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—— Short Notes

A = 90 原子核之殼層模型計算

Shell-Model Calculation for A=90 Nuclei

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Abstract —— Using a two-range central-plus-tensor potential for the effective interaction, a shell model calculation is performed for the A = 90 nuclei. The energy spectra of ^{90}Zr, ^{90}Y and ^{90}Sr can be well reproduced.

In the shell model calculation, an inert core is assumed and nucleons outside the core are treated as active nucleons in order to describe the low-lying levels of the nucleus. Usually a closed-shell nucleus can be adopted as such an inert core. There have been a number of shell model investigations for the nuclei in the vicinity of A = 90, in which ^{88}Sr is assumed as an inert core [1-4]. However, the above mentioned works all treated the two-body matrix elements of the residual interaction as free parameters which are varied until a best fit to the spectra is obtained. In this note, we consider a specific potential form for the effective interaction and treat the interaction strengths as free parameters. We will describe three A = 90 nuclei, i.e. ^{90}Zr, ^{90}Y and ^{90}Sr, in which two protons, two neutrons and one proton-one neutron are outside the closed core. The active protons are distributed in the $(p_{1/2}, g_{9/2})$ orbits and the active neutrons are distributed in the $(d_{5/2}, s_{1/2})$ orbits.

The wave function for each state in the above assumed configurations can be obtained by the diagonalization of the Hamiltonian H

$$H = H_o + V_{pp} + V_{nn} + V_{np}.$$

where H_o is the Hamiltonian of the single-particle, V_{pp}, V_{nn} and V_{np} represent the proton-proton, neutron-neutron and proton-neutron interactions respectively. The effective interaction is assumed to be a two-range central-plus-tensor potential which is proposed by Schiffer and True [5]. In a rather extensive analysis of the data over the entire nucleus chart, Schiffer and True have found an interesting similarity exists, in different multiplety between the angular distributions and the multipole coefficients of the effective two-body interactions. This remarkable discovery suggests the universality of the residual interaction. In the present calculation, the radial dependence of the potential is taken to be Yukawa type while the interaction ranges r_o = 1.415fm and r_1 = 2.0 fm similar to that used by Schiffer and True. The harmonic oscillator wave functions are employed with the oscillator constant, $\nu = 0.96\ A^{-1/3}$ fm^{-2}, where A = 90. With the above prescription, the interaction strengths consist of the T = 0, singlet-odd (CSO), triplet-even (CTE), and T = 1, singlet-even (CSE), triplet-odd (CTO) components of both ranges for central force, and the triplet-even (TTE), triplet-odd (TTO) components of short range for the tensor force. Recently, Chiang et al. [6] have investigated N = 28, 50 and 82 isotones using the same effection interaction as in this note and obtained a very good results. Therefore, we adopt their T = 1 interaction strengths and only consider the five T = 0 components as free parameters.

The binding energies of the ground states are not included in the χ^2 fit; therefore, the single particle energies of

$\pi\ p_{1/2}$ and $\nu\ d_{5/2}$ are fixed to be zero. The two single particle energies $\epsilon\ (\pi\ g_{9/2})$ and $\epsilon\ (\nu\ s_{1/2})$ are also trected as free parameters in the least-squares fit.

The five interaction strengths and the two single particle energies are varied in the least-squares search that includes 21 energy levels in the A = 90 nuclei. The results are shown in Fig. 1 (a), (b), (c). The general agreement is quite good except for very few levels. The root-mean-square deviation between the calculated and the observed energies is 0.251 Mev. The single-particle energies we obtained are $\epsilon\ (\pi g_{9/2}) = 0.952$ Mev and $\epsilon\ (\nu\ s_{1/2}) = 1.051$ Mev. These values are slightly larger than the corresponding experimental values 0.906 and 1.031 Mev deduced from the first excited $9/2^+$ and $1/2^+$ levels of ^{89}Y and ^{89}Sr.

