

含無法量得之狀態變數之最佳回饋控制系統某些結果之靈敏度分析
Sensitivity Analyses of Certain Results in the Optimal Control with
Incomplete State Feedback

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ABSTRACT — Sensitivity study is essential in the design of optimal control system with inaccessible state variables which must be estimated from noisy incomplete data. In this paper, the sensitivities of such systems given in the author's previous results and of the Kalman type optimal state feedback systems are investigated and compared.

1. Introduction

The optimal control law for a linear system with a quadratic performance index requires feedback of each and every state variable. Due to difficulty in practice of measuring every state variable, the practical usefulness of optimal control law is restricted. In these cases, either the method must be abandoned or a reasonable substitute for the state variables must be found.

Several recent papers have discussed the determination of the optimal control of the state system which cannot be measured completely [1-11]. The author [1] has presented two methods of obtaining optimal dynamic controllers when some of the state variables are not available for continuous measurements. Method A is based on the fact that the optimal control law can be differentiated a number of times and combined appropriately to obtain an equivalent control law requiring only those state variables which are measurable. Method B, on the other hand, is based on the idea that a model can be constructed to generate an approximation to the unknown state vector using the available outputs and the control inputs. One can then use a suitable transformation on the states of the model to obtain the optimal control law in terms of the output alone.

The main problem of optimal control with inaccessible state variables is that of estimating the state variables from noisy incom-

plete data. Therefore, a sensitivity analysis must be performed to account for the possible errors in the change of initial conditions and system parameters.

Recently, it has been demonstrated that for a single input linear time-invariant controllable n -th order system the sensitivity functions (or parameter influence coefficients) of the state with respect to any number of system parameters can be generated by an n -th order sensitivity model in addition to a system mode [12]. In this paper, we use the method of Kokotovic and Rutman [13,14], to generate the sensitivity functions for the optimal systems obtained in one of the author's previous publications. Only the second order systems are considered. The results are compared with that obtained for the Kalman type optimal state feedback systems by means of an example.

II. Sensitivity Analysis for Kalman Type Optimal State Feedback Systems

Consider the standard second-order system and quadratic performance index,

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -a_1 x_1 - a_2 x_2 + u \\ y &= x_1 \end{aligned} \quad (1)$$

and

$$J = \int_0^{\infty} \frac{1}{2} (x^T Q x + \rho u^2) dt$$

The optimal control is

$$u = -k_1 x_1 - k_2 x_2$$

By elimination, the system may now be reduced to:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -(a_1 + k_1) x_1 - (a_2 + k_2) x_2 \end{aligned} \quad (2)$$

The optimal system response may be found by solving Equation (2) and is given by

$$x_1^*(t) = e^{-\frac{B}{2}t} \left\{ (\cos rt + \frac{B}{2r} \sin rt) x_{10} + \frac{1}{r} \sin rt x_{20} \right\}$$

$$x_2^*(t) = e^{-\frac{B}{2}t} \left\{ -\frac{A}{r} \sin rt x_{10} + \left[\cos rt - \frac{B}{2r} \sin rt \right] x_{20} \right\} \quad (3)$$

where

$$A = a_1 + k_1$$

$$B = a_2 + k_2$$

and

$$r = \sqrt{A - \frac{B^2}{4}}$$

The performance index may be written in the form

$$J = \frac{1}{2} \int_0^{\infty} (\alpha_1 x_1^2 + \alpha_2 x_1 x_2 + \alpha_3 x_2^2) dt \quad (4)$$

where

$$\alpha_1 = Q_{11} + \rho k_1^2$$

$$\alpha_2 = 2Q_{12} + 2\rho k_1 k_2$$

$$\alpha_3 = Q_{22} + \rho k_2^2$$

Substituting Equation (3) into Equation (4) we obtain

$$J = \int_0^{\infty} \frac{1}{2} e^{-Bt} (\beta_1 \cos^2 rt + \beta_2 \sin^2 rt + \beta_3 \sin 2rt) dt$$

