta y)/\lambda f)$$

$$\cdot \exp(j2\pi\alpha(y_2'-\Delta y))$$

$$+(r_o/\lambda f) H^*((x_2'-\Delta x)/\lambda f, (y_2'-\Delta y)/\lambda f)$$

$$\cdot \exp(-j2\pi\alpha(y_2'-\Delta y))$$

Therefore, the field strength transmitted by the mask then obeys the proportionality

$$U_2 \alpha \ (1/\lambda f) G(x_2'/\lambda f, y_2'/\lambda f) t(x_2', y_2')$$

$$=(r_o^2/\lambda f) G(x_2'/\lambda f, y_2'/\lambda f) + 1/\lambda^3 f^3 |H((x_2'$$

$$-\Delta x)/\lambda f, (y_2'-\Delta y)/\lambda f)|^2 G$$

$$+(r_o/\lambda^2 f^2)G\cdot H(x_2'-\Delta x)/\lambda f,(y_2'-\Delta y)/\lambda f)$$

$$\cdot\exp(j2\pi\alpha(y_2'-\Delta y))$$

$$+(r_o/\lambda^2 f^2)G\cdot H^*((x_2'-\Delta x)/\lambda f,(y_2'-\Delta y)/\lambda f)$$

$$\cdot\exp(-j2\pi\alpha(y_2'-\Delta y))$$

And the amplitude distribution on the x_3-y_3 plane is

$$U_3=\mathcal{F}(U_2)$$

$$\because \quad \mathcal{F}(H(x-\Delta x,y-\Delta y))=h(-x,-y)\exp(-j2\pi(\Delta xX+\Delta yY)) \quad [3]$$

$$\therefore \quad U_3=r_o^2 g(x_3,y_3)+1/\lambda^2 f^2 h(x_3,y_3)\exp(j(2\pi/\lambda f)$$

$$\cdot(\Delta xx_3+ yy_3))*h^*(-x_3,-y_3)\exp(j(2\pi/\lambda f) \quad (18)$$

$$\cdot(\Delta xx_3+\Delta yy_3) \;*g(x_3,y_3)+(r_o/\lambda f)$$

$$\cdot\exp(-j2\pi\alpha\Delta y)\cdot g(x_3,y_3)*h(x_3,y_3)$$

$$\cdot\exp(j(2\pi/\lambda f)(\Delta xx_3+\Delta yy_3))*\delta(x_3,y_3+\alpha\lambda f)$$

$$+(r_o/\lambda f)\exp(j2\pi\alpha\Delta y)g(x_3,y_3)*h^*(-x_3,-y_3)$$

$$\cdot\exp(j(2\pi/\lambda f)(\Delta xx_3+\Delta yy_3))*\delta(x_3,y_3-\alpha\lambda f)$$

Compare Eq. (3) and Eq. (18), we see that the error influency due to Δx and Δy is to replace the $h(x,y)$ in Eq. (3) by $h(x,y)\exp(j(2\pi/\lambda f)(\Delta xx+\Delta yy))$. Because the width in y direction of $h(x,y)\exp(j(2\pi/\lambda f)(\Delta xx+\Delta yy))$ is the same as $h(x,y)$, therefore the fourth term which we are interested can pass the hole completly. Thus the final intensity distribution on the film may be written

$$I(x_4,y_4) \propto |\mathcal{F}((r_o/\lambda f)\exp(j2\pi\alpha\Delta y)g(x_3,y_3)*h^*(-x_3,-y_3)$$

$$\cdot\exp(j(2\pi/\lambda f)(\Delta xx_3+\Delta yy_3)*\delta(x_3,y_3-\alpha\lambda f)$$

$$+r_o'\exp(j2\pi\beta y_4)|^2$$

$$=|(r_o/\lambda^2 f^2)\exp(j2\pi\alpha\Delta y)G(x_4/\lambda f, y_4/\lambda f)H^*$$

$$((x_4-\Delta x)/\lambda f, (y_4-\Delta y)/\lambda f)$$

$$\cdot\exp(-j2\pi\alpha y_4)+r_o'\exp(j2\pi\beta y_4)|^2$$

If in the process of demodulation, the position of the modulated picture is not accurate. Let the error be Δx_r, Δy_r, then the intensity distribution of the modified picture may be written

