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間時共控候伺系統之研究

Discrete Time Common Control Delay Queuing System

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ABSTRACT—The paper presents a detailed study of a discrete time common control delay queuing system. The system considered in the paper has a single server and single common control operator. Customers arrive following a binomial distribution, both the operating time of the common control service and service time of server are independent random processes with general probability distribution. Queue discipline is first-come first-served. The generating functions of the stationary state probabilities of the system on the regeneration points being defined and of the waiting time probabilities are presented. The mean queue length and mean waiting time are also obtained.

I. Introduction

The study of the congestion phenomenon in a crossbar switching system [1] used in modern telephone system, in which a number of common control circuits are employed to control several link devices, formed the subject of the common control queuing system. A common control queuing system differs from the ordinary queuing system in that every customer first goes to the common control operator and then to a server. There is no queue between the common control operator and the server. Another important application of the common control queuing system is the so-called No. 1 ESS system[2] where a digital computer is employed as common central processor. Two different types of common control loss system were studied by Jacobaeus [3] and Rodenburg[4]. While Syski[5] made a systematic and comprehensive presentation of a delay common control queuing system. Recently, Chen and his co-workers have considered a combined delay and loss common control queuing system [6]. All the previous works on common control queuing system are essentially concerned with the continuous time situation in which the Poisson input process and negative exponential operating time of the common control service and service time of the server were employed. However, discrete time

III. The Equations of the System

To describe the system quatitatively, let the random variables, $N_q(t)$, $N_s(t)$, $N_c(t)$ denote the number of customers who are waiting in queue, served by server, served by common control device, respectively, at time t. Since only the single common control device, single server queuing system is considered, both of $N_s(t)$ and $N_c(t)$ are binary random functions. Moreover, it is impossible that boty $N_s(t)$ and $N_c(t)$ are equal to 1 at any time t.

Let

$$P_{oo}(t)=P_{rob}\{N_q(t)=0,\ N_c(t)=0,\ N_s(t)=0\}$$

$$P_i(t)=P_{rob}\{N_q(t)=i,\ N_c(t)=0,\ N_s(t)=1\} \quad i=0,1,2,....$$

$$Q_j(t)=P_{rob}\{N_q(t)=j,\ N_c(t)=1,\ N_s(t)=0\} \quad j=0,1,2,....$$

which are subject, for all time t, to the normalizing condition:

$$P_{oo}(t)+\sum_{i=0}^{\infty} P_i(t)+\sum_{J=0}^{\infty} Q_j(t)=1 \qquad (4)$$

Now, let τ_{ck}, τ_{sk} be the time instants when the service of the operating time, and service time, respectively, of the kth customer is completed. Of course, all the times τ_{ck} and τ_{sk} are on some discrete time instant considered. In fact, these departure epoches $\{(\tau_{ck}-0, \tau_{sk}-0)|k=1,2, ...\}$ constitute a sequence of regeneration points, that is, on these departure epoches, the system states form a Markov chain. Thus, we can employ the imbedded Markov chain method to solve the state probability distribution of the system on these regeneration points. It is clear that, on these regeneration points,

$$P_{oo}(\tau_{ck}-0)=P_i(\tau_{ck}-0)=P_{oo}(\tau_{sk}-0)=Q_i(\tau_{sk}-0)=0$$

for all i and k.

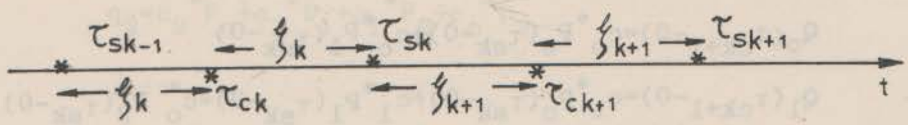
Thus, on these regeneration points, the normalizing condition, equation (4) becomes

$$\sum_{i=0}^{\infty} P_i(\tau_{sk}-0)=1 \qquad (5)$$

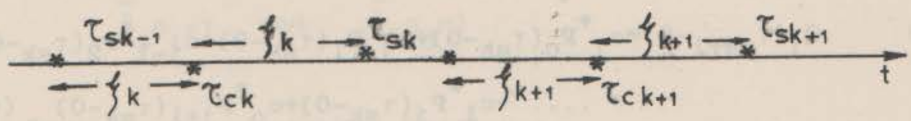
and $$\sum_{j=0}^{\infty} Q_j(\tau_{ck}-0)=1$$