where

$$\beta_1 = \alpha_1 x_{10}^2 + \alpha_2 x_{10} x_{20} + \alpha_3 x_{20}^2$$

$$\beta_2 = \frac{1}{4r^2} [\alpha_1 (Bx_{10} + 2x_{20})^2 - \alpha_2 (2Ax_{10} + Bx_{20})(Bx_{10} + 2x_{20}) + \alpha_3 (2Ax_{10} + Bx_{20})^2]$$

$$\beta_3 = \frac{1}{2r} [\alpha_1 x_{10} (Bx_{10} + 2x_{20}) + \alpha_2 (x_{20}^2 - Ax_{10}^2) - \alpha_3 x_{20} (2Ax_{10} + Bx_{20})]$$

Setting $\theta = rt$ and $\lambda = \frac{B}{r}$, we deduce immediately that the performance is given by

$$\begin{aligned} J^* &= \frac{1}{2r} \int_0^{\infty} e^{-\lambda\theta} (\beta_1 \cos^2 \theta + \beta_2 \sin^2 \theta + \beta_3 \sin 2\theta) d\theta \\ &= \frac{1}{8\lambda B} \{ (B^2 + 2r^2) \beta_1 + 2r^2 \beta_2 + 2rB \beta_3 \} \end{aligned} \quad (5)$$

Substituting β_i into Equation (5), yields

$$J^* = \frac{1}{4AB} \{ [(A+B^2)\alpha_1 - AB\alpha_2 + A^2\alpha_3]x_{10}^2 + 2B\alpha_1x_{10}x_{20} + (\alpha_1 + A\alpha_3)x_{20}^2 \} \quad (6)$$

From the mathematics view-point, $J^* = J(a_1, a_2, x_{10}, x_{20})$, the change of performance index, ΔJ , can be written as

$$\Delta J = \frac{\partial J^*}{\partial a_1} \Delta a_1 + \frac{\partial J^*}{\partial a_2} \Delta a_2 + \frac{\partial J^*}{\partial x_{10}} \Delta x_{10} + \frac{\partial J^*}{\partial x_{20}} \Delta x_{20} \quad (7)$$

Then

$$\begin{aligned} \frac{\Delta J}{J^*} &= \frac{2\Delta x_{10}}{x_{10}} \{ [(A+B^2)\alpha_1 - AB\alpha_2 + A^2\alpha_3]x_{10}^2 + B\alpha_1x_{10}x_{20} \} / \Delta_1 \\ &+ \frac{2\Delta x_{20}}{x_{20}} \{ B\alpha_1x_{10}x_{20} + (\alpha_1 + A\alpha_3)x_{20}^2 \} / \Delta_1 - \frac{\Delta a_1}{A} \{ (B^2\alpha_1 - A^2\alpha_3)x_{10}^2 \\ &+ 2B\alpha_1x_{10}x_{20} + \alpha_1x_{20}^2 \} / \Delta_1 - \frac{\Delta a_2}{B} \{ [(A-B^2)\alpha_1 + A^2\alpha_3]x_{10}^2 \\ &+ (\alpha_1 + A\alpha_3)x_{20}^2 \} / \Delta_1 \end{aligned} \quad (8)$$

where

$$\begin{aligned} \Delta_1 &= [(A+B^2)\alpha_1 - AB\alpha_2 + A^2\alpha_3]x_{10}^2 + B\alpha_1x_{10}x_{20} \\ &+ (\alpha_1 + A\alpha_3)x_{20}^2 \end{aligned}$$

For the sake of simplicity, consider the second-order system which has been considered by Liou et al. [1]:

$$\frac{d^2c}{dt^2} + 2\xi\frac{dc}{dt} + c = u \quad (9)$$

$$y = c$$

and the performance index

$$J(u) = \frac{1}{2} \int_0^{\infty} [c^2(t) + \rho u^2(t)] dt$$

with a given initial conditions $c(0) = c_0$, $\frac{dc(0)}{dt} = 0$.