$$I(x_1', y_1')=|(r_o/\lambda^2 f^2)\exp(j2\pi\alpha\Delta y)G((x_1'-\Delta x_r)/\lambda f,$$

$$\cdot(y_1'-\Delta y_r)/\lambda f)H^*((x_1'-\Delta x_r-\Delta x)/\lambda f,$$

$$\cdot(y_1'-\Delta y_r-\Delta y)/\lambda f)\exp(-j2\pi\alpha(y_1'-\Delta y_r))$$

$$+r_o'\exp(j2\pi\beta(y_1'-\Delta y))|^2$$

The same, in the process of demodulation, the position of the "optical key" may produce error, let the error be Δx_ℓ, Δy_ℓ, then the transmittance of the "optical key" may be written

$$t(x_3', y_3')=t(x_3'-\Delta x_\ell, y_3'-\ y\ )\propto r_o^2$$

$$+1/\lambda^2 f^2|H(x_3'-\Delta x_\ell)/\lambda f, (y_3'\ -\Delta y_\ell)/\lambda f)|^2$$

$$+(r_o/\lambda f)H((x_3'-\Delta x_\ell)/\lambda f,\ (y_3'-\Delta y_\ell)/\lambda f)$$

$$\cdot\exp(j2\pi\alpha(y_3'-\Delta y_\ell))$$

$$+(r_o/\lambda f)H^*((x_3'-\Delta x\ )/\lambda f, (y_3'-\Delta y_\ell)/\lambda f)$$

$$\cdot\exp(-j2\pi\alpha(y_3'-\Delta y_\ell))$$

Therefore in the demodulation process, the amplitude distribution of the field pass through the x_3-y_3 plane may be written

$$U_3(x_3',y_3') \propto I(x_3',y_3')t(x_3',y_3')$$

$$=(|(r_o/\lambda^2f^2)\exp(j2\pi\alpha\Delta y)G((x_3'-\Delta x_r)/\lambda f,$$

$$(y_3'-\Delta y_r)/\lambda f)H^*((x_3'-\Delta x_r-\Delta x)/\lambda f,(y_3'-\Delta y_r-\Delta y)/\lambda f)$$

$$\cdot\exp(-j2\pi\alpha(y_3'-y_r))+r_o'\exp(j2\pi\beta(y_3'-\Delta y)|^2)$$

$$\cdot(r_o^2+1/\lambda^2f^2|H((x_3'-\Delta x_\ell)/\lambda f,(y_3'-\Delta y_\ell)/\lambda f)|^2$$

$$+(r_o/\lambda f)H((x_3'-\Delta x_\ell)/\lambda f,(y_3'-y_\ell)/\lambda f)$$

$$\cdot\exp(j2\pi\alpha(y_3'-y_\ell))+(r_o/\lambda f)H^*((x_3'-x_\ell)/\lambda f,$$

$$(y_3'-\Delta y_\ell)/\lambda f)\exp(-j2\pi\alpha(y_3'-\Delta y_\ell))$$

Thus, the field strength on the x_4-y_4 plane may be wirtten

$$U_4(x_4,y_4)=\mathcal{F}(U_3(x_3',y_3'))$$

Noting that

$$\mathcal{F}(G((x_3'-\Delta x_r)/\lambda f,(y_3'-\Delta y_r)/\lambda f))=g(x_4,y_4)\exp(j(2\pi/\lambda f)\cdot(\Delta x_r x_4+\Delta y_r y_4)) \quad (19)$$

$$\mathcal{F}(H((x_3'-\Delta x_r-\Delta x)/\lambda f,\ (y_3'-\Delta y_r-\Delta y)/\lambda f)) \quad (20)$$

$$=h(x_4,y_4)\exp(j(2\pi/\lambda f)((x_r+x)x_4+(\Delta y_r+\Delta y)y_4))$$

$$\mathcal{F}(\exp(-j2\pi\alpha(y_3'-\Delta y_r))=\exp(j2\pi\alpha\Delta y_r)\delta(x_4,y_4-\alpha\lambda f) \quad (21)$$

$$\mathcal{F}(\exp(j2\pi\beta(y_3'-\Delta y))=\exp(-j2\pi\beta\Delta y)\delta(x_4,y_4+\beta\lambda f) \quad (22)$$