Considering any two successive regeneration points as shown in Figure

2:



(a) The k+1st customer arrives before the departure of the kth customer



(b) The k+1st customer arrives after the departure of the kth customer

Figure 2: Regeneration Points of the Discrete Time Common Control Delay Queuing System

1. If at time $\tau_{ck}-0$, the system is in the state Q_j and during the service time of the kth customer, there are i customer arrivals. Then the system becomes in the state P_{i+j} at time $\tau_{sk}-0$.
2. If at time $\tau_{sk}-0$, the system is in the state P_o and during the operating time of the k+1th customer, there are j customer arrivals. Then the system becomes in the state Q_j at time $\tau_{ck+1}-0$.
3. If at time $\tau_{sk}-0$, the system is in the state P_i with $i \neq 0$ and during the operating time of the k+1th customer, there are j customer arrivals. Then the system becomes in the state Q_{i+j-1} at time $\tau_{ck+1}-0$.

Concluding the only possible transitions of the system states shown in above, we get the state equations of the system:

$$P_o(\tau_{sk}-0)=b_o{}^*Q_o(\tau_{ck}-0)$$

$$P_1(\tau_{sk}-0)=b_1{}^*Q_o(\tau_{ck}-0)+b_o{}^*Q_1(\tau_{ck}-0)$$

$$P_2(\tau_{sk}-0)=b_2{}^*Q_o(\tau_{ck}-0)+b_1{}^*Q_1(\tau_{ck}-0)+b_o{}^*Q_2(\tau_{ck}-0)$$

....

$$P_i(\tau_{sk}-0)=b_i{}^*Q_o(\tau_{ck}-0)+b_{i-1}{}^*Q_1(\tau_{ck}-0)+\ldots+b_1{}^*Q_{i-1}(\tau_{ck}-0)+b_o{}^*Q_i(\tau_{ck}-0)$$

....

and

$$Q_o(\tau_{ck+1}-0)=c_o^* P_o(\tau_{sk}-0)+c_o^* P_1(\tau_{sk}-0)$$

$$Q_1(\tau_{ck+1}-0)=c_1^* P_o(\tau_{sk}-0)+c_1^* P_1(\tau_{sk}-0)+c_o^* P_2(\tau_{sk}-0)$$

$$Q_2(\tau_{ck+1}-0)=c_2^* P_o(\tau_{sk}-0)+c_2^* P_1(\tau_{sk}-0)+c_1^* P_2(\tau_{sk}-0) +c_o^* P_3(\tau_{sk}-0)$$

....

$$Q_j(\tau_{ck+1}-0)=c_j^* P_o(\tau_{sk}-0)+c_j^* P_1(\tau_{sk}-0)+c_{j-1}^* P_2(\tau_{sk}-0) + \dots +c_1^* P_j(\tau_{sk}-0)+c_o^* P_{j+1}(\tau_{sk}-0) \quad (6)$$

....

and these state equations (6) are subject to the normalizing condition equations (5).

Now consider the steady state, since the service times are identical independent random process, the system states $P_i(t)$ and $Q_j(t)$ will be independent of k.

Let

$$p_i=P_i(\tau_{sk}-0)$$

$$q_j=Q_j(\tau_{ck}-0) \qquad \text{for all i, j, and k.}$$

Then the state equations (6) and normalizing condition equations (5) become

$$p_o=b_o^* q_o$$

$$p_1=b_1^* q_o+b_o^* q_1$$

$$p_2=b_2^* q_o+b_1^* q_1+b_o^* q_2 \qquad (7)$$

......

$$p_i=b_i^* q_o+b_{i-1}^* q_1+ \dots +b_1^* q_{i-1}+b_o^* q_i$$

......

and

$$q_o=c_o^* p_o+c_o^* p_1$$

$$q_1=c_1^* p_o+c_1^* p_1+c_o^* p_2$$

$$q_2=c_2^*p_o+c_2^*p_1+c_1^*p_2+c_o^*p_3 \tag{8}$$

.....

$$q_j=c_j^*p_o+c_j^*p_1+c_{j-1}^*p_2+\dots+c_1^*p_j+c_o^*p_{j+1}$$

......

which are subjected to the normalizing conditions:

$$\sum_{i=0}^{\infty}p_i=1 \quad \text{and} \quad \sum_{j=0}^{\infty}q_j=1 \tag{9}$$

