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論電阻條片行列與平面波之交互作用——入射磁場垂直於諸條片與入射面

Interaction of an Infinite Array of Resistive Stripes and the Plane Wave with the Magnetic Field Perpendicular to the Stripes and the Plane of Incidence

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Abstract —— A normalized impedance $\hat{z}_s$ is obtained for an array of resistive stripes illuminated obliquely by a plane waves having its magnetic vector normal to the plane of incidence and perpendicular to the stripes. It is found that $\hat{z}_s$ is composed of a partial impedance $\hat{z}_{s1}$ similar to the perfectly conducting striped-grid plus another partial impedance $\hat{z}_{s2}$ which is account for the effect of the stripe resistance. For the case of interest, $s << \lambda$, where s is the stripe spacing, it is found that $\hat{z}_{s2}$ is linearly proportional to the stripe resistance, the aspect ratio and is nearly indepent of frequency in contrast to $\hat{z}_{s1}$ which is proportional to s/λ.

I. Introduction

The interaction of plane electromagnetic waves and a plane grid of perfectly conducting stripes have drawn attentions of many investigators from time to time in the past for various reasons, for example this configuration is one of those few amenable for analytical solution [1], Still more important it has been found that such a structure is useful in antanna and other application in microwave engineering [2]. While there are large volume of literatures on the perfectly conducting strip grating [3,4,5], it appears to be less information available regarding to the electrical properties of a similar structure of finite conductivity. Recently it was noticed that the latter configuration may find application in waveguide components and waveguide transmission [6,7]. The purpose of this paper is to analyze the electrical behavior of that structure when the incident wave is polarized in the plane of incidence as shown in Fig. 1. It is noted that the intersection of the plane of incidence and the plane of stripes coincides with the axis of a stripes. For convenience of analysis we number this stripe as the zeroth stripe. The adjacent stripes are numbered in order. In this paper we consider that the stripes are uniformly spaced and are identical in width. s and d are the spacing between neighboring stripe and stripe width, respectively. θ_i is the angle of incidence.

The so-called resistive stripes can be made of metal film depositing on a dielectric substrate. The film thickness are of the order of a few hundredth of a micrometer, hence one may ignore that thickness and characterizes the film with a film resistance R_f [8]. A principle goal of this paper is to investigate the effects of R_f on the electrical properties of the thin film metal stripes.

Since the metal film is supported by a dielectric substrate, hence one needs to include the effects of the dielectric layer in obtaining the electrical behaviors of the array of resistive stripes. However for limiting the scope of the present paper, we confine our attention to the stripes themselves and leave the effects of the dielectric substrate to the subsequent report.

II. Currents in the Stripes and the Associated Electromagnetic Field

In this section we aim to find the field components associated with the currents in the stripes of finite conductivity. One notices from Fig. 1 that the plane of stripes coincides with the plane x=0, hence the current density in any stripe can be resolved into two parts, namely $J_{nz}\hat{z}$ and $J_{ny}\hat{y}$ where subscript n denotes that the current densities belong to the nth stripe. $\hat{y}$ and $\hat{z}$ indicates the unit vectors in y and z direction, respectively. The field components associated with those current densities can be obtain via the vector potential $\vec{A}$ [9] as

$$\vec{E} = -j\omega\mu\vec{A} + \frac{1}{j\omega\epsilon}\nabla(\nabla\cdot\vec{A}) \tag{1}$$

$$\vec{H} = \nabla \times \vec{A} \tag{2}$$

In reference to Fig. 1 the incident plane wave can be expressed as

$$E_z^i = E_o \cos\theta_i e^{jk_o x\cos\theta_i} e^{-jk_o z\sin\theta_i} \tag{3}$$

$$E_x^i = \tan\theta_i E_z^i \tag{4}$$

$$H_y^i = E_z^i/\eta\cos\theta_i \tag{5}$$

where E_o, θ_i, k_o, η are the amplitude of the incident wave, the angle of incident, the free space wave number and the free space intrinsic impedance, respectively. As the incoming wave impinging upon the resistive stripes, the current densities J_{ny} and J_{nz} are induced. Hence we may anticipate that those current densities have the identical z dependence as (3) to (5). The symmetries of the array configuration and the plane wave excitation imply that those current densities are identical in every stripes, i.e.,