The width in the y direction of Eqs. (19), (20), (21), (22) are the same as the ideal case. Thus the width of the various output terms in the x_4-y_4 plane is the same as Fig. 5, but the information we get in the x_4-y_4 plane will be replace by follows:

$$\text{finanal information}= \mathcal{F}[(r_o/\lambda^2 f^2)\exp(j2\pi\alpha\Delta y)G((x_3'-\Delta x_r)/\lambda f,(y_3'-\Delta y_r)/\lambda f)\cdot H^*((x_3'-\Delta x_r-\Delta x)/\lambda f,(y_3'-\Delta y_r-\Delta y)/\lambda f)\exp(-j2\pi\alpha(y_3'-\Delta y_r))\cdot r_o'\exp(-j2\pi\beta(y_3'-\Delta y)\cdot(r_o/\lambda f)\cdot H(x_3'-\Delta x_\ell)/\lambda f,(y_3'-\Delta y_\ell)/\lambda f)\exp(j2\pi\alpha(y_3'-\Delta y_\ell))] \quad (23)$$

$$= \mathcal{F}[(r_o^2 r_o'/\lambda^3 f^3)\exp(j2\pi\alpha(\Delta y+\Delta y_r-\Delta y_\ell))\cdot\exp(j2\pi\beta\Delta y)\cdot G((x_3'-\Delta x_r)\lambda f,(y_3'-\Delta y_r)/\lambda f)\cdot H^*((x_3'-\Delta x_r-\Delta x)/\lambda f,(y_3'-y_r-\Delta y)/\lambda f)\cdot H(x_3'-\Delta x_\ell)/\lambda f,(y_3'-\Delta y_\ell)/\lambda f)\cdot\exp(-j2\pi\beta y_3')]$$

Noting that:

$$\mathcal{F}(H^*((x_3'-\Delta x_r-\Delta x)/\lambda f,(y_3'-\Delta y_r-\Delta y)/\lambda f))$$

$$=h^*(-x_4,-y_4)\exp(-j(2\pi/\lambda f)(x_4(\Delta x_r+\Delta x)+y_4(\Delta y_r+\Delta y))$$

$$\mathcal{F}(H((x_3'-\Delta x_\ell)/\lambda f,\ (y_3'-\Delta y_\ell)/\lambda f))$$

$$=h(x_4,y_4)\exp(-j(2\pi/\lambda f)(x_4\ x\Delta+y_4\Delta y_\ell))$$

$$\therefore\ \mathcal{F}[H^*((x_3'-\Delta x_r-\Delta x)/\lambda f,(y_3'-\Delta y_r-\Delta y)/\lambda f).H((x_3'$$

$$-\Delta x_\ell)/\lambda f,(y_3'-y_\ell)/\lambda f)] \quad (24)$$

$$=h^*(-x_4,-y_4)\exp(-j(2\pi/\lambda f)(x_4(\Delta x_r+\Delta x)$$

$$+y_4(\Delta y_r+\Delta y)*h(x_4,y_4)\exp(-j(2\pi/\lambda f)(x_4\Delta x_\ell+y_4\Delta y_\ell))$$

$$=\int\int h(\xi,\eta)\exp(-j(2\pi/\lambda f)(\xi\Delta x_\ell+\eta\Delta y_\ell)\cdot h^*(\xi-x,\eta-y)$$

$$\cdot\exp(-j(2\pi/\lambda f)((x-\xi)(\Delta x_r+\Delta x)+(y-\eta)(\Delta y_r+\Delta y)))d\xi d\eta$$

$$=\exp(-j(2\pi/\lambda f)(x(\Delta x_r+\Delta x)+y(\Delta y_r+\Delta y)))\int\int h(\xi,\eta)h^*(\xi$$

$$-x,\eta-y)\exp(-j(2\pi/\lambda f)(\xi(\Delta x_\ell-\Delta x_r-\Delta x)+\eta(\Delta y_\ell-\Delta y_r$$

$$-\Delta y)))d\xi d\eta$$

If we can adjust Δx_ℓ, Δx_r, Δx, Δy_ℓ, Δy_r, Δy such that in the integration range (that is the range of function $h(x,y)$)

$w_x/\lambda f\ (\Delta x_\ell-\Delta x_r-\Delta x)<<1$ where w_x is the maximum width of h in x direction (25)