Then

$$\frac{\Delta J}{J^*} = 2 \frac{\Delta c_o}{c_o} - \frac{2}{B} \cdot \frac{(A-B^2)\alpha_1 + A^2\alpha_3}{(A+B^2)\alpha_1 - AB\alpha_2 + A^2\alpha_3} \Delta \xi \quad (10)$$

where

$$\begin{aligned} A &= 1+k_1 \\ B &= 2\xi+k_2 \\ \alpha_1 &= 1+\rho k_1^2 \\ \alpha_2 &= 2\rho k_1 k_2 \\ \alpha_3 &= \rho k_2^2 \end{aligned}$$

III. Sensitivity Analysis for Liou's Method A Type Optimal Output Feedback Systems

Consider the same system as in Equation (9). The dynamic control law obtained by Liou's Method A [1] is given by

$$\dot{u} + L u = M \dot{y} + N y \quad (11)$$

where

$$\begin{aligned} L &= k_2 + 2\xi \\ M &= -k_1 \\ N &= k_2 - 2\xi k_1 \end{aligned}$$

Take Laplace transform, Equations (9) and (11) yield

$$(s^2 + 2\xi s + 1) c(s) = u(s) + (s + 2\xi) c_o \quad (12)$$

$$\begin{aligned} (s+L) u(s) &= (Ms+N) c(s) + u_o - M c_o \\ &= (Ms+N) c(s) - (M+k_1) c_o \end{aligned}$$

$$(s+L) u(s) = (Ms+N) c(s) \quad (13)$$

Combination of Equations (12) and (13) yields

$$(s^2 + 2\xi s + 1) (s+L) c(s) = (Ms+N) c(s) + (s+2\xi) (s+L) c_o$$

$$c(s) = \frac{[s^2 + (2\xi + L)s + 2\xi L] c_0}{s^3 + (L + 2\xi)s^2 + (1 + 2\xi L - M)s + (L - N)}$$

$$u(s) = \frac{[Ms^2 + (2\xi M + N)s + 2\xi N] c_0}{s^3 + (L + 2\xi)s^2 + (1 + 2\xi L - M)s + (L - N)}$$

Since

$$\int_0^{\infty} f^2(t) dt = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} F(s)F(-s) ds$$

When $F(s)$ is rational, the value of this complex integral has been tabulated in terms of the polynomial coefficients by Newton et al. [15]

Then

$$\begin{aligned} J^* &= \frac{1}{2} \int_0^{\infty} [c^2(t) + \rho u^2(t)] dt \\ &= \frac{c_0^2}{2\phi_1} \{ (1 + \rho M^2) d_0 d_1 + [(2\xi - L)^2 + \rho(2\xi M - N)^2] d_0 \\ &\quad + 4\xi^2 d_2 (L^2 + \rho N^2) \} \end{aligned}$$

where

$$d_2 = L + 2\xi, \quad d_1 = 1 + 2\xi L - M, \quad d_0 = L - N$$

and

$$\phi_1 = 2d_0 (d_1 d_2 - d_0)$$

Now consider

$$J^* = J^*(c_0, \xi)$$

$$\Delta J = \frac{\partial J^*}{\partial c_0} \Delta c_0 + \frac{\partial J^*}{\partial \xi} \Delta \xi$$

Then

$$\frac{\Delta J}{J^*} = 2 \frac{\Delta c_0}{c_0} +$$

$$\frac{2L(1 + \rho M^2) d_0 + [8\xi(1 + \rho M^2) - 4(L + \rho MN)] d_0 + 8\xi(L^2 + \rho N^2)(\xi + d_2)}{(1 + \rho M^2) d_0 d_1 + [4\xi^2(1 + \rho M^2) - 4\xi(L + \rho MN) + (L^2 + \rho N^2)] d_0 + 4\xi^2(L^2 + \rho N^2) d_2}$$

$$-\frac{\phi_1'}{\phi_1} \Delta \xi$$

where $\phi_1' = 4d_0 (d_1 + Ld_2)$

IV. Sensitivity Analysis for Liou's Method B Type Optimal Output Feedback Systems

Consider the same system as in Equation (9). The dynamic control law obtained by Liou's Method B [1] is given by