$$J_{nz} = j_z(y)e^{-jk_o z\sin\theta_i} \tag{6}$$

$$J_{ny} = j_y(y)e^{-jk_o z\sin\theta_i} \tag{7}$$

where y is in $ns-d/2 \le y \le ns + d/2$ and J_{nz} and J_{ny} vanish outside the stripes. For convenience of analysis we introduce the local coordinates h defined as

$$h = y - ns \qquad -d/2 \le h \le d/2 \tag{8}$$

To obtain the reflected field and the transmitted field we turn our attention to the axial current density $j_z(h)$ in nth stripe. Considering an element strip of width dh the differential vector potential $d\vec{A} = \hat{z}dA_z$ is [10]

$$dA_z = \frac{j_z(h)dh}{4j} e^{-jk_o z\sin\theta_i} H_o^{(2)}[k_o\cos\theta_i \sqrt{(ns+h-y)^2+x^2}] \tag{9}$$

where $H_o^{(2)}[\quad]$ is the zeroth order Hankel function of second kind. The potential A_z due to current density $j_z(h)$ in all stripes can be obtain as

$$A_z = \frac{1}{4j} \sum_{n=-\infty}^{n=\infty} \int_{-d/2}^{d/2} j_z(h)dh H_o^{(2)}\{k_o\cos\theta_i[(ns+h-y)^2 + x^2]^{1/2}\} \cdot e^{-jk_o z\sin\theta_i} \tag{10}$$

Interchanging the order of integration and summation and making use of the relation

$$\sum_{n=-\infty}^{n=\infty} H_o^{(2)}[k_o\cos\theta_i \sqrt{(ns+h-y)^2+x^2}]$$

$$= \frac{2}{k_o s \cos\theta_i} e^{\mp jk_o x\cos\theta_i} + \frac{4j}{s} \sum_{m=1}^{\infty} \frac{e^{\mp q_m x}}{q_m} \cos\frac{2\pi m}{s}(y-h) \tag{11}$$

where $$q_m = \sqrt{\left(\frac{2\pi m}{s}\right)^2 - k_o^2\cos^2\theta_i} \tag{12}$$

we have

$$A_z = \frac{1}{j2k_o s\cos\theta_i} e^{\mp jk_o x\cos\theta_i} \int_{-d/2}^{d/2} j_z(h)dh$$

$$+ \frac{1}{s}\sum_{m=1}^{\infty} q_m^{-1} e^{\mp q_m x} \int_{-d/2}^{d/2} j_z(h)\cos\frac{2\pi mh}{s}(y-h)dh \tag{13}$$

The relation (11) can be derived from well known Poisson's formula as indicated in [11]. The z-directed and y-directed electric components due to $j_z(h)$ can be attained from (13) and (1) as

$$e_{zz} = -\frac{\eta\cos\theta_i}{2s} e^{\mp jk_o x\cos\theta_i} \int_{-d/2}^{d/2} j_z(h')dh'$$

$$-\frac{j\omega\mu\cos^2\theta_i}{s} \sum_{m=1}^{\infty} q_m^{-1} e^{\mp q_m x} \int_{-d/2}^{d/2} j_z(h')\cos\frac{2\pi m}{s}(y-h')dh' \tag{14}$$

$$e_{yz} = \frac{\eta\sin\theta_i}{s} \sum_{m=1}^{\infty} \frac{2\pi m}{sq_m} e^{\mp q_m x} \int_{-d/2}^{d/2} j_z(h')\sin\frac{2\pi m}{s}(y-h')dh' \tag{15}$$