$w_y/\lambda f\ (\Delta y_\ell-\Delta y_r-\Delta y)<<1$ where w_y is the maximum width of h in y direction (26)

then the exponatial term in the Eq. (24) can be treated as approximate 1, thus Eq. (24) is equal to

$$\exp(-j(2\pi/\lambda f)(x(\Delta x_r+\Delta x)+y(\Delta y_r+\Delta y))\int\int h(\xi,\eta) \quad (27)$$

$$\cdot h^*(\xi-x,\eta-y)d\xi d\eta$$

$$=\exp(-j(2\pi/\lambda f)(x(\Delta x_r+\Delta x)+y(\Delta y_r+\Delta y))h_*h^*$$

$$=\exp(-j(2\pi/\lambda f)(x(\Delta x_r+\Delta x)+y(\Delta y_r+\Delta y))\mathcal{F}(H.H^*)$$

$$=\exp(-j(2\pi/\lambda f)(x(\Delta x_r+\Delta x)+y(\Delta y_r+\Delta y))\delta(x,y)$$

$$=\delta(x,y)$$

that means: If the condition of Eq. (25) and Eq. (26) is satisfied, then $H^*((x_3'-\Delta x_r-\Delta x)/\lambda f,\ (y_3'-\Delta y_r-\Delta y)/\lambda f)\cdot H((x_3'-\Delta x_\ell)/\lambda f,(y_3'-\Delta y_\ell)/\lambda f)$ is approximate constant.

But the spectrum of H and G has cutoff frequency. Let $x_{CH}/\lambda f$, $y_{CH}/\lambda f$ is the cutoff frequency of H, and let $x_{CG}/\lambda f$, $y_{CG}/\lambda f$ is the cutoff frequency of G. If we want to get the undistortional original information, it should be the cutoff frequency of H larger than the cutoff frequency of G,

that is $$x_{CG}/\lambda f < x_{CH}/\lambda f,\quad y_{CG}/\lambda f < y_{CH}/\lambda f \qquad (28)$$

In these condition, Eq. (27) is not equal to a data function, therefore $H^*((x_3'-\Delta x_r-\Delta x)/\lambda f,(y_3'-\Delta y_r-\Delta y)/\lambda f)\cdot H((x_3'-\Delta x_\ell)/\lambda f,(y_3'-\Delta y_\ell)/\lambda f) \simeq \text{constant}\cdot\text{rect}((x_3'-\Delta x_r-\Delta x)/2x_{CH})\cdot\text{rect}((y_3'-\Delta y_r-\Delta y)/2y_{CH})$ then Eq. (23) can be reduced by

$$\text{Constant}\cdot\mathcal{F}(G((x_3'-\Delta x_r)/\lambda f,(y_3'-\Delta y_r)/\lambda f\cdot\text{rect}$$
$$((x_3'-\Delta x_r-\Delta x)/2x_{CH})\cdot\text{rect}((y_3'-\Delta y_r-\Delta y)/2y_{CH}$$
$$\cdot\exp(-j2\pi\beta y_3')) \qquad (29)$$

In order to exame the Eq.(29), we study the spectrum curve of Fig. 7.

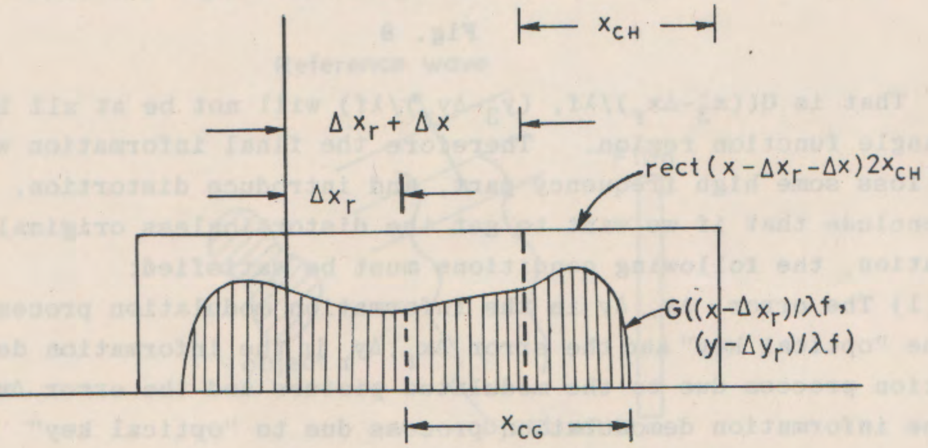

Fig. 7 Spectral density of G and H.