IV. The Stationary State Probability Distribution

To solve the stationary state probabilities of the queuing system, let us first define the following generating functions:

$$\Phi_p(x)=\sum_{i=0}^{\infty}p_ix^i$$

$$\Phi_q(x)=\sum_{i=0}^{\infty}q_ix^i$$

$$B(x)=\sum_{i=0}^{\infty}b_ix^i$$

$$C(x)=\sum_{i=0}^{\infty}c_ix^i$$

$$B^*(x)=\sum_{i=0}^{\infty}b_i^*x^i$$

$$C^*(x)=\sum_{i=0}^{\infty}c_i^*x^i$$

It is clear that

$$\Phi_p(1)=\Phi_q(1)=B(1)=C(1)=B^*(1)=C^*(1)=1 \tag{10}$$

Now, multiplying x^i to the ith equation of equations (7) and then summing all the results, we get

$$\Phi_p(x)=B^*(x)\Phi_q(x) \tag{11}$$

Multiplying x^i to the ith equation of equations (8) and then summing all the results, we get

$$\Phi_q(x)=C^*(x)[p_o-\frac{p_o}{x}+\frac{1}{x}\Phi_p(x)] \tag{12}$$

Substituding equation (11) into equation (12), we get

$$\Phi_q(x)=\frac{x-1}{x-B^*(x)C^*(x)}p_oC^*(x) \tag{13}$$

By equation (10), we get

$$\Phi_q(1)=1=\lim_{x\to 1}\frac{x-1}{x-B^*(x)C^*(x)}p_oC^*(x)$$

$$=\frac{p_o}{1-B^{*'}(1)-C^{*'}(1)}$$

or

$$p_o=1-B^{*'}(1)-C^{*'}(1) \tag{14}$$

Substituding equation (2) into the definition of $B^*(x)$, we get

$$B^*(x)=\sum_{i=0}^{\infty} b_ix^i$$

$$=\sum_{i=0}^{\infty}\sum_{m=0}^{\infty}\binom{m}{i}a^i(1-a)^{m-i}b_mx^i$$

$$=\sum_{m=0}^{\infty}\sum_{i=0}^{\infty}\binom{m}{i}(ax)^i(1-a)^{m-i}b_m$$

$$=\sum_{m=0}^{\infty}(ax+(1-a))^mb_m$$

$$=B(ax+(1-a)) \tag{15}$$

By the same procedure, substituding equation (3) into the definition of $C^*(x)$, we get

$$C^*(x)=C(ax+(1-a)) \tag{16}$$

It follows from equations (15) and (16), that

$$\begin{aligned}B^{*'}(x)&=aB'(ax+(1-a))\\ B^{*''}(x)&=a^2B''(ax+(1-a))\\ C^{*'}(x)&=aC'(ax+(1-a))\\ C^{*''}(x)&=a^2C''(ax+(1-a))\end{aligned} \tag{17}$$

By the definitions of B(x) and C(x), it is clear that

$$B'(1)=\frac{1}{\Delta t}E[\zeta]$$

$$C'(1)=\frac{1}{\Delta t}E[\xi]$$

$$B^{*'}(1)=\frac{a}{\Delta t}E[\zeta]$$

$$C^{*'}(1)=\frac{a}{\Delta t}E[\xi]$$

$$B''(1)=\frac{1}{\Delta t^2}E[\zeta(\zeta-\Delta t)] \tag{18}$$

$$C''(1)m=\frac{1}{\Delta t^2}E[\xi(\xi-\Delta t)]$$

$$B^{*''}(1)=\frac{a^2}{\Delta t^2}E[\zeta(\zeta-\Delta t)]$$

$$C^{*''}(1)=\frac{a^2}{\Delta t^2}E[\xi(\xi-\Delta t)]$$

Applying equations (17) and (18) to equation (14), we get

$$p_o=1-\rho$$

where

$$\rho=\lambda(E[\zeta]+E[\xi]) \tag{19}$$

and

$$\lambda=\frac{a}{\Delta t}$$

Substituding equation (19) into equation (13), we get

$$\Phi_q(x)=(1-\rho)\frac{(x-1)C(ax+(1-a))}{x-B(ax+(1-a))C(ax+(1-a))} \tag{20}$$

Substituding equation (20) into equation (11), we get

$$\Phi_p(x)=(1-\rho)\frac{(x-1)C(ax+(1-a))B(ax+(1-a))}{x-B(ax+(1-a)C(ax+(1-a))} \tag{21}$$