respectively. We note that e_{yz} and e_{zz} indicate the component electric field of the waves emanated from the stripes, hence the upper and lower sign of ($\mp$) seen in (14) and (15) will be employed for the region $x > 0$ and $x < 0$ respectively. In this paper we aim to consider the striped-grid as an impedance sheet which has the property of macroscopically uniformity. To achieve that purpose one chooses $s \ll \lambda$ and in turn q_m can be approximated as $2\pi m/s$. Apparently in (14) and (15) the first term of (14) represents the only propagating waves. Comparing with (3) to (15) one may regard that term as part of the reflected and the transmitted waves in the $x > 0$ and $x < 0$ half space, respectively. The partial reflected ratio due to $j_z(h)$ in the stripe-grid can be obtained readily as

$$\rho_L = -\frac{\eta}{2sE_o} \int_{-d/2}^{d/2} j_z(h')dh' \tag{16}$$

The presence of e_{yz} will cause a current component $j_y(h)$ in the stripes. Similar to the previous analysis we can also obtain a vector potential A_y and using (1) we arrive at the component electric field due to $j_y(h)$ as

$$e_{zy} = \frac{\eta\sin\theta_i}{s} \sum_{m=1}^{\infty} \frac{2\pi m}{sq_m} e^{\mp q_m x} \int_{-d/2}^{d/2} j_y(h')\sin\frac{2\pi m}{s}(y-h')dh' \tag{17}$$

and

$$e_{yy} = -\frac{\eta}{2\sin\theta_i} e^{\mp jk_o x\cos\theta_i} \int_{-d/2}^{d/2} j_y(h')dh'$$

$$-\frac{j\omega\mu}{s} \sum_{m=1}^{\infty} q_m^{-1}\left[1-\left(\frac{2\pi m}{k_o s}\right)^2\right] e^{\mp q_m x} \int_{-d/2}^{d/2} j_y(h')\cos\frac{2\pi m}{s}(y-h')dh' \tag{18}$$

with $e^{-jk_o z\sin\theta_i}$ delected for shortening the equations. It is of interests to note from (18) that a depolorized plane wave propagating outward from the stripes can be excited if the total y-directed current in a stripe is of non-vanishing value.

III. Coupled Integral Equations for the Current Densities $j_z(h)$ and $j_y(h)$

In the forgoing section we indicated that there are three types of the electromagnetic fields surrounding the array of resistive stripes, namely the incident field, the fields due to the longitudinal current density $j_z(h)$ and the cross-stripe current density $j_y(h)$. Further on the plane of the stripes the total electric fields components tangentical to that plane must satisfy the following boundary conditions

$$E_z^i(y,h') + e_{zz}(y,h') + e_{zy}(y,h') = j_z(y)R_f \tag{19}$$

$$e_{yz}(h) + e_{yy}(h) = j_y(h)R_f \tag{20}$$

for x=0, $-d/2 \le h' \le d/2$, $ns-d/2 \le y \le ns+d/2$. We insert (3), (14) and (17) in (19) and arrive at

$$E_o\cos\theta_i - \frac{\eta\cos\theta_i}{2s}\int_{-d/2}^{d/2} j_z(h')dh'$$

$$-\frac{j\omega\mu\cos^2\theta_i}{s}\sum_{m=1}^{\infty} q_m^{-1}\int_{-d/2}^{d/2} j_z(h')\cos\frac{2\pi m}{h}(y-h')dh'$$

$$+\frac{\eta\sin\theta_i}{s}\sum_{m=1}^{\infty}\frac{2\pi m}{sq_m}\int_{-d/2}^{d/2} j_y(h')\sin\frac{2\pi m}{s}(y-h')dh'$$

$$= R_f j_z(y). \tag{21}$$

Similarly one substitutes (15) and (18) into (20) and attains

$$\frac{\eta\sin\theta_i}{s}\sum_{m=1}^{\infty}\frac{2\pi m}{sq_m}\int_{-d/2}^{d/2} j_z(h')\sin\frac{2\pi m}{s}(y-h')dh'$$

$$-\frac{\eta}{2s\cos\theta_i}\int_{-d/2}^{d/2} j_y(h')dh'$$

$$-\frac{j\omega\mu}{s}\sum_{m=1}^{\infty} q_m^{-1}\left[1-\left(\frac{2\pi m}{k_o s}\right)^2\right]\int_{-d/2}^{d/2} j_y(h')\cos\frac{2\pi m}{s}(y-h')dh'$$

