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# 相關不良產品檢驗對生產管制影響的推廣

# The Generalization of the Effect of Correlated Defective Inspection Scheme for Process Control

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**Abstract** -- A production process with two quality levels and four states is presented. A sequence (k) sampling plan in which samples of k (k > 0) consecutively produced items are taken periodically, and a simple decision rule based on the result of inspecting each sample are applied to control the process. It is assumed taht each item has two correlated attributes X and Y. In practice, the cost of inspection of X is higher than the cost of inspection of Y and the result of inspecting X determines the item is good (acceptable) or bad (unacceptable). The problem of whether to use the inspection of X or Y is studied and some approximation results are obtained to compare the two inspection costs corresponding to X and Y.

## I. Introduction

Although X-chart is still used widely in industries, because of its simplicity and ease for operation, recently the theory of control charts has been developed to use Cumulative Sum Charts (CUSUM charts) and Moving Average Charts. Many important results have been obtained by Page [4], Chiu [1], Lai [3], Reynolds [5] etc. However, from the economic point of view, the consideration of the inspection cost on the design of control charts could improve the efficiency of the control chart. A particular case which is similar to the ratio estimation in Sampling Theoy has been studied by Huang and Yang [2]. In this paper, we shall generalize the results of Huang and Yang [2] to samples of size k, k > 1.

## II. The production Process and the Sampling Plan

Let the production process produces batches of k items, k > 1, continuously. Let us sample a batch and, after skipping a certain fixed number of batches, we sample another batch again. We repeat this sampling procedure when the production process continues to run. All k items in each sampled batch are inspected. The number of defectives in each sample is plotted on the control chart and the manager will make a decision, based on this information, to take corrective action. As in [2], we assume the process has two quality levels. Define $\lambda_1$ and $\lambda_2$ to be the fractions of defectives produced at level 1 and 2 respectively. If the number of defectives in a sample is found to be greater than a properly assigned integer, say J, the corrective action will be taken and the production process is stopped obviously, the probability of taking the corrective action depends on $\lambda_1$, $\lambda_2$, and J.

Define a unit time as the time interval in which the production process produces k items. For computational simplicity, it is assumed that the process shift (see [1]) will happen only at the end of each unit time interval. In other words, the process shift is not likely to happen within each unit time interval. Let $\alpha$ be the probability of the process shifting from quality level 1 at time m to quality level 2 at time m + 1, where m is a positive integer. The value $\alpha$ is independent of time m. Suppose the attribute X of each sampled item is inspected and it can determine whether each sampled item is defective or not. If the item is defective, a cost C will be counted. Let $n_x$ be the previous J in this case. Then the probability of having the number of defectives greater than or equal to $n_x$ in an inspected sample can be shown to be:

$$q_1 = \sum_{i=n_x}^{k} \binom{k}{i} \lambda_1^i (1-\lambda_1)^{k-i} \quad \text{for quality level 1,}$$

and

$$q_2 = \sum_{i=n_x}^{k} \binom{k}{i} \lambda_2^i (1-\lambda_2)^{k-i} \quad \text{for quality level 2.}$$

## III. Expected Cost Per Unit Time By Inspecting Attribute X

Let $\beta = 1-(1-\alpha)^m$, when m is a positive integer, and let N be the stopping random variable corresponding to the time of taking corrective action. From the formulas given in [2], it is found that the expected stopping time E(N) is

$$E(N) = \frac{m(q_2 + \beta - q_2\beta)}{q_2(q_1 + \beta - q_1\beta)} \tag{1}$$

Let $C_I$ be the cost of inspecting the attribute X of one item and $C_s$ be the cost of taking the corrective action. Then it is not hard to see that the expected cost per unit time $R_1$ in running the process is

$$R_1 = \lambda_2 kC + \frac{C_s}{E(N)} + \frac{kC_I}{m} + \frac{q_2 L_b \ \beta(\alpha-1)}{m\alpha(q_2 + \beta - q_2\beta)} \tag{2}$$

where $L_b = (\lambda_2 - \lambda_1)kC$. Using the method given in [2], we conclude that the approximation $R_a$ to the optimal cost per unit time $R_o$, is given by:

$$R_a = \lambda_2 kC + L_b \left[ \frac{\sqrt{\alpha g}\, q_2 L_b + 2\sqrt{e\left(1-\frac{q_2}{2}\right)\left(L_b(3\alpha-1)+3\alpha C_s\right)}}{\sqrt{\alpha g}\,(4-3q_2)L_b + 2\sqrt{e\left(1-\frac{q_2}{2}\right)\left(L_b(1-2\alpha) - 2\alpha C_s \frac{q_2-q_1}{q_2}\right)}} \right] \tag{3}$$

where $g = kC_I + q_1 C_s$ and $e = q_2 L_b(1-\alpha) - \alpha C_s(q_2 - q_1)$. The approximation error is $R_a - R_o = O(\alpha)$.