Equations (20) and (21) give the explicit forms of the generating functions of the stationary state probabilities on those regeneration points. To get the state probabilities on those regeneration points, one can easily obtain by the formulas:

$$p_k=\frac{1}{k!}\frac{d^k}{dx^k}\Phi_p(x)|_{x=0}$$

$$q_k=\frac{1}{k!}\frac{d^k}{dx^k}\Phi_q(x)|_{x=0}$$

The mean queue length of the system on those regeneration points can be obtained by taking the derivative of $\Phi_q(x)$ and $\Phi_p(x)$ as x equals to 1:

$$L_p = \frac{d}{dx}\,\Phi_p(x)\Big|_{x=1}$$

$$=\rho + \frac{\lambda^2\{E[\zeta(\zeta-\Delta t)]+E[\xi(\xi-\Delta t)]+2E[\zeta]E[\xi]\}}{2(1-\rho)}$$

$$L_q = \frac{d}{dx}\,\Phi_q(x)\Big|_{x=1} \tag{22}$$

$$=\lambda E[\xi] + \frac{\lambda^2\{E[\zeta(\zeta-\Delta t)]+E[\xi(\xi-\Delta t)]+2E[\xi]E[\zeta]\}}{2(1-\rho)}$$

Note that the state probabilities obtained above are only on the re-generation points even in Markovian processes.

V. Waiting Time Probability Distribution Function

To find the waiting time distribution function, let ω be the random variable denoting the waiting time of a certain customer. Also let δ be the random variable denoting the completion time of the customer, that is, the time from his arrival to his departure. It is clear that

$$\delta=\omega+\xi+\zeta \tag{23}$$

Define the waiting time probability distribution

$$\omega_n = P_{rob}\{\omega = n\Delta t\}$$

and the completion time probability distribution

$$d_n = P_{rob}\{\delta = n\Delta t\}$$

Let A_d be the random variable denoting the number of arrivals during the completion time of the customer and define the probability distribution

$$d_n^* = P_{rob}\{A_d = n\}$$

Then it follows from equation (1)

$$d_n^* = \sum_{m=0}^{\infty} P_{rob}\{A_m = n\} P_{rob}\{\delta = m\Delta t\}$$

$$= \sum_{m=0}^{\infty} \binom{m}{n} a^n (1-a)^{m-n} d_n \tag{24}$$

Defining the generating function as follows:

$$\Psi(x)=\sum_{n=0}^{\infty}\omega_n x^n$$

$$D(x)=\sum_{n=0}^{\infty}d_n x^n$$

$$D^*(x)=\sum_{n=0}^{\infty}d_n^* x^n$$

However, the number of arrivals during the completion time of the kth customer is equal to the number of customers waiting in queue at the time instant $\tau_{sk}-0$. Thus we get

$$d_n^*=P_n \qquad D^*(x)=\Phi_p(x) \tag{25}$$

Applying equations (25) and (24) to the definitions of $D(x)$ and $D^*(x)$, we get

$$\Phi_p(x)=D^*(x)=\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\binom{m}{n}a^n(1-a)^{m-n}d_n x^n = D(ax+(1-a)) \tag{26}$$

Rewrite equation (21) as

$$\Phi_p(x)=(1-\rho)\frac{\frac{ax+(1-a)-1}{a}B(ax+(1-a))C(ax+(1-a))}{\frac{ax+(1-a)-(1-a)}{a}-B(ax+(1-a))C(ax+(1-a))} \tag{27}$$

Comparing equations (26) and (27), we get

$$D(x)=(1-\rho)\frac{\frac{x-1}{a}B(x)C(x)}{\frac{x-(1-a)}{a}-B(x)C(x)} = (1-\rho)\frac{(x-1)B(x)C(x)}{(x-1)+a(1-B(x)C(x))} \tag{28}$$