$$= R_f j_y(y). \tag{22}$$

Eqs. (21) and (22) are the coupled integral equations for the problem outlined in Fig. 1. In the last pair of the integral equations, y indicates a point of any stripe in the array , we focus our attention to the nth stripe. On account of the symmetry between the in-coming plane wave and the array configuration one anticipates that $j_z(h)$ is symmetric with respect to the center line of that stripe as seen in Fig. 2(a). Upon substituting the local coordinate h defined in (8) for the nth stripe in (21) one observes that $j_y(h)$ is anti-symmetric with respect to h. Taking advantages of the above observation the coupled integral equations (21) and (22) can be reduced to the form

$$E_o\cos\theta_i - \frac{\eta\cos\theta_i}{2s}\int_{-d/2}^{d/2} j_z(h')dh'$$

$$-\frac{j\omega\mu\cos^2\theta_i}{s}\sum_{m=1}^{\infty} q_m^{-1}\cos\frac{2\pi mh}{s}\int_{-d/2}^{d/2} j_z(h')\cos\frac{2\pi mh'}{s}\,dh'$$

$$-\frac{\eta\sin\theta_i}{s}\sum_{m=1}^{\infty}\frac{2\pi m}{sq_m}\cos\frac{2\pi m}{s}h\int_{-d/2}^{d/2} j_y(h')\sin\frac{2\pi mh'}{s}\,dh'$$

$$= R_f\, j_z(h) \tag{23}$$

$$\frac{\eta\sin\theta_i}{s}\sum_{m=1}^{\infty}\frac{2\pi m}{sq_m}\sin\frac{2\pi mh}{s}\int_{-d/2}^{d/2} j_z(h')\cos\frac{2\pi mh'}{s}\,dh'$$

$$-j\frac{\omega\mu}{s}\sum_{m=1}^{\infty} q_m^{-1}\left(1-\frac{4\pi^2m^2}{k_o^2 s^2}\right)\sin\frac{2\pi mh}{s}\int_{-d/2}^{d/2} j_y(h')\sin\frac{2\pi mh'}{s}\,dh'$$

$$= R_f\, j_y(h) \tag{24}$$

where $\quad -d/2 \leq h, h' \leq d/2$

Exact solutions to the above integral equations are difficult to attain. For engineering application one may choose a certain solution forms according to the above-mentioned properties of $j_z(h)$ and $j_y(h)$ such as

$$j_z(h) \cong A_o + \sum_{m=1}^{M} A_m\cos\frac{2\pi m}{s}h, \tag{25}$$

$$j_y(h) \cong \sum_{m=1}^{M} B_m\sin\frac{2\pi m}{s}h, \tag{26}$$

and the unknown coefficients A_n and B_n can be determined using the well known Galerkin's method [12]. It is noted that the above choice of solution forms are in line with the field behaviors near the edge of a dielectric wedge [13].

