# 時序機器是否線性之測試

# On Testing the Linearity of Sequential Machines

張祖玄 Tsu-Shuan Chang

Department of Electronics Engineering

(Received August 3, 1976)

ABSTRACT — A new method for testing whether a given sequential machine is linear is presented. This test algorithm is different from any existing identification method in that it is applied before the characterizing matrices having been found, hence a tedious calculation can be avoided.

Linear sequential machine is important in its applications in computer control circuitry, digital communication systems, etc.. Hence identification of linear machine is an important topic. Several tests in the identification procedure is used to test the linearity and the characterizing matrices are found after a tedious calculation during the procedure [1]. But the final test is to verify the found equations indeed represent the given machine under all input and state combinations. All the calculations are useless if the final test is failed. Such case is avoided if the presented method is used, because linearity is tested before the other existing method being used to find the charactizing matrices.

The response of a linear sequential machine to an arbitrary input sequence can be found from its impulse response by superposition principle. In other words, if the response of any given input sequence can be determined from its impulse response, then the machine must be linear. If the given machine is nonlinear, then an input-output pair may be found by intuition and only a single test is required just as in example 1. If the given machine is linear, then the test must include all the input-output pairs and it is impossible of course. But if we can find an input-output pair which uniquely defines a given machine, then only a single test is required for any given machine. Hence a checking sequence which uniquely defines a given machine is just fitted for this purpose. Because any sequence with length k is a distinguishing sequence [1], hence a checking sequence of a given machine does exist [2]. In this presented testing procedure, the checking sequence is used to test the linearity of a given machine.

Testing Procedure:

(1) Construct a distinguishing table as in Reference 1;

(2) Find the impulse response from the given machine

(3) Find a checking sequence of the given machine by the method in Reference 2

(4) Determine the response of the machine due to the input of the found

checking sequence in step 3 from the impulse response in step 2 by superposition principle

(5) Compare the output sequence found in step 4 with the output of the found checking sequence in step 3. If they are the same, then the machine is linear.

*Example 1:* (The given machine $M_1$ is taken from Reference 2.)

Choose C as the initial state, then it's an inert machine. The impulse response is 10000...... If the machine is linear, then the output due to 101 must be 101. But the true output sequence is 100 from the given table, hence it's nonlinear. By intuition, nonlinearity can be determined by only a single test.

*Example 2:* (Machine $M_2$ is taken from Reference 1.)

(1) Choose D as the initial state, then it's an inert machine with the impulse response 101010......

A checking sequence found by the method in Reference 2 is given below:

Input 000100001000100100100

Output 000101011111010000101

The same input-output pair can be determined from the impulse response by superposition principle, hence it's linear.

(2) If the initial state is A, then it's not an inert machine. The response to zero input sequence is 101010.... and the response to impulse input is 00000......, hence the true impulse response is 01010101...... A checking sequence is shown;

Input 0000100010010001001100

Output 1010000010111110100101

The same output can be determined from the true impulse response and zero input response for the input of this checking sequence. Hence the given machine is linear.

From this example, we can observe the choice of the initial state is not important, hence one can choose the initial state for his convenience. After determining the given machine is linear, then the characterizing matrices can be found by any existing method.

*Example 3:* ($M_3$ is an N-input machine with N=2.)

Choose A as the initial state, then it's an inert machine.

(1) Construct a distinguishing table

| | A | B | C | D |
|---|---|---|---|---|
| Z(0) | 0 | 1 | 0 | 1 |
| Z(1) | 0 | 1 | 1 | 0 |

(2) Let $Z_1$ be the impulse response due to $X_1$,

i.e. $X_1$ 100000......

$X_2$ 000000......

then the output sequence is

$Z_1$ 10101010......

Similarly, we can find $Z_2$ by setting $X_1$ to zero sequence and let $X_2$ be an impulse, then $Z_2$ 01111111....

(3) By the method in Reference 2, we can find a checking sequence

$X_1$ 00000 01000 00000 10010 00001 00000 01001 00100 10001 00

$X_2$ 00010 01000 00100 00000 01000 00010 01001 00100 10010 00

Z 00001 10010 10110 00010 10011 11110 01100 11001 10000 10

(4) The same output sequence Z due to the input sequence $X_1$ & $X_2$ in (3) can be found by using the impulse response found in (2), applying the superposition principle. Therefore, the machine is linear. Indeed, the given machine can be expressed as follows;

$$Y(t)=\begin{bmatrix}1&0\\1&1\end{bmatrix}y(t)+\begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}X_1\\X_2\end{bmatrix}$$ with four internal states

$$Z(t)=\begin{bmatrix}0&1\end{bmatrix}y(t)+\begin{bmatrix}1&0\end{bmatrix}\begin{bmatrix}X_1\\X_2\end{bmatrix}\quad A=\begin{bmatrix}0\\0\end{bmatrix}\ B=\begin{bmatrix}1\\1\end{bmatrix}\ C=\begin{bmatrix}0\\1\end{bmatrix}\ D=\begin{bmatrix}1\\0\end{bmatrix}$$

In this example, we show that the same procedure can be applied to multi-input machine.

The basic idea in this method is to put all state transitions and outputs in one input-output pair, hence only a single test is required. Since the test procedure is simple and regular, hence it's suitable for computer's work. Furthermore, the linearity is determined first, hence useless calculation can be avoided.

| PS | NS,Z 0 | 1 |
|---|---|---|
| A | A,1 | B,0 |
| B | C,0 | A,0 |
| C | B,0 | C,1 |

Machine $M_1$

| PS | NS,Z 0 | 1 |
|---|---|---|
| A | B,1 | D,0 |
| B | A,0 | C,1 |
| C | C,1 | A,0 |
| D | D,0 | B,1 |

Machine $M_2$

| PS | NS,Z $X_1X_2$ 0 0 | 0 1 | 1 0 | 1 1 |
|---|---|---|---|---|
| A | A,0 | B,0 | C,1 | D,1 |
| B | B,1 | A,1 | D,0 | C,0 |
| C | D,0 | C,0 | B,1 | A,1 |
| D | C,1 | D,1 | A,0 | B,0 |

Machine $M_3$

**Acknowledgment**

This article is under the direction of Prof. Chi-Tsong Chen. The author especially wish to thank Prof. Min-Wen Du for his helpful discussions and encouragement.

## Appendix Superposition principle

An example is given here to explain the superposition principle. For the same machine used in example 2, case 1

the impulse response response is 101010........

If the input is given as follows:

```
                 1st impulse
                 :    2nd impulse
                 :    :
Input      0001000001000100100100
                 :    :    : : 101 --responding sequence due to the 5th impulse
                 :    :    : 101010
                 :    :    101010101 --responding sequence due to the 3rd impulse
                 :    1010101010101
                 101010101010101010 --responding sequence due to the 1st impulse
Output     0001010111110100000101
```

By adding the five responding sequences, we find the output sequence. It is the same as the output sequence in the checking sequence.

## References

1. Zvi Kohavi: "Switching and Finite Automata Theory", New York, McGraw-Hill, 1970, pp. 530, Computer Science Series.
2. F. C. Hennie : "Fault Dectecting Experiments for Sequential Circuits" Proc. Fifth Ann. Symp. Switching Circuit Theory and Logical Design, Princeton, N. J., November, 1964. pp 95-110.

# 最少無静態障礙組合邏輯線路之計算機傾向合成法

# A Note on Computer Oriented Synthesis of Minimum Static Hazard-Free Combinational Logic CKT

陳 正 Cheng Chen

Department of Computer Science, N.C.T.U.

(Received August 12, 1976)