Applying equations (23) to the definition of $D(x)$, we get

$$D(x)=\Psi(x)\,B(x)\,C(x) \tag{29}$$

Substituding equation (28) into equation (29), we get

$$\Psi(x)=\frac{(1-\rho)(x-1)}{(x-1)+a(1-B(x)\,C(x))} \tag{30}$$

Equation (30) gives the explicit form of the generating function of the waiting time probability distribution. One can then obtain the waiting time probability distribution by the formula:

$$\omega_n=(d^n/dx^n)\Psi(x)|_{x=0}$$

The mean waiting time can be obtained by taking the derivative of $\Psi(x)$ as x equals to 1 and multiplied by Δt

$$W=\Delta t\frac{d}{dx}\Psi(x)|_{x=1}$$

$$=\Delta t\frac{a[B''(1)+2B'(1)C'(1)+C''(1)]}{2(1-\rho)}$$

By equation (18), we get

$$W=\frac{\lambda\{E[\zeta(\zeta-\Delta t)]+2E[\zeta]E[\xi]+E[\zeta(\xi-\Delta t)]\}}{2(1-\rho)} \qquad (31)$$

where ρ and λ are defined in equations (19).

VI. Special Cases

The results obtained in equations (19), (20), (21), (22) and (30), (31) may be regarded as an extension of some previous works in the literature:

Case 1. Suppose that the operating times of all customers are equal to zero, that is,

$$c_n=E[\xi]=E[\xi(\xi-\Delta t)]=0 \qquad n>0$$

$$c_o=C(x)=C^*(x)=1$$

Then equations (19), (20), (22) and (30), (31) reduce to

$$p_o=1-\rho$$

$$\rho=\lambda E[\zeta]$$

$$\lambda=\frac{a}{\Delta t}$$

$$\Phi_p(x)=(1-\rho)\frac{(x-1)B(ax+(1-a))}{x-B(ax+(1-a))}$$

$$L_p=\rho+\frac{\lambda^2E[\zeta(\zeta-\Delta t)]}{2(1-\rho)}$$

$$\Phi(x)=\frac{(1-\rho)(x-1)}{(x-1)+a(1-B(x)}$$

$$W=\frac{\lambda E[\zeta(\zeta-\Delta t)]}{2(1-\rho)}$$

These results were reported by Meisling[7] as the solution of ordinary queuing system.

Moreover, it follows from equations (21) and (22) that

$$\Phi_q(x)=(1-\rho)\frac{(x-1)}{x-B(ax+(1-a))}$$

$$L_q(x)=\frac{\lambda^2 E[\zeta(\zeta-\Delta t)]}{2(1-\rho)}$$

These are the generating function of the stationary state probabilities of the system and the mean queue length of the system at the beginning of the service of a certain customer. Note that L_q is less than L_p by ρ which is mean number of customers arrived during the service of a customer, since during the service of a customer, all the arrivals must join into queue.

If we let the service time be zero instead of the operating time, then the system also becomes the ordinary queuing system, the same results will be obtained except that in this case:

$$\Phi_q=\Phi_p$$

$$L_q=L_p$$

Since in this case, τ_{ck} and τ_{sk} are at the same instant.

Case 2. Suppose that both of the operating time and service time have independent geometric distribution, that is,

$$b_n=(1-\mu)\mu^n \qquad n=0,1,2,\ \ldots.$$

$$c_n=(1-\eta)\eta^n$$

It is clear that

$$B(x)=\frac{(1-\mu)}{1-\mu x}$$

$$C(x)=\frac{(1-\eta)}{1-\eta x}$$

$$E[\zeta]=\mu\Delta t/(1-\rho)$$

$$E[\xi]=\eta\Delta t/(1-\rho)$$

$$E[\zeta(\zeta-\Delta t)]=2\mu^2\Delta t^2/(1-\rho)^2=2E[\zeta]^2$$

$$E[\xi(\xi-\Delta t)]=2\eta^2\Delta t^2/(1-\rho)^2=2E[\xi]^2$$

Thus equations (22) and (31) become

$$\rho=\lambda\{E[\zeta]+E[\xi]=\frac{a[\mu+\eta-2\mu\eta]}{(1-\mu)(1-\eta)}$$

$$L_p=\rho+\frac{\lambda^2\{E[\zeta]^2+E[\zeta]E[\xi]+E[\xi]^2\}}{1-\rho}$$

$$=\rho+\frac{a^2\{\frac{\mu^2}{(1-\mu)^2}+\frac{\mu\eta}{(1-\mu)(1-\eta)}+\frac{\eta^2}{(1-\eta)^2}\}}{1-\rho}$$

$$L_q=\lambda E[\xi]+\frac{\lambda^2\{E[\zeta]^2+E[\zeta]E[\xi]+E[\xi]^2\}}{1-\rho} \qquad (32)$$