From Fig. 7, we can see that if $|\Delta x| < x_{CH}-x_{CG}$

$|\Delta y| < y_{CH}-y_{CG}$

then $G((x_3'-\Delta x_r)/\lambda f,(y_3'-\Delta y_r)/\lambda f)$ is also in the rectangle function region, therefore Eq. (29) is equal to Constant$\cdot\mathcal{F}(G((x_3'-x_r)/\lambda f,$ $(y_3'-\Delta y_r)/\lambda f)\exp(-j2\pi\beta y_3'))$=Const $\cdot\, g(x_4,y_4)\exp(-j(2\pi/\lambda f)(\Delta x_r x_k+\Delta y_r y_k))$ $*\,\delta(x_4,y_4-\beta\lambda f)$=Const$\cdot g(x_4,y_4-\beta\lambda f)\exp(-j(2\pi/\lambda f)(\Delta x_r x_4+\Delta y_r(y_4-\beta\lambda f))$

(30)

The absolute value of Eq. (30) is the original information we desire.

But if $|\Delta x|\not< x_{CH}-x_{CG}$, or $|\Delta y|\not< y_{CH}-y_{CG}$, then the spectrum curve will be explained in Fig. 8.

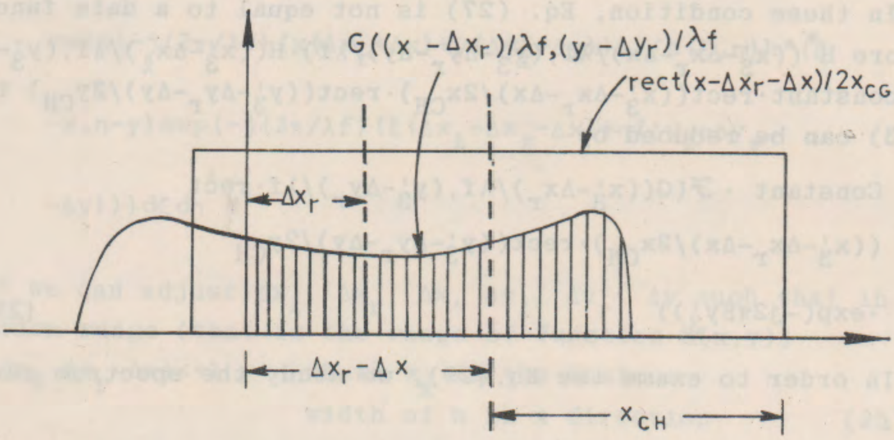

Fig. 8

That is $G((x_3'-\Delta x_r)/\lambda f,\ (y_3'-\Delta y_r)/\lambda f)$ will not be at all in the rectangle function region. Therefore the final information we get will loss some high frequency part, and introduce distortion. Thus we conclude that if we want to get the distortionless original information, the following conditions must be satisfied:

(1) The error Δx, Δy in the information modulation process due to the "optical key" and the error Δx_r, Δy_r in the information demodulation process due to the modulated picture and the error Δx_ℓ, Δy_ℓ in the information demodulation process due to "optical key" must satisfy $|\Delta x_\ell-\Delta x_r-\Delta x| \ll \lambda f/w_x$, $|\Delta y_\ell-\Delta y_r-\Delta y| \ll \lambda f/w_y$.

(2) The cutoff frequency of H must larger than the cutoff frequency of G.

(3) The error $\Delta x, \Delta y$ in the information modulation process due to "optical key" should satisfy $|\Delta x|<x_{CH}-x_{CG}$, $|\Delta y|<y_{CH}-y_{CG}$.

3. The Size of the Modulated Picture

The information store in the modulated picture is given by

$$\cdot I(x_4, y_4) = |\mathcal{F}(r_o/\lambda f) h^*(-x_3, -y_3) * g(x_3, y_3)$$

$$* \delta(x_3, y_3 - \alpha\lambda f) + r_o' \exp(j2\pi\beta y_4)|^2 \qquad (31)$$

$$= |(r_o/\lambda^2 f^2) H^*(-x_4/\lambda f, -y_4/\lambda f)$$

$$\cdot G(x_4/\lambda f, y_4/\lambda f) \exp(-j2\pi\beta y_4)$$

$$+ r_o' \exp(j2\pi\beta y_4)|^2$$

Because the cutoff frequency of G is less than the cutoff frequency of H, therefore the useful size of Eq. (31) is determined by the size $G(x_4/\lambda f,\ y_4/\lambda f)$, the size of $G(x_4/\lambda f, y_4/\lambda f)$ is determined by its cutoff frequency f_{xc}, f_{yc}, therefore the size of G is given by $x_m = \lambda f \cdot f_{xc}$ $y_m = \lambda f \cdot f_{yc}$. Thus the modulated picture has its size approximate to $(\lambda f f_{xc}) x (\lambda f f_{yc})$.