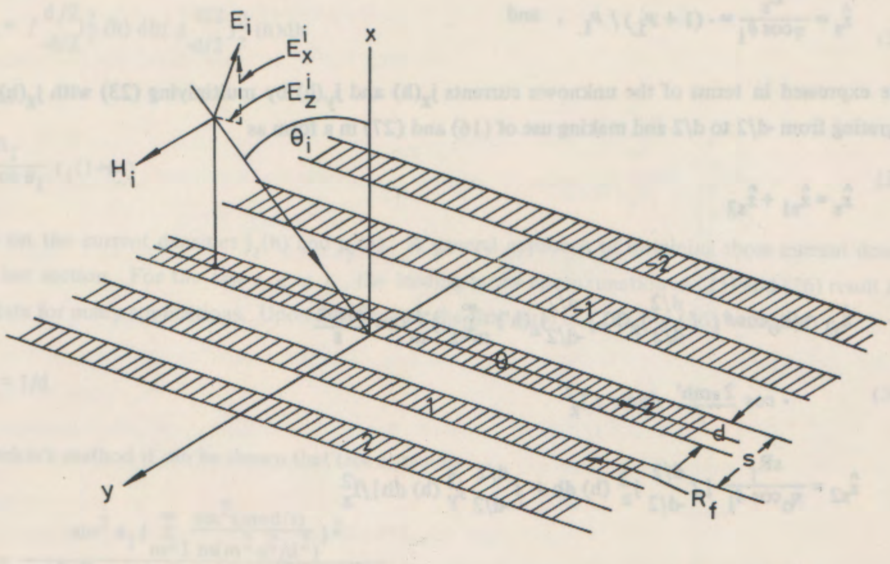

Fig. 1 Interaction of plane wave and the array of resistive stripes.

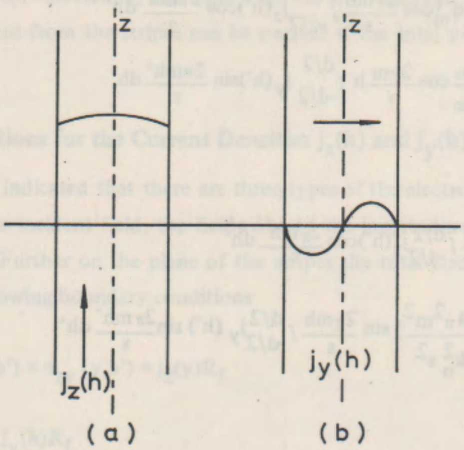

Fig. 2 Current distributions in the nth stripe.

IV. Impedance of the Array of Resistive Stripe

In last section we obtain a pair of coupled integral equations for the unknown stripe currents $j_z(h)$ and $j_y(h)$. A method of approach for an approximate solutions is also indicated. However we will not pursue any further on the currents, rather we choose to study the electrical behaviors via an impedance representation similar to the wave-guide obstacle problems [14]. In reference to Fig. 1 we choose to represent the x-component of the incident wave with a transmission line of characteristic impedance $\eta_o \cos\theta_i$ and propagation constant $k_o \cos\theta_i$. The presence of the array of resistive stripes causes a reflected wave with the reflection coefficient ρ_L defined in (16). Thus electrically the striped-grid can be thought equivalent to a normalized shunt impedance defined as

$$\hat{z}_s = \frac{Z_s}{\eta \cos\theta_i} = -(1+\rho_L)/\rho_L, \text{ and} \tag{27}$$

$\hat{z}_s$ can be expressed in terms of the unknown currents $j_z(h)$ and $j_y(h)$ by multiplying (23) with $j_z(h)$, (24) with $j_y(h)$, integrating from -d/2 to d/2 and making use of (16) and (27) in a form as

$$\hat{z}_s = \hat{z}_{s1} + \hat{z}_{s2} \tag{28}$$

where

$$\hat{z}_{s1} = jk_o\cos\theta_i \int_{-d/2}^{d/2} j_z(h) \int_{-d/2}^{d/2} j_z(h') \sum_{m=1}^{\infty} q_m^{-1} \cos\frac{2\pi mh}{s} \cdot \cos\frac{2\pi mh'}{s}\, dhdh' / I_z^2 \tag{29}$$

$$\hat{z}_{s2} = \frac{sR_f}{\eta_o \cos\theta_i}\left[\int_{-d/2}^{d/2} j_z^2(h)\, dh + \int_{-d/2}^{d/2} j_y^2(h)\, dh\right]/I_z^2 \tag{30}$$

$$I_z = \int_{-d/2}^{d/2} j_z(h)\, dh \tag{31}$$

It is of interest to note that $\hat{z}_{s1}$ is in a form similar to the impedance obtained for the array of perfectly conducting stripes with the incident electric field polarized in the direction of the stripes [15]. For the case of interest $s << \lambda$ one may let,

$$q_m \approx \frac{2\pi m}{s}, \tag{32}$$

and with aid of the relation

$$\sum_{m=1}^{\infty} \frac{\cos(2\pi mh/s)\cos(2\pi mh'/s)}{m}$$

$$= -\tfrac{1}{4} \ln[4(\cos \frac{2\pi h}{s} - \cos \frac{2\pi h'}{s})^2] \tag{33}$$

$$\cos \frac{2\pi h}{s} = \cos^2(\frac{\pi d}{2s}) + \sin^2(\frac{\pi d}{2s}) \cos \theta \tag{34}$$