$$=\frac{a\eta}{1-\eta}+\frac{a^2\{\frac{\mu^2}{(1-\mu)^2}+\frac{\mu\eta}{(1-\mu)(1-\eta)}+\frac{\eta^2}{(1-\eta)^2}\}}{1-\rho}$$

$$W=\frac{\lambda\{E[\zeta]^2+E[\zeta]E[\xi]+E[\xi]^2\}}{(1-\rho)}$$

$$=\frac{a\Delta t\{\frac{\mu^2}{(1-\mu)^2}+\frac{\mu\eta}{(1-\mu)(1-\eta)}+\frac{\eta^2}{(1-\eta)^2}\}}{1-\rho}$$

And equations (20), (21), and (30) become

$$\Phi_q(x)=\frac{(1-\rho)(1-\eta)[1-\mu+a\mu-a\mu x]}{(1-\mu)(1-\eta)-a[\mu(1-\eta)+\eta(1-\mu)+a\mu\eta]x+a^2\mu\eta x^2}$$

$$\Phi_p(x)=\frac{(1-\rho)(1-\mu)(1-\eta)}{(1-\mu)(1-\eta)-a[\mu(1-\eta)+\eta(1-\mu)+a\mu\eta]x+a^2\mu\eta x^2} \qquad (33)$$

$$\Psi(x)=\frac{(1-\rho)(1-\mu x)(1-\eta x)}{1-a(\mu+\eta)+a\mu\eta+[a\mu\eta-(\mu+\eta)]x+\mu\eta x^2}$$

Expanding equations (33) as the infinite series of x, we get

$$p_i=\frac{(1-\rho)}{\alpha_2-\alpha_1}\{\alpha_2^{i+1}-\alpha_1^{i+1}\} \qquad i\geq 0$$

$$q_i=\frac{(1-\rho)}{(1-\mu)(\alpha_2-\alpha_1)}\{(1-\mu+a\mu)(\alpha_2^{i+1}-\alpha_1^{i+1})-a\mu(\alpha_2^{i}-\alpha_1^{i})\}$$

$$i\geq 0$$

$$\omega_i=\frac{1-\rho}{(1-a(\mu+\eta)+a\mu\eta)(\beta_2-\beta_1)}\{(\beta_2^{i+1}-\beta_1^{i+1}) \qquad (34)$$

$$-(\mu+\eta)(\beta_2{}^{i}-\beta_1{}^{i})+\mu\eta(\beta_2{}^{i-1}-\beta_1{}^{i-1})\} \qquad i \geq 1$$

$$\omega_o = \frac{1-\rho}{1-a(\mu+\eta)+a\mu\eta}$$

where α_i are the roots of the characteristic equation:

$$(1-\mu)(1-\eta)\alpha^2-a[\mu(1-\eta)+\eta(1-\mu)+a\mu\eta]\alpha+a^2\mu\eta=0$$

and β_i are the roots of the characteristic equation:

$$[1-a(\mu+\eta)+a\mu\eta]\beta^2+[a\mu\eta-(\mu+\eta)]\beta+\mu\eta=0$$

As $\mu=0$, then

$$\rho = \frac{a\eta}{1-\eta}$$

$$\alpha_1=\beta_1=0$$

$$\alpha_2 = \rho$$

$$\beta_2 = \frac{\eta}{1-a\eta}$$

Thus equations (32) and (34) reduce to

$$L_p=L_q = \frac{\rho}{1-\rho}$$

$$W = \frac{\rho^2}{\lambda(1-\rho)} = \frac{\rho^2}{a(1-\rho)}\Delta t$$

$$p_i=q_i=(1-\rho)\rho^i$$

$$\omega_o = \frac{1-\eta}{1-a\eta}$$

$$\omega_i=(1-\rho)a\left(\frac{\eta}{1-a\eta}\right)^{i+1} \qquad i > 0$$

These are the result of ordinary queuing system with binomial input process and geometric service reported by Meisling [7].