4. Holographic Confidential Picture-Taking Technology

If the desired confidential information is a three dimension picture, we can use the holographic technology as illustrated in Fig. 9 to record the amplitude and phase of the object[4]. Thus if

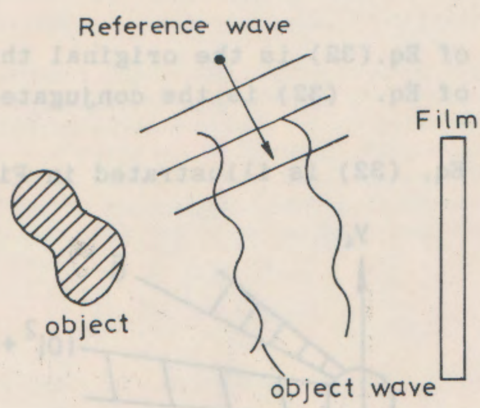

Fig. 9

$$\tilde{O}(x,y) = O(x,y)\exp(-j\phi(x,y))$$

represents the wavefront to be recorded, and

$$\tilde{A}(x,y) = A\exp(-2\pi a y)$$

represents the "reference" wavefront, the intensity distribution of the film is given by

$$I(x,y)=g(x,y)=|O(x,y)\exp(-j\phi(x,y)+A\exp(-2\pi ay)|^2$$

Then we use the interference film $g(x,y)$ as our desired modulated picture, and follows the same process we can modulate the holographic picture. In the demodulation process. In order to get the holographic sense, at x_4-y_4 plane we set a screen, and at the coordinate $(0,\beta\lambda f)$ of x_4-y_4 plane open a hole such that the hole can pass the term $h*h^**g$ and cut off all other terms (the minimum width in y direction of the hole will be $2w_h+w_g$).

Then at x_4-y_4 plane we can get

$$\begin{aligned} g(x_4,y_4+f) &= |O(x_4,y_4+\alpha\lambda f)\exp(-j\phi(x_4,y_4+\alpha\lambda f)) \\ &\quad +A\exp(-j2\pi a(y_4+\alpha\lambda f)|^2 \\ &= |O|^2+A^2+A.O(x_4,y_4+\alpha\lambda f) \\ &\quad .\exp(j\phi(x_4,y_4+\alpha\lambda f))\cdot\exp(-j2\pi a(y_4+\alpha\lambda f)) \\ &\quad +A.O(x_4,y_4+\alpha\lambda f)\exp(-j\phi(x_4,y_4+\alpha\lambda f)) \\ &\quad .\exp(j2\pi a(y_4+\alpha\lambda f)) \end{aligned} \qquad (32)$$

The fourth term of Eq.(32) is the original three dimension object, the third term of Eq. (32) is the conjugate of the original object.

The spectrum of Eq. (32) is illustrated in Fig. 10.

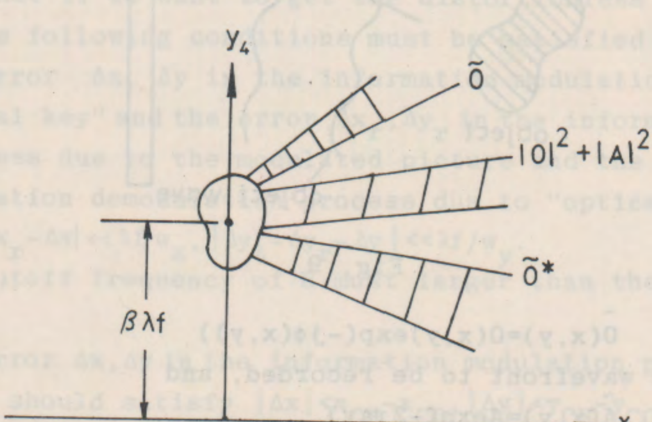

Fig. 10 Spectrum of the three dimension object on the x_4-y_4 plane.

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