$$z + \bar{M}z = \bar{N}y$$

where

$$z = u + Ty$$

$$\bar{M} = 1 + k_2$$

$$\bar{N} = \bar{M}T - k_1 - k_2$$

$$T = k_1 + (1 - 2\xi)k_2$$

and

$$z(0) = (T - k_1)c_0$$

Define

$$\bar{z} = [c, c, z]^T$$

Then

$$\bar{z}(s) = \frac{1}{\Delta_3} \begin{bmatrix} s^2 + (2\xi + \bar{M})s + 2\xi\bar{M} & | & s + \bar{M} & | & 1 \\ -(1+T)s - \bar{M}(1+T) + \bar{N} & | & s^2 + \bar{M}s & | & s \\ \bar{N}(s + 2\xi) & | & \bar{N} & | & s^2 + 2\xi s + (1+T) \end{bmatrix}$$

where

$$\Delta_3 = s^3 + (2\xi + \bar{M})s^2 + (1 + T + 2\xi\bar{M})s + (\bar{M} + \bar{M}T - \bar{N})$$

or

$$c(s) = \frac{c_0}{\Delta_3} \{s^2 + (2\xi + \bar{M})s + (2\xi\bar{M} + T - k_1)\}$$

$$u(s) = z(s) - Tc(s)$$

$$= \frac{c_0}{\Delta_3} \{-k_1 s^2 + (k_2 - k_1 - 2\xi k_1)s + [T - k_1 - 2\xi(k_1 - k_2)]\}$$

Then, $J^* = \frac{c_0}{2\phi_2} \{\alpha_1 d_0 d_1 + (4\alpha_1 \xi^2 + \delta_1) d_0 + (4\xi^2 \delta_2 + 4\xi \delta_3 + \delta_4) d_2\}$

where

$$\alpha_1 = 1 + \rho k_1^2$$

$$\delta_1 = \delta_2 - 2(T - k_1)(1 - \rho k_1)$$

$$\delta_2 = (1+k_2)^2 + \rho(k_1-k_2)^2$$

$$\delta_3 = (T-k_1)[\bar{M} - \rho(k_1-k_2)] = (T-k_1)[(1-\rho k_1) + k_2(1+\rho)]$$

$$\delta_4 = (T-k_1)^2(1+\rho)$$

$$d_2 = \bar{M} + 2\xi$$

$$d_1 = 2\xi\bar{M} + 1 + T$$

$$d_0 = \bar{M} + \bar{M}T - \bar{N} = 1 + k_1$$

$$\phi_2 = 2d_0 d_1 d_2$$

since

$$\Delta J = \frac{\partial J^*}{\partial c_0} \Delta c_0 + \frac{\partial J^*}{\partial \xi} \Delta \xi$$

Then

$$\frac{\Delta J}{J^*} = 2 \frac{\Delta c_0}{c_0} + \left\{ \frac{2\bar{M}\alpha_1 d_0 + 8\alpha_1 \xi d_0 + (8\xi\delta_2 + 4\delta_3)d_2 + 2(4\xi^2\delta_2 + 4\xi\delta_3 + \delta_4)}{\alpha_1 d_0 d_1 + (4\alpha_1 \xi^2 + \delta_1)d_0 + (4\xi^2\delta_2 + 4\xi\delta_3 + \delta_4)d_2} - \frac{\phi_2'}{\phi_2} \right\} \Delta \xi$$

where $\phi_2' = 4d_0(d_1 + \bar{M}d_2)$

From above analysis, we get $\frac{\Delta J}{J^*} = 2 \frac{\Delta c_0}{c_0}$ for all $\Delta \xi = 0$. Now we take a simple example to investigate the effect of $\frac{\Delta J}{J^*}$ by small change of $\Delta \xi$.