Case 3. Suppose that both the operating time and service time are fixed constants $N\Delta t$ and $M\Delta t$, respectively, that is

$$b_i=0 \qquad i \neq M$$

$$b_M=1$$

$$c_j=0 \qquad j \neq N$$

$$c_N=1$$

It is clear that

$$B(x)=x^M$$

$$C(x)=x^N$$

$$E[\zeta]=M\Delta t$$

$$E[\xi]=N\Delta t$$

$$E[\zeta(\zeta-\Delta t)]=M(M-1)\Delta t^2$$

$$E[\xi(\xi-\Delta t)]=N(N-1)\Delta t^2$$

Thus equations (19), (22) and (31) reduce to

$$p_o=1-\rho$$

$$\rho=a(N+M)$$

$$L_p=\frac{\rho(2-\rho-a)}{2(1-\rho)}$$

$$L_q=\frac{2aN+\rho a[M-N-1]}{2(1-\rho)}$$

$$W=\frac{\rho(M+N-1)}{2(1-\rho)}\Delta t$$

If M=0, then

$$\rho=aN$$

$$L_p=L_q=\frac{\rho(2-\rho-a)}{2(1-\rho)}$$

$$W=\frac{\rho(N-1)}{2(1-\rho)}\Delta t$$

These are the results of ordinary queuing system with binomial input process and constant service time reported by Meisling [7].

Case 4. Suppose that the operating time is constant and the service time is geometric. That is

$$c_i=0 \qquad i\neq N$$

$$c_N=1$$

and

$$b_j=(1-\mu)\mu^j \qquad j=0,1,2,\ \ldots$$

It is clear that

$$B(x)=\frac{1-\mu}{1-\mu x}$$

$$C(x)=x^N$$

$$E[\zeta]=\frac{\mu\Delta t}{1-\mu}$$

$$E[\xi]=N\Delta t$$

$$E[\zeta(\zeta-\Delta t)]=2\mu^2\Delta t^2/(1-\mu)^2=2E[\zeta]^2$$

$$E[\xi(\xi-\Delta t)]=N(N-1)\Delta t^2$$

Thus equations (19), (22) and (31) reduce to

$$p_o=1-\rho$$

$$\rho=a(N+\frac{\mu}{1-\mu})$$

$$L_p=\rho+\frac{a^2\{2E[\zeta]^2+2NE[\zeta]+N(N-1)\}}{2(1-\rho)}$$

$$P_q=aN+\frac{a^2\{2E[\zeta]^2+2NE[\zeta]+N(N-1)\}}{2(1-\rho)}$$

$$W=\frac{a\{2E[\zeta]^2+2NE[\zeta]+N(N-1)\}}{2(1-\rho)}$$

Case 5. Suppose that the operating time is geometric and service time is constant $M\Delta t$, that is

$$b_i=0 \qquad i \neq M$$

$$b_M=1$$

and

$$c_j=(1-\eta)\eta^j \qquad j=0,1,2,\ldots$$

It is clear that

$$B(x)=x^M$$

$$C(x)=\frac{1-\eta}{1-\eta x}$$

$$E[\zeta]=M\Delta t$$

$$E[\xi]=\frac{\eta}{1-\eta}\Delta t$$

$$E[\zeta(\zeta-\Delta t)]=M(M-1)\Delta t^2$$

$$E[\xi(\xi-\Delta t)]=2\eta^2\Delta t^2/(1-\eta)^2=2E[\xi]^2$$

Thus equations (19), (22) and (31) reduce to

$$p_o=1-\rho$$

$$\rho=a(\frac{\eta}{1-\eta}+M)$$

$$L_p=\rho+\frac{a^2\{2E[\xi]^2+2ME[\xi]+M(M-1)\}}{2(1-\rho)}$$

$$L_q=aM+\frac{a^2\{2E[\xi]^2+2ME[\xi]+M(M-1)\}}{2(1-\rho)}$$

$$W=\frac{a\{2E[\xi]^2+2ME[\xi]+M(M-1)\}}{2(1-\rho)}\Delta t$$

VII. Conclusion and Suggestions for Further Research

A discrete time common control delay queuing system with binomial distribution input, first-come first-served queue discipline, general distributions of both operating time and service time, single common control operator and single server is studied. The imbedded Markov chain technique is generalized to apply a set of regeneration points in which different types of events were included for this study. It is shown that some results of the mean queue length and mean waiting time reported in an ordinary queuing system can be obtained as special cases from that obtained in this paper.

For the single common control single server discrete time delay systems, the studies of the system with general input process, the system with priority queue have not yet been explored. Common control queuing systems with multiple servers for both continue time and discrete time situations are also deserved for further investigation.