The interaction strengths we obtained are shown in Table 1, in which the results of Schiffer and True are also displayed for comparison. Generally speaking the magnitudes and signs of the strengths of these two works are similar to each other except for TTO of short range, which has positive sign in our case. This may be related to the fact that the TTE and CSE of short range are more attractive than that of Schiffer and True.

In conclusion, the experimental spectra of the low-lying states of ^{90}Zr, ^{90}Y and ^{90}Sr are well reproduced by the truncated shell-model calculation by using a two-range central-plus-tensor potential for the effective interaciton.

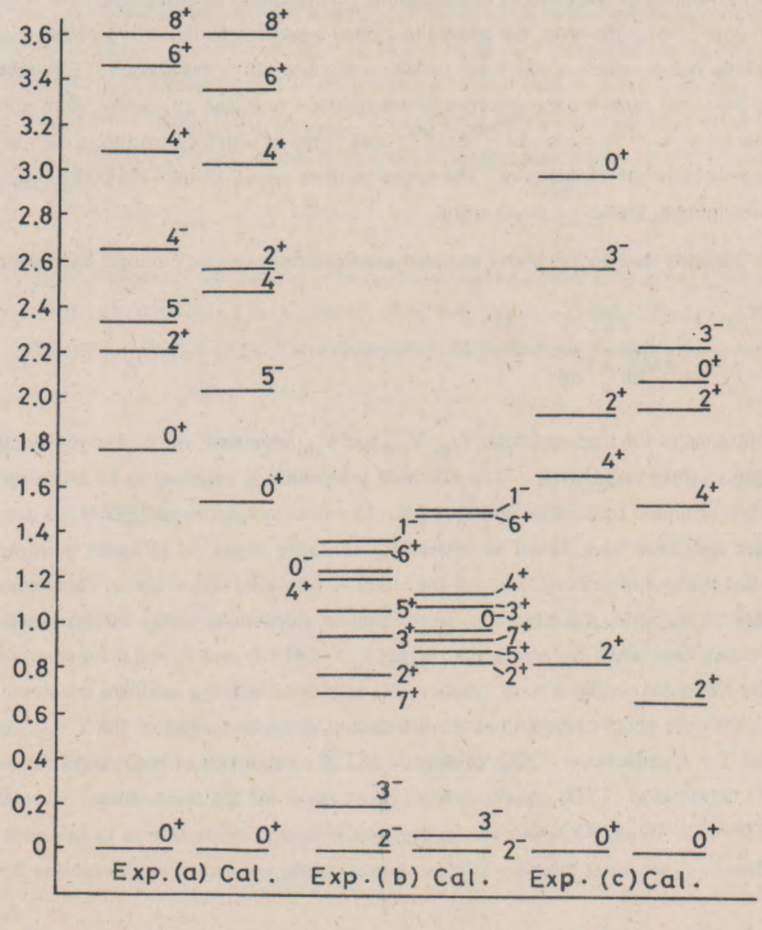

Fig. 1. calculated and experimented energy spectra for (a) $^{90}_{40}Zr_{50}$ (b) $^{90}_{39}Y_{51}$ (c) $^{90}_{38}Sr_{52}$.

Table 1.

Interaction strengths (in Mev) of the best fit potential parameters compared with those of Schiffer and True (ST).

	Range	Ours	ST
CSE	short	-100.33	- 49.32
	long	28.75	15.47
	long/short	- 0.29	- 0.31
CTO	short	-216.23	-155.82
	long	90.68	62.06
	long/short	- 0.41	- 0.40
CSO	short	132.54	125.53
	long	- 50.51	- 44.37
	long/short	- 0.38	- 0.35
CTE	short	-121.62	-118.09
	long	44.23	27.27
	long/short	- 0.36	- 0.23
TTO	short	19.00	- 6.10
TTE	short	- 60.12	- 42.52

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