One may reduce $\hat{z}_{s1}$ to the form

$$\hat{z}_{s1} \approx \frac{jk_o s \cos \theta_i}{2\pi} \ln[\csc(\frac{\pi d}{2s})] \tag{35}$$

with the higher order terms neglected. We notice that $\hat{z}_{s1}$ depends only upon the configuration parameters as d/s and s/ λ and it is inductive in nature.

The effects of R_f, the resistance of the stripes contain in the partical impedance $\hat{z}_{s2}$. For convenience of further discussions we let

$$r_1 = \int_{-d/2}^{d/2} j_z^2 (h)\, dh / I_z^2 \tag{36}$$

$$r_2 = \int_{-d/2}^{d/2} j_y^2 (h)\, dh / \int_{-d/2}^{d/2} j_z^2 (h) dh \tag{37}$$

and $\hat{z}_{s2}$ becomes

$$\frac{sR_f}{\eta \cos \theta_i} r_1(1+r_2) \tag{38}$$

which depends on the current densities $j_z(h)$ and $j_y(h)$. A general approach of obtaining those current densities is pointed out in last section. For the case $s << \lambda$, the leading terms approximation in (25) and (26) result in sufficient accurate data for many applications. Upon substituting the first term of (25) into (36) we attain

$$r_1 = 1/d \tag{39}$$

With aid of Galerkin's method it can be shown that (see appendix)

$$r_2 = \frac{\sin^2 \theta_i \left[\sum_{m=1}^{\infty} \frac{\sin^2(m\pi d/s)}{m(m^2 - s^2/d^2)} \right]^2}{2\left[\frac{\pi^2}{2} \frac{R_f}{\eta} (\frac{d}{s})^2 - j \frac{\lambda}{s} \sum_{m=1}^{\infty} \frac{m \sin^2(m\pi d/s)}{(m^2 - s^2/d^2)^2} \right]^2} \tag{40}$$

Inserting (39) in (38) results in

$$z_{s2} = \frac{(s/d)R_f}{\eta \cos\theta_i}(1 + r_2) \qquad (41)$$

In Fig. 3 we plot $|r_2|$ versus d/s with R_f/η as the parameter and $s/\lambda = 0.2$. In Fig. 4 we plot $|r_2|$ versus d/s with s/λ as the parameter, $R/\eta = 1$, we consider the extreme one $\theta_i = \pi/2$. It is noted that $|r_2|$ is about two order of magnitudes smaller than unity when the stripe spacing is less than a quarter of a free space wave-length. Hence one may conclude that $\hat{z}_{s2}$ is basically resistive in nature and it is nearly independent of frequency. One may further observes that z_{s2} is inversely proportional to the cosine of incident angle. On the other hand z_{s1} is linearly proportional to s/λ and $\cos\theta_i$.

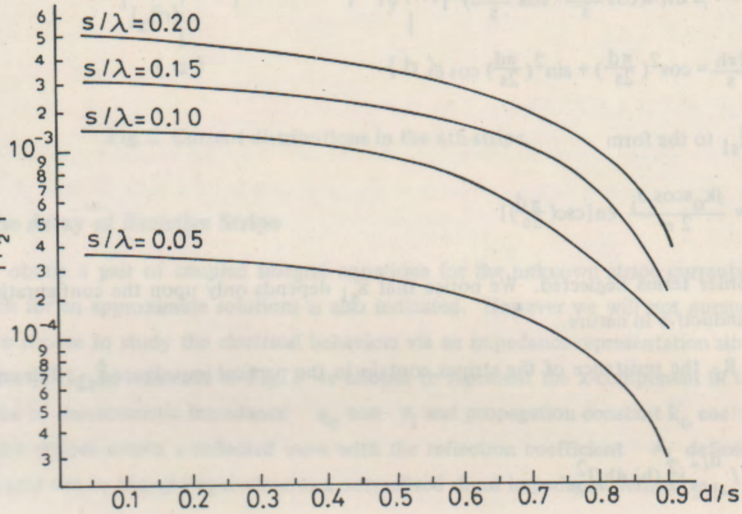

Fig. 3 $|r_2|$ versus d/s with s/λ as the parameter $R_f/\eta = 1.$, $\theta_i = \pi/2$.