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List of symbols

t_k	= discrete time epoch
Δt	= (t_k, t_{k+1}), small fixed time interval
a	= probability of an arrival in the system at each t_i
ξ	= random variable denoting the operating time of the common control service
ζ	= random variable denoting the service time of the server

$\{b_m\}$ = service time distribution

$\{c_m\}$ = operating time distribution

A_1 = random variable denoting the number of customers arrived in the system in a semi-open time interval of length $1\Delta t$

A_s = random variable denoting the number of customers arrived in the system during the service time of a certain customer

A_c = random variable denoting the number of customers arrived in the system during the operating time of a certain customer

$\{b_n^*\}$ = probability distribution of A_s

$\{c_n^*\}$ = probability distribution of A_c

$N_q(t)$ = random variable denoting the number of waiting customers in the queue at time t

$N_c(t)$ = random variable denoting the state of common control operator at time t

$N_s(t)$ = random variable denoting the state of server at time t

$P_{oo}(t)$ = $P_{rob}\{N_q(t)=0,\ N_c(t)=0,\ N_s(t)=0\}$

$P_i(t)$ = $P_{rob}\{N_q(t)=i,\ N_c(t)=0,\ N_s(t)=1\}$

$Q_j(t)$ = $P_{rob}\{N_q(t)=j,\ N_c(t)=1,\ N_s(t)=0\}$

τ_{ck} = The time instant when the service of common control the operating of the kth customer is completed

τ_{sk} = The time instant when the service of server of the kth customer is completed. This is the departure time of the kth customer.

$\{(\tau_{ck}-0,\ \tau_{sk}-0)\,|\,k=1,2,\ \ldots.\}$: regeneration points

p_i = $P_i(\tau_{sk}-0)$

q_j = $Q_j(\tau_{ck}-0)$

$\Phi_p(x)$ = $\sum_{i=0}^{\infty} p_i x^i$

$\Phi_q(x)$ = $\sum_{i=0}^{\infty} q_i x^i$

$B(x)$ = $\sum_{i=0}^{\infty} b_i x^i$

$C(x) = \sum_{i=0}^{\infty} c_i x^i$

$B^*(x) = \sum_{i=0}^{\infty} b_i{}^* x^i$

$C^*(x) = \sum_{i=0}^{\infty} c_i{}^* x^i$

$\rho = 1-\lambda(E[\xi]+E[\zeta])$

$\lambda = \frac{a}{\Delta t}$

L_p = mean queue length on the departure time τ_{sk}

L_q = mean queue length on the time τ_{ck}

ω = random variable denoting the waiting time

δ = $\omega+\xi+\zeta$, random variable denoting the completion time

A_d = random variable denoting the number of arrivals during the completion time of a certain customer

$\{\omega_n\}$ = probability distribution of waiting time ω

$\{d_n\}$ = probability distribution of δ

$\{d_n{}^*\}$ = probability distribution of A_d

$\Psi(x) = \sum_{n=0}^{\infty} \omega_n x^n$

$D(x) = \sum_{n=0}^{\infty} d_n x^n$

$D^*(x) = \sum_{n=0}^{\infty} d_n{}^* x^n$

W = mean waiting time

$1-\eta$ = probability of the completion of operating time of the customer being served by common control operator at each t_i

$1-\mu$ = probability of the completion of service time of the customer being served by server at each t_i

$N\Delta t$ = constant operating time

$M\Delta t$ = constant service time

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印製中文表格之新語言
A New Language for Tabulation–CHITAL

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ABSTRACT—This paper defines a new kind o
is called CHITAL — CHInese table TAbula
purpose of the CHITAL is used for descri
Chinese characters. The purpose of this
that a fairly complicate table and even a
cribed. In addition to defining the langu
bes the implementation of an interprete
illustrated results.

I. Introduction

Three main problems exist in the de
processing system. They are (1) input o
data processing of Chinese characters, and
racters. There are two alternative appro
The first method uses a graphic-display
characters. This device can be a storage
tronix 4010) or a non-storage display CR
Using a graphic-display device, a hard c
by taking picture directly from the scr
print' hard copy machine. Alternatively,
used to plot the Chinese characters. A sui
this job is the Versatec Matrix electrost
quality of the electrostatic plotter is be
phic-display device. In this paper, a Vers
will be chosen as output device for Chine
has a nib density of 1600 dots with 160 d

Usually, the final product of a Chin
system is in the form of a table. The C
arranged inside the table. Between these
rizontal or vertical lines of various wid
ches to generate a table. The first appro