## IV. The Correlated Attribute Inspection

Suppose we have another attribute Y and we inspect Y instead of X. Let us assume that there are some probabilities of misclassifying bad items (defectives) and good items, and

$$P(Y = \text{good} \mid X = \text{good}) = p_1,$$

$$P(Y = \text{bad} \mid X = \text{bad}) = p_2.$$

Now we sample k items (a batch) each time as before but we inspect Y instead of X. The corrective action is taken if the number of bad items (based on Y) is greater than or equal to a posoitive integer $n_y$, where $n_y \leq k$ and $n_y$ may be greater than $n_x$ subject to practical considerations. Let $\lambda_1^*$ be the probability of an inspected item being bad according to Y when the process is in level 1. Then $\lambda_1^* = \lambda_1 p_2 + (1-\lambda_1)(1-p_1)$. Similarly $\lambda_2^* = \lambda_2 p_2 + (1-\lambda_2)(1-p_2)$. Then the probability of having the number of bad items, based on Y, greater than or equal to $n_y$

in an inspected sample is

$$q_1^* = \sum_{i=n_y}^{k} \binom{k}{i} \lambda_1^{*i} (1-\lambda_1^*)^{k-i} \qquad \text{for level 1,}$$

and

$$q_2^* = \sum_{i=n_y}^{k} \binom{k}{i} \lambda_2^{*i} (1-\lambda_2^*)^{k-i} \qquad \text{for level 2.}$$

As before, it is found that the expected stopping time $E(N^*)$ for this new procedure is

$$E(N^*) = \frac{m(q_2^* + \beta - q_2^* \beta)}{q_2^* (q_1^* + \beta - q_1^* \beta)}$$

The expected cost per unit time $R_1^*$ in running the process is given by

$$R_1^* = \lambda_2 kC + \frac{kC_I^*}{m} + \frac{C_s}{E(N^*)} + \frac{q_2^* L_b \beta(\alpha-1)}{m\alpha(q_2^* + \beta - q_2^* \beta)} .$$

where $C_I^*$ is the new inspection cost of inspecting attribute Y of an item. As before, we found that the approximation $R_a^*$ to the optimal cost per unit time $R_o^*$ is given by:

$$R_a^* = \lambda_2 kC + L_b \left[ \frac{\sqrt{\alpha g^*}\, q_2^* L_b + 2\sqrt{e^*(1-q_2^*/2)}\,(L_b(3\alpha-1)+3\alpha C_s)}{\sqrt{\alpha g^*}\,(4-3q_2^*) L_b + 2\sqrt{e^*(1-q_2^*/2)}\,(L_b(1-2\alpha) - 2\alpha C_s \frac{q_2^* - q_1^*}{q_2^*})} \right] \qquad (4)$$

where $g^* = kC_I^* + q_1^* C_s$ and $e^* = q_2^* L_b(1-\alpha) - \alpha C_s (q_2^* - q_1^*)$. The approximation error is $R_a^* - R_o^* \sim O(\alpha)$.

## V. The Relationship Between Two Inspection Costs

Let $C_I$ be fixed and let $\xi$ be the exact solution of $C_I^*$ in the equation $R_o - R_o^* = 0$. Let $\xi_a$ be the solution of $C_I^*$ in the equation $R_a - R_a^* = 0$ (i.e. $\xi_a$ is an approximation to $\xi$ ). Then from (3) and (4), it is found that

$$\xi_a = \frac{e^*}{k}\left(1 - \frac{q_2^*}{2}\right) H^2 + \frac{1}{k} q_1^* C_s \qquad (5)$$

and

$$\xi - \xi_a = O(\alpha)$$

where

$$H = \frac{\sqrt{g}\,[\alpha C_s(3 - \frac{q_2}{4}) - L_b(1 - (3 - \frac{q_2}{4})\alpha)] + \sqrt{\alpha e(1 - \frac{q_2}{2})}\,(q_1 q_2^* - q_1^* q_2)\, C_s / q_2 q_2^*}{\sqrt{\alpha g}\,(\frac{q_2 - q_2^*}{2}) L_b + [\alpha C_s(3 - \frac{q_2^*}{4}) - L_b(1 - (3 - \frac{q_2^*}{4})\alpha)]\sqrt{e(1 - \frac{q_2}{2})}} .$$

Equation (5) is a good approximation formula for $\xi$ when $\lambda_1$ and $\alpha$ are small. For the fast production process, $\alpha$ is small and equation (5) is valid for cost consideration. Thus the comparison between the two inspection costs is given by (5). For example, if we have the correlated inspection cost $C_I^*$ less than $\xi_a$ calculated by (5) for a given $C_I$, then we should use the correlated inspection scheme (i.e. inspecting Y) since it will have less optimal expected running cost per unit time.

## References


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