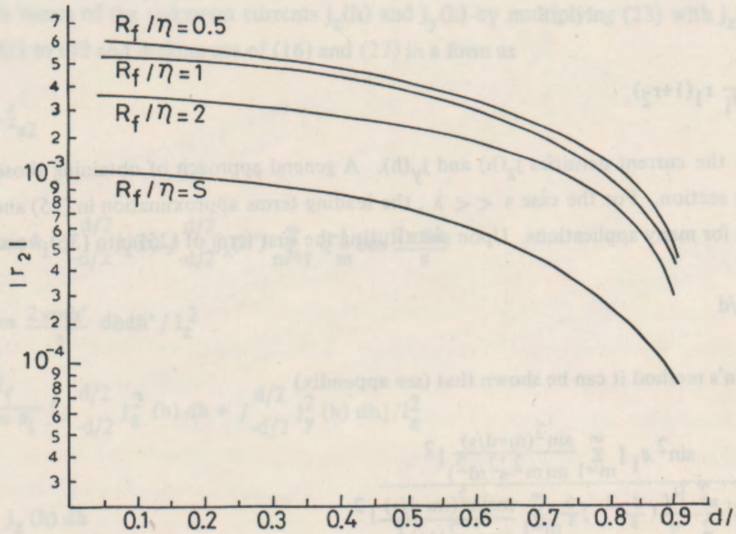

Fig. 4 $|r_2|$ versus d/s with R_f/η as the parameter, $s/\lambda = 0.2$, $\theta_i = \pi/2$.

V. Conclusions

In this paper we study the electrical behaviors of an array of resistive stripes illuminated obliquely by a plane wave having its magnetic field normal to the plane of incidence and perpendicular to the stripes. An impedance $\hat{z}_s$ consisting of two parts is obtained. One part z_{s1} is similar to the classical results and is inductive in nature, the other part z_{s2} is basically a resistive term, and is approximately linear function of the stripe resistance, the aspect ratio s/d. z_{s2} differs from z_{s1} in that the z_{s1} is nearly linear with s/λ and the z_{s2} is weekly dependent on s/λ, further z_{s1} is proportical to $\cos\theta_i$ while z_{s2} is inversely proportional to $\cos\theta_i$.

Appendix Derivation of (40)

In the text we define

$$r_2 = \int_{-d/2}^{d/2} j_y^2(h)\,dh \Big/ \int_{-d/2}^{d/2} j_z^2(h)\,dh \tag{A-1}$$

The solution forms of $j_z(h)$ and $j_y(h)$ are given by (25) and (26), respectively. Using the leading term approximation we have

$$r_2 = \tfrac{1}{2}\left(\frac{B_1}{A_o}\right)^2 \tag{A-2}$$

where A_o and B_1 are the coefficients of the first terms of (25) and (26), respectively. Multiplying (24) with $\sin\frac{2\pi h}{d}$ and inserting the leading terms of (25) and (26) in (24) yields

$$\frac{B_1}{A_o} = \frac{\sin\theta_i \sum\limits_{m=1}^{\infty} \dfrac{\sin^2(\frac{m\pi d}{s})}{(m(m^2-s^2/d^2)}}{\dfrac{\pi^2 R_f}{2\eta}(d/s)^2 - j\dfrac{\lambda}{s}\sum\limits_{m=1}^{\infty}\dfrac{m\sin^2(\frac{m\pi d}{s})}{(m^2-s^2/d^2)^2}} \tag{A-3}$$

Upon substituting (A-3) into (A-2) we obtain (40) of the text.

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