Example

Consider the control system which has been considered by Liou et al. [1].

$$\frac{d^2 c}{dt^2} + \frac{3}{2} \frac{dc}{dt} + c = u$$

and the performance index

$$J(u) = \frac{1}{2} \int_0^\infty [c^2(t) + \frac{1}{8} u^2(t)] dt$$

Then

$$\rho = \frac{1}{8}, \quad \xi = \frac{3}{4}$$

and $k_1 = 2, \quad k_2 = 1$

(1) For Kalman type system:

$$A=1+k_1=3$$

$$B=2\xi+k_2=\frac{5}{2}$$

$$\alpha_1=1+\rho k_1^2=\frac{3}{2}$$

$$\alpha_2=2\rho k_1 k_2=\frac{1}{2}$$

$$\alpha_3=\rho k_2^2=\frac{1}{8}$$

$$\text{Then } \frac{\Delta J}{J^*} = 2 \frac{\Delta c_0}{c_0} + 0.2 \frac{\Delta \xi}{\xi} .$$

(2) For Method-A system:

$$L=\frac{5}{2}, \quad M=-2, \quad N=-2$$

$$d_2=4, \quad d_1=\frac{27}{4}, \quad d_0=\frac{9}{2}$$

$$\phi_1=\frac{405}{2}, \quad \phi_1'=-\frac{603}{2}$$

$$\text{Then } \frac{\Delta J}{J^*} = 2 \frac{\Delta c_0}{c_0} + 2.15 \frac{\Delta \xi}{\xi} .$$

(3) For Method-B system:

$$\bar{M} = 2, \quad \bar{N} = 2, \quad T = \frac{3}{2}$$

$$d_2 = \frac{7}{2}, \quad d_1 = \frac{11}{2}, \quad d_0 = 3$$

$$\alpha_1 = \frac{3}{2}, \quad \phi_2 = \frac{231}{2}, \quad \phi_2' = 150$$

$$\delta_1 = \frac{39}{8}, \quad \delta_2 = \frac{33}{8}, \quad \delta_3 = -\frac{15}{16}, \quad \delta_4 = \frac{9}{32}$$

$$\text{Then } \frac{\Delta J}{J^*} = 2 \frac{\Delta c_0}{c_0} + 0.38 \frac{\Delta \xi}{\xi} .$$

V. Conclusion

The sensitivities of the Kalman type optimal state feedback control systems and the optimal control systems with inaccessible state feedback designed via the author's previously published methods are investigated and compared. For the particular example considered, the Kalman type system is the least sensitive and the Liou's method-B type system is less sensitive than the Liou's method-A type system, to the change of system parameters. However, all three types of systems have exactly the same sensitivity with respect to the change in the initial output. The incremental sensitivity with respect to the initial output, $\frac{\Delta J}{J^*} \frac{\Delta c_0}{c_0}$, is independent of the initial output and

the system parameters, and is equal to 2 when the initial rate of output change is zero. That is

$$\frac{\Delta J}{J^*} = 2 \frac{\Delta C_0}{C_0} \quad \text{for} \quad \Delta \xi = 0 \quad \text{and} \quad \dot{C}(0) = 0$$

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ABSTRACT — The I-V characteristics of over 100 bulk InSb single crystals at 70°K have been measured. These samples, from a few to about 1 cm in length and 0.4 to 1.0 cm in diameter, were of either the p- or n-type with carrier concentrations ranging from 1.5×10^{13} to 1.5×10^{16} /cm³. These multi-valued I-V characteristics were found to be highly non-linear, with as much as four distinct regions each of which can be attributed to a different transport mechanism. Furthermore, these characteristics were found to be dependent not only on the carrier type and doping density of the sample but also on the contact potential barrier height. A theoretical explanation of the observed behavior will be presented.

1. Introduction

Although high frequency generation in the range from a few MHz [1-3] to some 100 GHz [3-4] in InSb has been reported by a number of investigators [5-6] a totally satisfactory explanation of the associated phenomena has yet to be formulated [7]. As to the investigation of the static properties of InSb in the absence of a magnetic field, very little has been reported; Bartram [8] has performed some static experiments with a sample 3 cm in length. In order to better understand the transport mechanisms in InSb, an extended and extensive investigation of the quasi-static properties. Brief reports of our investigation have been presented previously [9-10].

The I-V characteristics of 104 bulk InSb single crystals, from a few to about 1 cm in length and 0.4 to 1.0 cm in diameter, with a or p-type carrier densities ranging from 1.5×10^{13} to 1.5×10^{16} /cm³, have been measured at 70°K. For each combination of doping, carrier type,

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